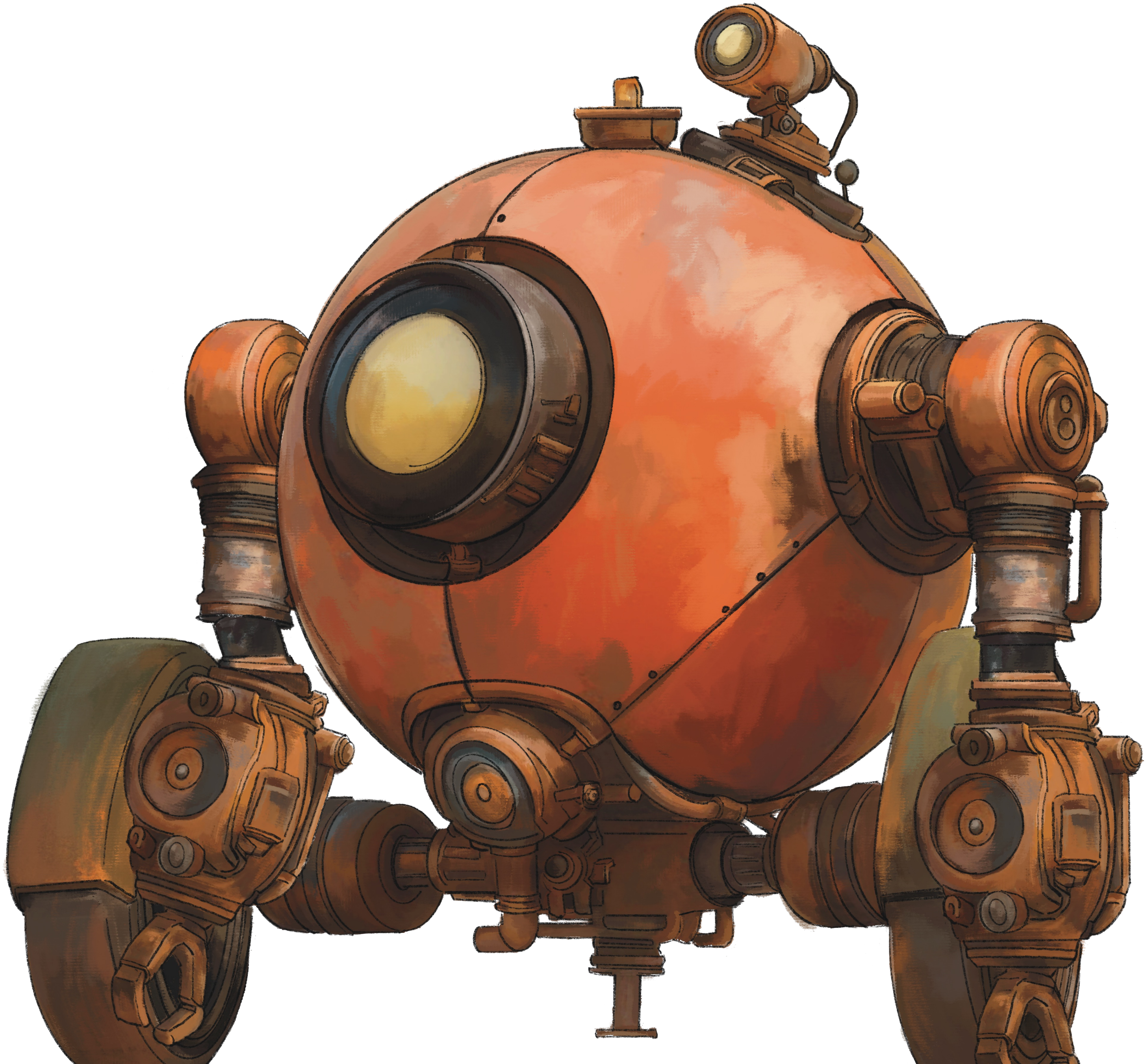
CS 3630, Fall 2025

Lecture 13:

A Logistics Robot:

Sensing



Logistics Robots

Sensing

So far, we’ve seen simple sensor models:

- Discrete measurements (conductivity, light)
- Univariate Gaussians (weight/scale)

