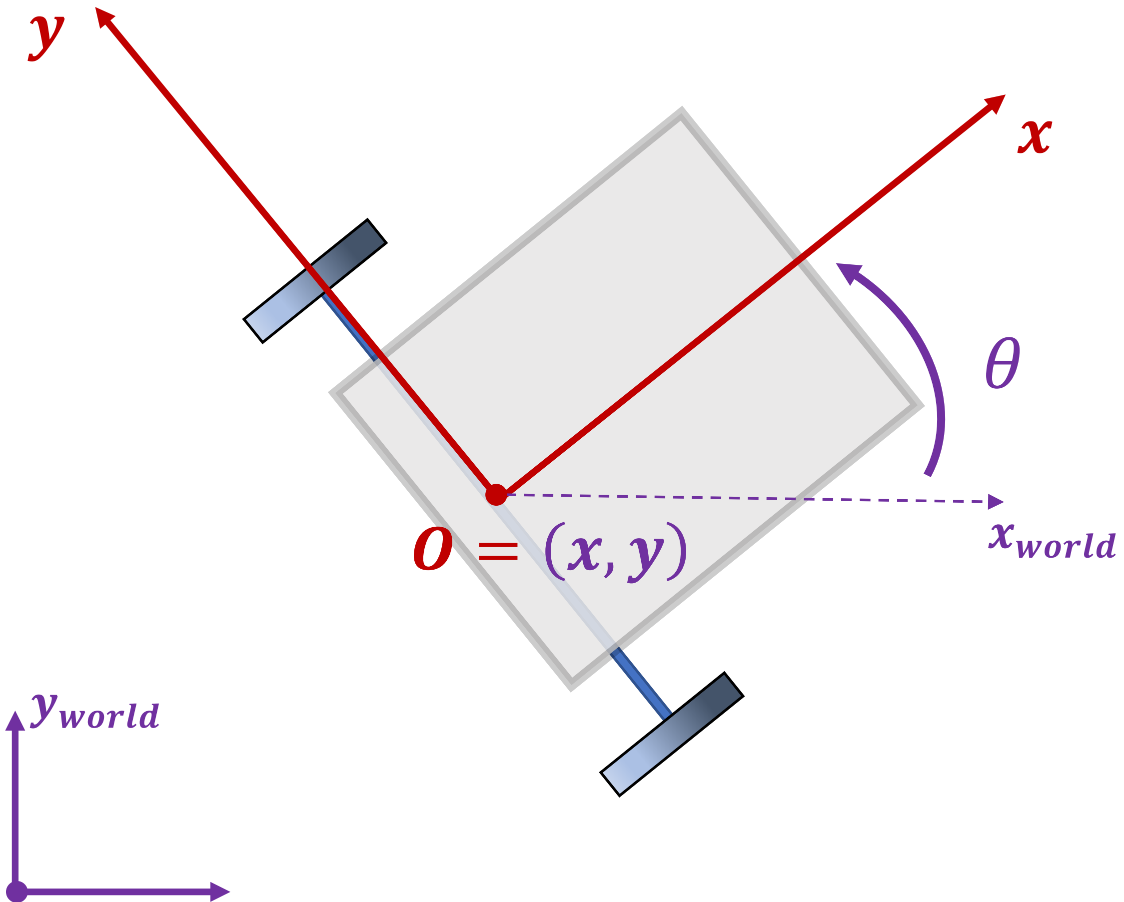
# CS 3630!

*Lecture 18:*
*Differential Drive Robots:*
*Motion Planning*

# *Diff Drive Recap*

# Configuration Space

- A ***configuration*** is a complete specification of the position of every point in a robot system.
- The ***configuration space*** is the set of all configurations.
- We use $q$ to denote a point in a configuration space $Q$.




Because our DDR can rotate in the plane, it is necessary to know both the position and the orientation of the body-attached frame to specify a configuration:

$$Q = \mathbb{R}^2 \times [0, 2\pi)$$

$$q = (x, y, \theta) \in Q$$

If we know the configuration, $q = (x, y, \theta)$, we can compute the location of any point on the robot.

*Path Planning*

# Path Planning

- Find a collision-free path from the starting configuration $q_{init}$ to a goal configuration $q_{goal}$.
- Collision checking between the robot and obstacles can be computationally heavy, so we deal with this by mapping obstacles in the world to the robot's configuration space.
- In the configuration space, we now have the problem of finding a path for a single point (which represents the configuration of the robot).
- In general, computing this mapping can be difficult computationally, but it's not so bad for robots that translate in the plane.

# Obstacles in C-Space

- Let $q$ denote a point in a configuration space $\mathcal{Q}$
- The path planning problem is to find a mapping $\gamma$: $[0,1] \rightarrow \mathcal{Q}$ s.t. no configuration along the path intersects an obstacle.
- Denote the i-th workspace obstacle by $\mathcal{O}_i$*, and by* $R(q)$ *the volume occupied by the robot at configuration* $q$.
- A *configuration space obstacle* $\mathcal{QO}_i$ is the set of configurations $q$ at which the robot intersects $\mathcal{O}_i$

$$\mathcal{QO}_i = \{\, q \in \mathcal{Q} \mid R(q) \cap \mathcal{O}_i \neq \emptyset\}$$

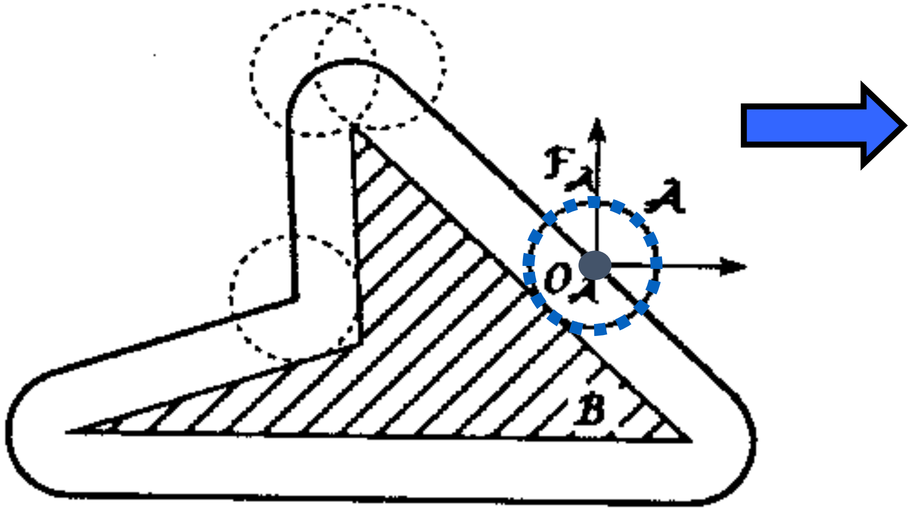
- The *free configuration space* (or just *free space*) $\mathcal{Q}_{free}$ *is*

$$\mathcal{Q}_{free} = \mathcal{Q} - \cup_i \mathcal{QO}_i$$

- A *free path* is a mapping $\gamma$: $[0,1] \rightarrow \mathcal{Q}_{free}$.