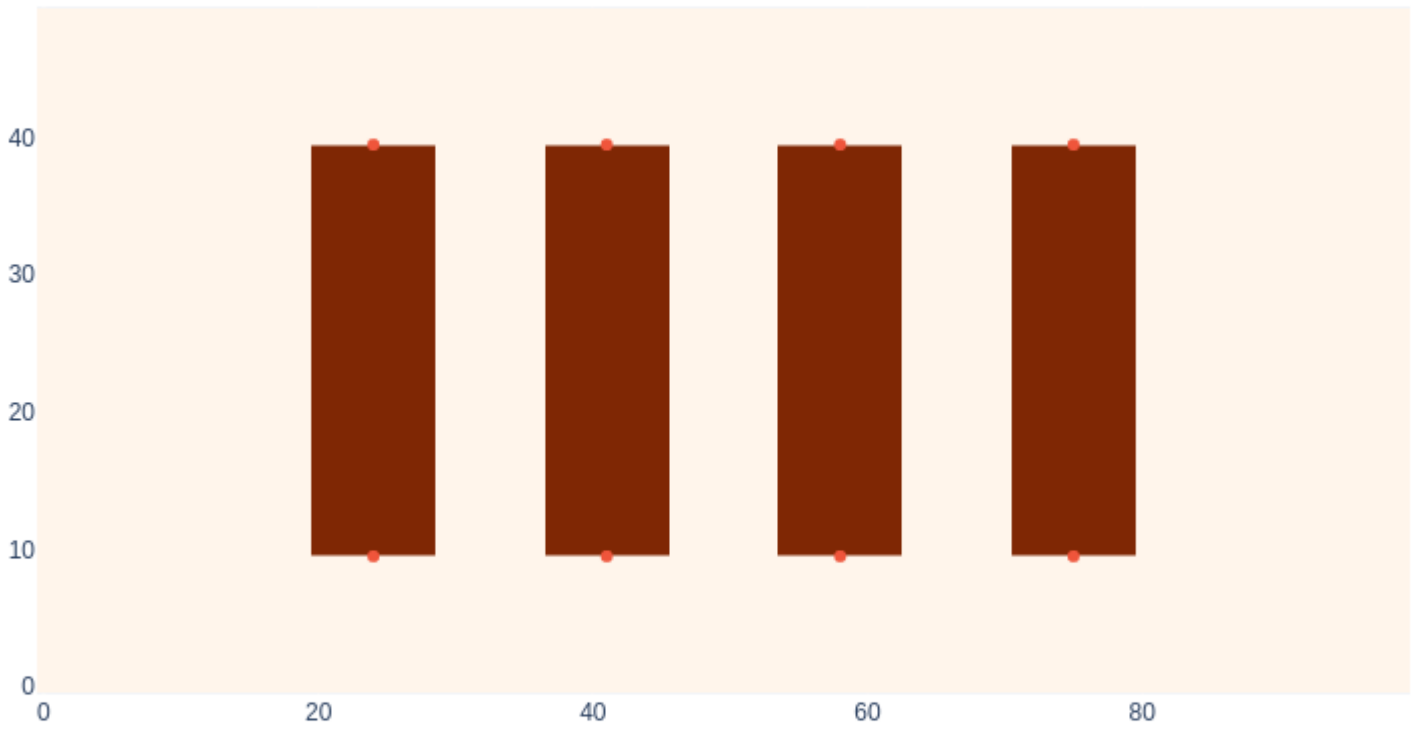
In this chapter, we’ll see more realistic sensor models:

- Proximity (object detection, binary)
- Range (distance to a beacon, Gaussian)
- Pseudo-GPS (2D coordinates, bi-variate Gaussian)

For Perception, we’ll require more sophisticated computational tools that exploit efficient and effective approximation schemes.

Warehouse Environment

- Sensors measure various features of the environment.
    - Geometric aspects of the environment (e.g., location of obstacles)
    - Artifacts placed in the environment (e.g., QR Code, RFID transmitters, GPS)
    - Visual features in the environment.



Environment:

- Warehouse is an enclosed 100x50m space
- Four shelving units
- Eight beacons (for range sensor)

Sensors:

- Proximity sensor detects walls and shelves
- Range sensor measures distance to the nearest beacon
- Pseudo-GPS sensor gives 2D coordinates of the robot in the warehouse.

An Ideal Proximity Sensor

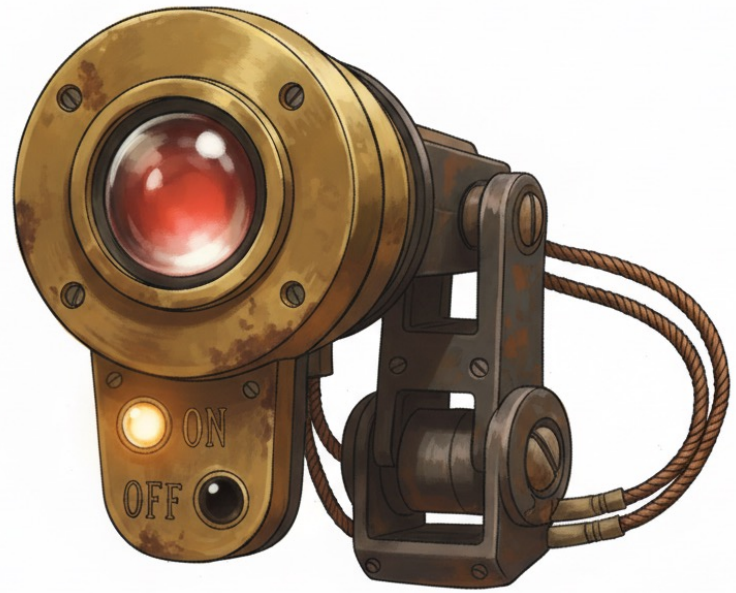
- Binary sensor that detects obstacles.
- Sensor returns measurement $z_k \in \{ON, OFF\}$
- Denote by X_{obs} the obstacle region (includes shelves and walls)
- Distance to nearest obstacle is defined by

$$d(x) = \min_{x' \in X_{obs}} \|x - x'\|^{\frac{1}{2}}$$

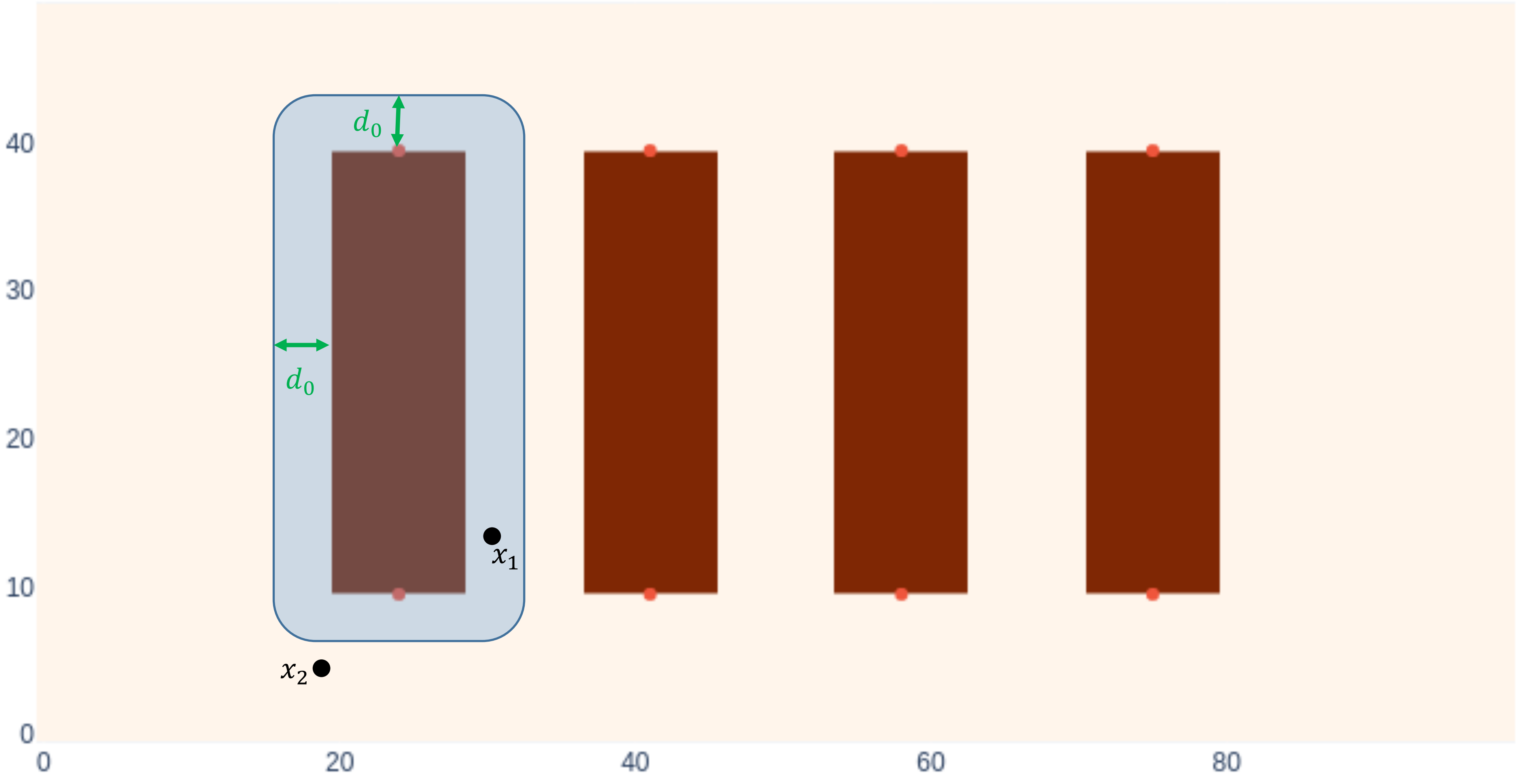
- If $d(x_k) \leq d_0$ (for some predetermined distance d_0), the sensor triggers:

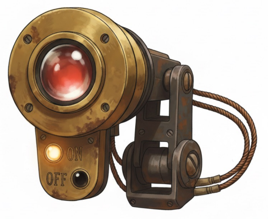
$$z_k = ON$$

- If $d(x_k) > d_0$, $z_k = OFF$.



Ideal Proximity Sensor



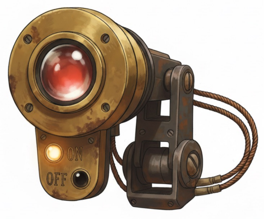


In this example,

- $z_1 = ON$
- $z_2 = OFF$

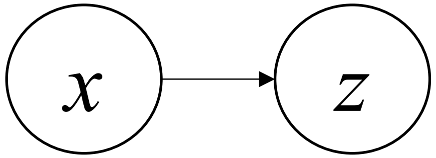
- This figure illustrates how the proximity sensor works for one of the shelves.
- Similar blue regions exist for all four shelves and the four walls.

Likelihood for the Ideal Proximity Sensor



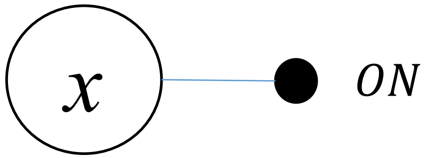
- We can model an ideal proximity sensor using the measurement model:

$$P(z_k = ON \mid x_k) = \begin{cases} 1 & d(x_k) \le d_0 \\ 0 & otherwise \end{cases}$$

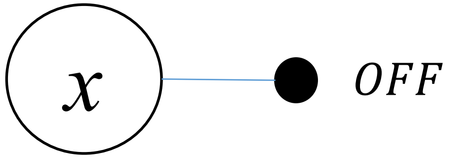


- The ***likelihood*** for this sensor is given by:

$$\mathcal{L}(x_k; z_k = ON) = \begin{cases} 1 & d(x_k) \le d_0 \\ 0 & otherwise \end{cases}$$



$$\mathcal{L}(x_k; z_k = OFF) = \begin{cases} 0 & d(x_k) \le d_0 \\ 1 & otherwise \end{cases}$$

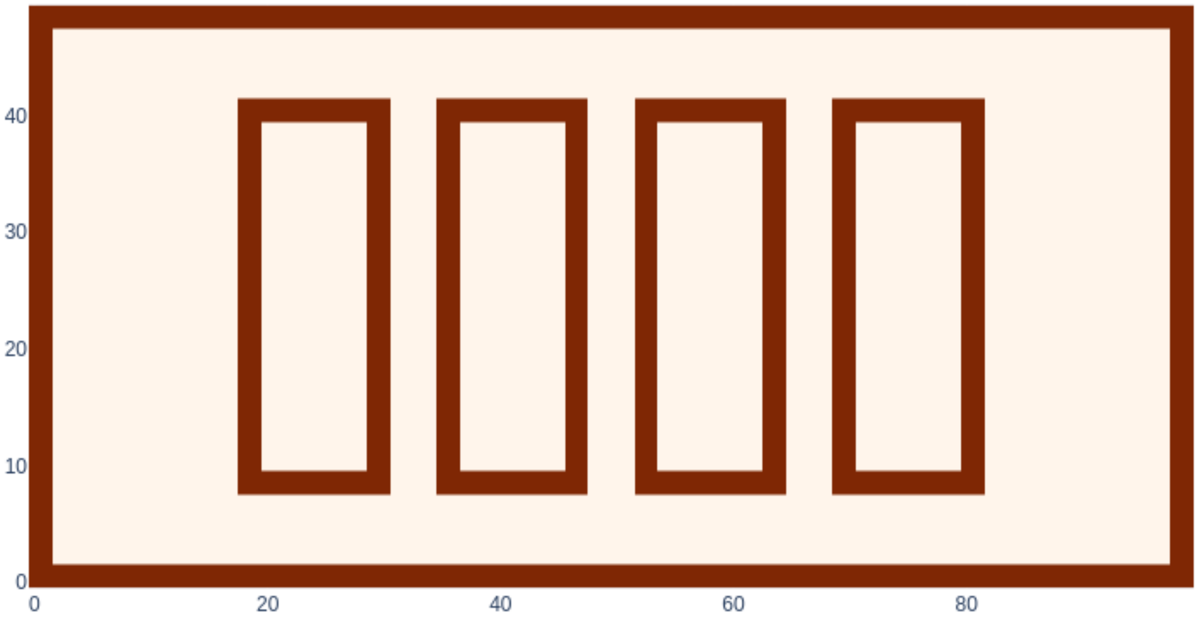


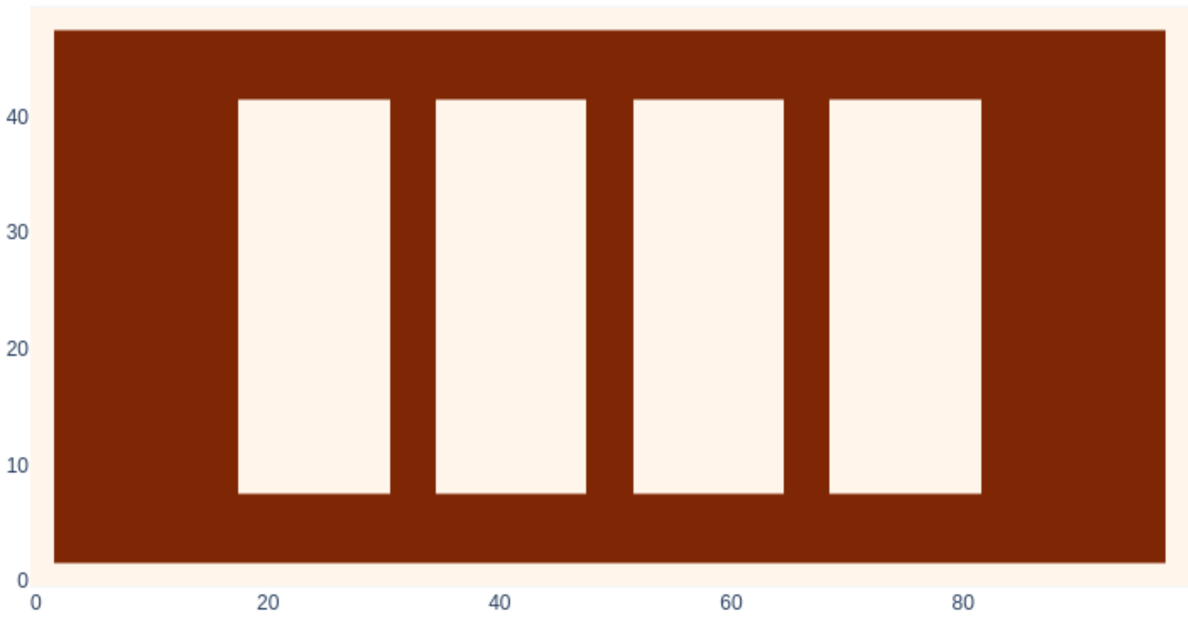
Likelihood for the Ideal Proximity Sensor

- We can plot the ***likelihood*** for each possible value of z_k.

$$\mathcal{L}(x_k; z_k = ON) = \begin{cases} 1 & d(x_k) \le d_0 \\ 0 & otherwise \end{cases}$$

$$\mathcal{L}(x_k; z_k = OFF) = \begin{cases} 0 & d(x_k) \le d_0 \\ 1 & otherwise \end{cases}$$



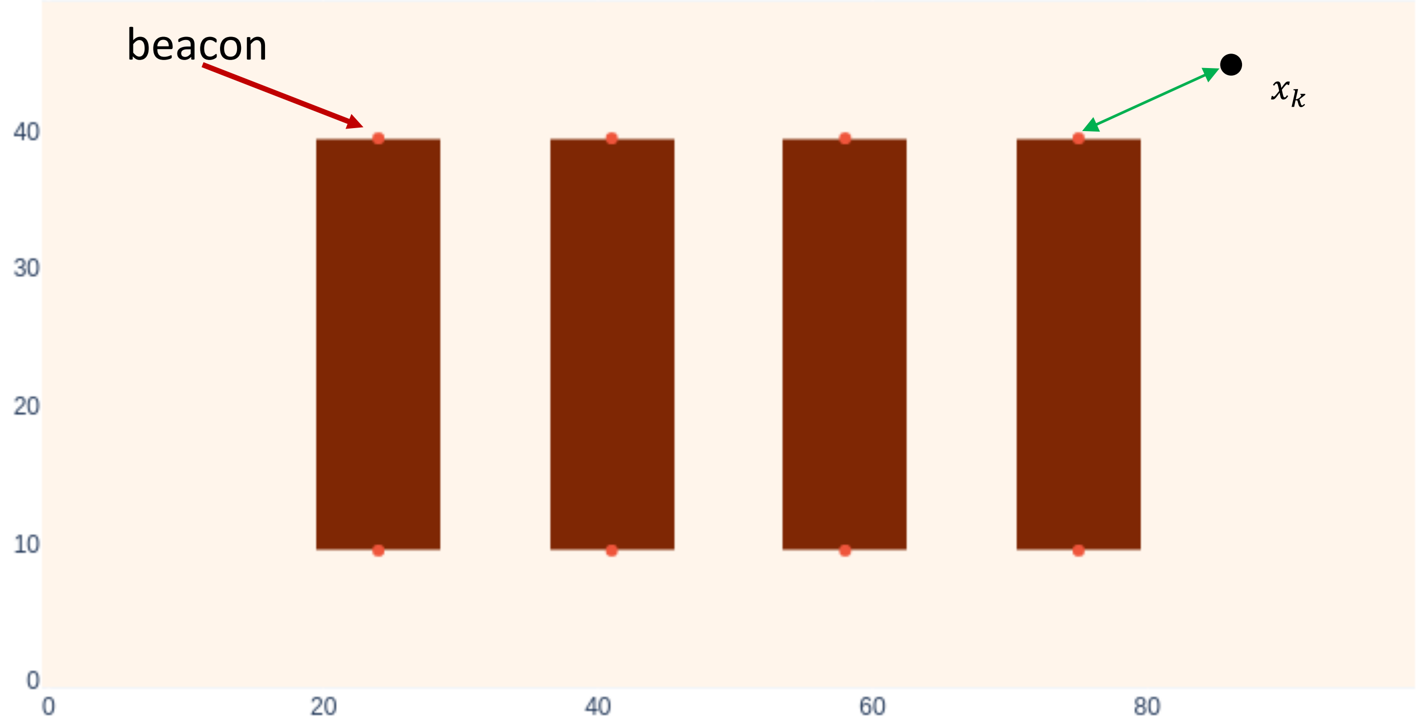


- **The likelihood is a function of x_k. It is *not* a probability distribution!**
- **The specific form of the likelihood depends on which value of z_k was observed.**

An Ideal Range Sensor

- Eight beacons, at locations $b_0, \dots, b_7$.
- The range sensor returns the distance to the beacons:

$$h(x_k; b_i) = \|x_k - b_i\| = \sqrt{(x_k - b_i)^T (x_k - b_i)}$$



- This sensor can be realized using RFID technology.

- Of course the beacon range is finite, so when $\|x_k - b_i\| > d_{\max}$ for all i, we set

$$h(x_k; b_i) = \textbf{inf}$$

A Noisy Range Sensor

- We often assume that sensor measurements are corrupted by additive noise. In this case, our range sensor returns a noisy measurement:

$$z_k = h(x_k; b_i) + w_k = \|x_k - b_i\| + w_k$$

in which w_k is the noise term.

- We'll assume i.i.d. zero-mean Gaussian noise, $f_{W_k}(w_k) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{w_k^2}{2\sigma^2}}$
- The resulting conditional pdf for the measurement (given x_k and b_i) is given by

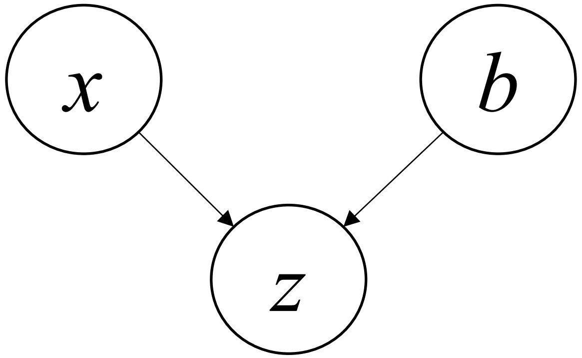
$$f_{Z_k}(z_k | x_k, b_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left(z_k - h(x_k; b_i)\right)^2}{2\sigma^2}}$$

➢ Given the state and the beacon ID, the range measurement is a Gaussian R.V. whose mean is equal to the true range.

Measurement Model

- The sensor measurement model is a conditional pdf:

$$f_{Z_k}(z_k|x_k, b_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left(z_k - h(x_k; b_i)\right)^2}{2\sigma^2}}$$

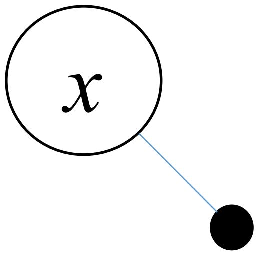


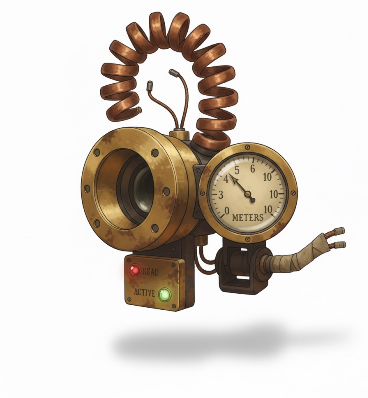
- This pdf describes the behavior the a r.v. z_k when x_k and b_i are known.
- As such, we can expect f_{Z_k} to behave like any other pdf, e.g.,

$$\int_{-\infty}^{\infty} f_{Z_k}(z_k|x_k, b_i) dz_k = 1$$

Measurement Likelihood

- The measurement ***likelihood*** is a function of x_k





$$\mathcal{L}(x_k; z_k, b_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left(z_k - h(x_k; b_i)\right)^2}{2\sigma^2}}$$

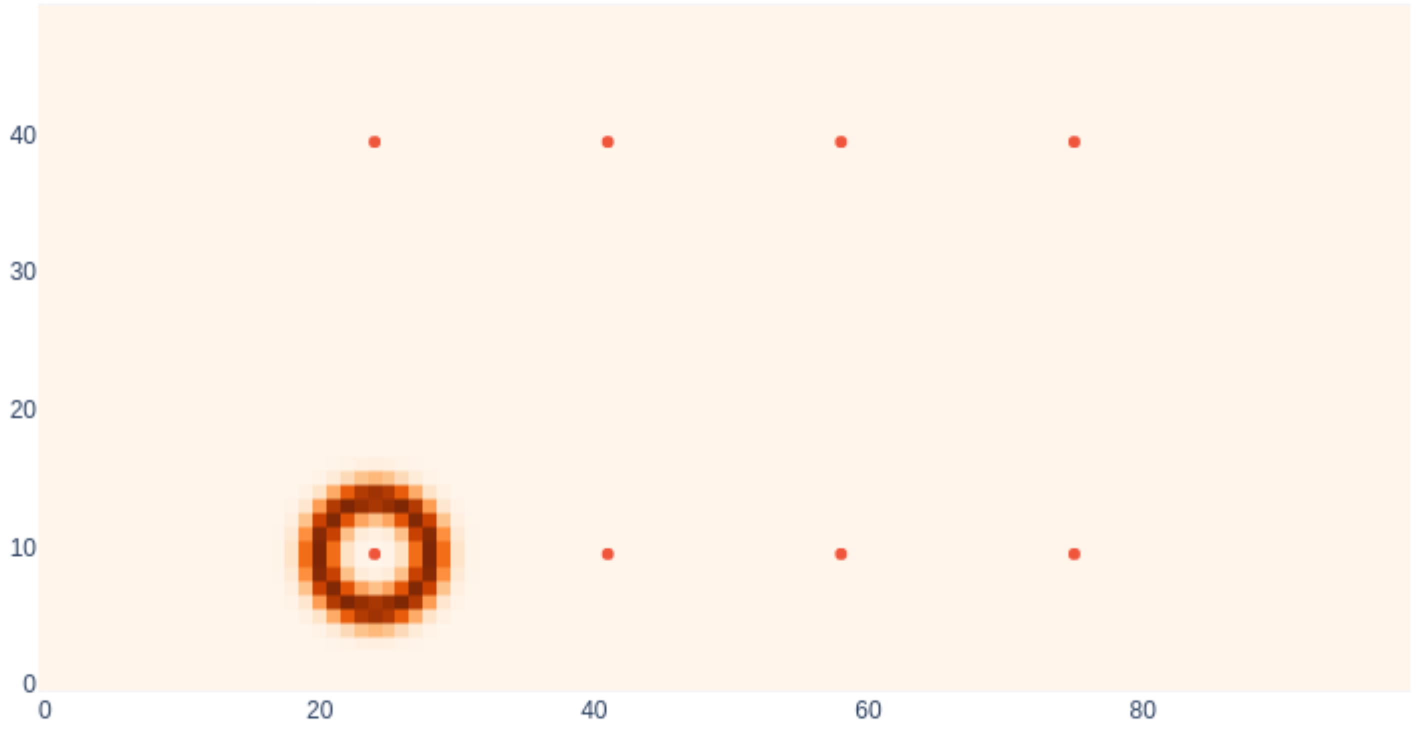
- This likelihood is ***not*** a probability. For example,

$$\int_{-\infty}^{\infty} \mathcal{L}(x_k; z_k, b_i)\, dx_k \neq 1$$

- The likelihood tells us something about how likely it would be to see various values for x_k, but it does not tell us probabilities.

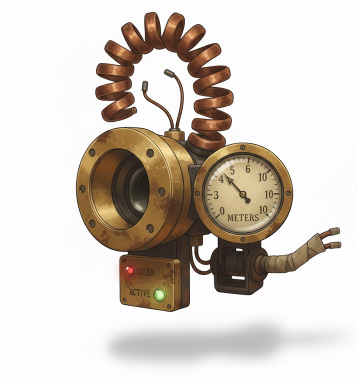
Measurement Likelihood

- For a given measurement z_k and specific beacon b_i, we can plot the likelihood function on our warehouse map.
- For the case $b_i = b_0$ and $z_k = 4.03$, we obtain the plot for $\mathcal{L}(x_k; 4.03, b_0)$ shown below (and in book).

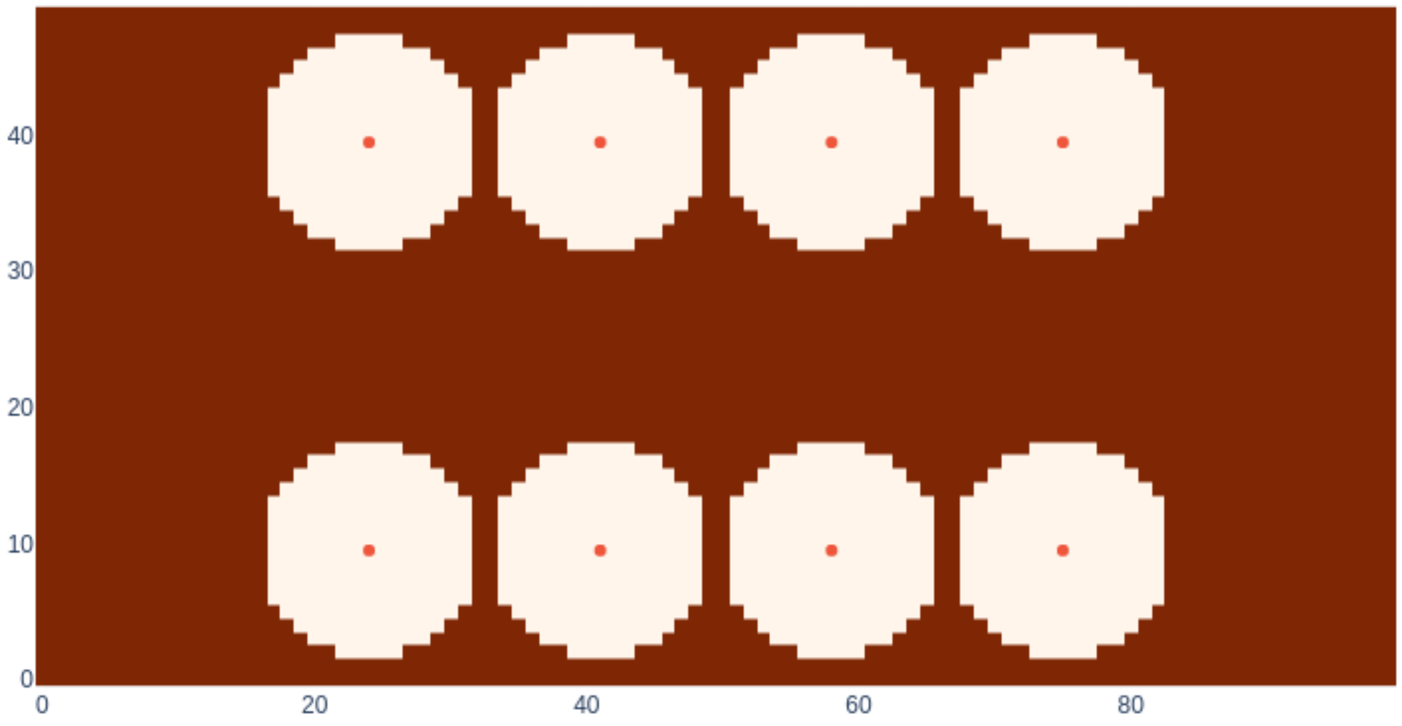


- The likelihood achieves its maximum on the circle of radius 4.03, centered on beacon b_0.
- The value of $\mathcal{L}(x_k; 4.03, b_0)$ looks like a Gaussian curve along any radial line extended from beacon b_0.

Out-of-range Measurements



- If in range, sensor provides distance to a specific beacon.
- If ***all beacons out of range***, i.e., $\|x_k - b_i\| > d_{max}$ for all i $\rightarrow$ $h(x_k; b_i) = \mathbf{inf}$.
- We can construct a likelihood for this case: $\mathcal{L}(x_k; z_k = \text{inf}, b_i = \text{NONE})$



$$\mathcal{L}(x_k; z_k = \text{inf}, b_i = \text{NONE}) = \begin{cases} 1 & h(x_k; b_i) > d_{max}, i = 0 \dots 7 \\ 0 & otherwise \end{cases}$$

A Pseudo-GPS Sensor

- GPS-like sensors return the coordinates in a global frame.
- In the simplest case, we have $z_k = h(x_k) = x_k$.
- Not unusual to have different units, e.g., centimeters.
- In these cases, we scale the measurement: $z_k = h(x_k) = Cx_k$
- Consider additive noise, then our measurement model is:

$$z_k = h(x_k) + w_k = Cx_k + w_k$$

- If w_k is zero-mean Gaussian noise (as usual), we again have a ***conditional Gaussian probability density***:

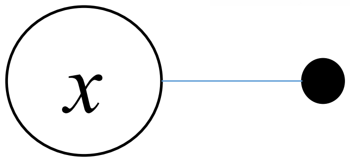
$$f_{Z_k}(z_k|x_k) = \frac{1}{\sqrt{|2\pi\Sigma|}} exp\left\{-\frac{1}{2}(z_k - Cx_k)^T \Sigma^{-1}(z_k - Cx_k)\right\}$$

$$x \longrightarrow z$$

GPS-style Likelihoods

- The likelihood for our GPS-like sensor is given by

$$\mathcal{L}(x_k; z_k) = \frac{1}{\sqrt{|2\pi\Sigma|}} exp\left\{-\frac{1}{2}(z_k - Cx_k)^T\Sigma^{-1}(z_k - Cx_k)\right\}$$

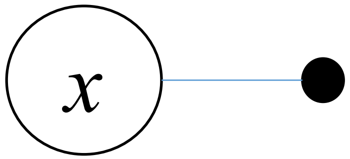


- Let's work on the exponent: $(z_k - Cx_k)^T$

$$(z_k - Cx_k) = C(C^{-1}z_k - x_k) \rightarrow (z_k - Cx_k)^T = [C(C^{-1}z_k - x_k)]^T = (C^{-1}z_k - x_k)^T C^T$$

- Therefore, we can write the likelihood as:

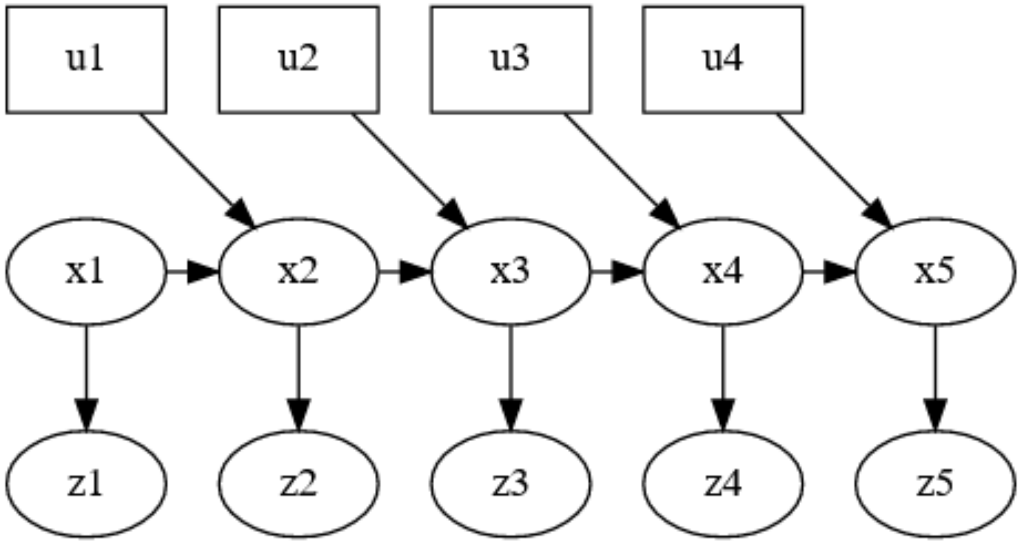
$$\mathcal{L}(x_k; z_k) = \frac{1}{\sqrt{|2\pi\Sigma|}} exp\left\{-\frac{1}{2}(x_k - C^{-1}z_k)^T C^T\Sigma^{-1}C(x_k - C^{-1}z_k)\right\}$$



which has the form of a Gaussian with mean $C^{-1}z_k$ and inverse covariance $C^T\Sigma^{-1}C$.

Simulating States and Measurements

- Given a control tape $u_1, \dots, u_{n-1}$ and a prior distribution for X_1, it's easy to generate a sample trajectory $x_1, \dots, x_n$ along with a sample measurement history $z_1, \dots, z_n$.



1. Generate a sample for x_1 by sampling from the prior $P(X_1 = x_1)$.
2. Generate a sample measurement z_1 by sampling from the measurement model $p(Z_1|x_1)$
3. For each i:
    1. Generate a sample for x_i by sampling from the transition distribution $p(X_i|x_{i-1}, u_{i-1})$
    2. Generate a measurement sample z_i by sampling from the measurement model $p(Z_i|x_i)$

Next Time…

Perception

- Bayes Filter
- Markov Localization
- Monte Carlo Localization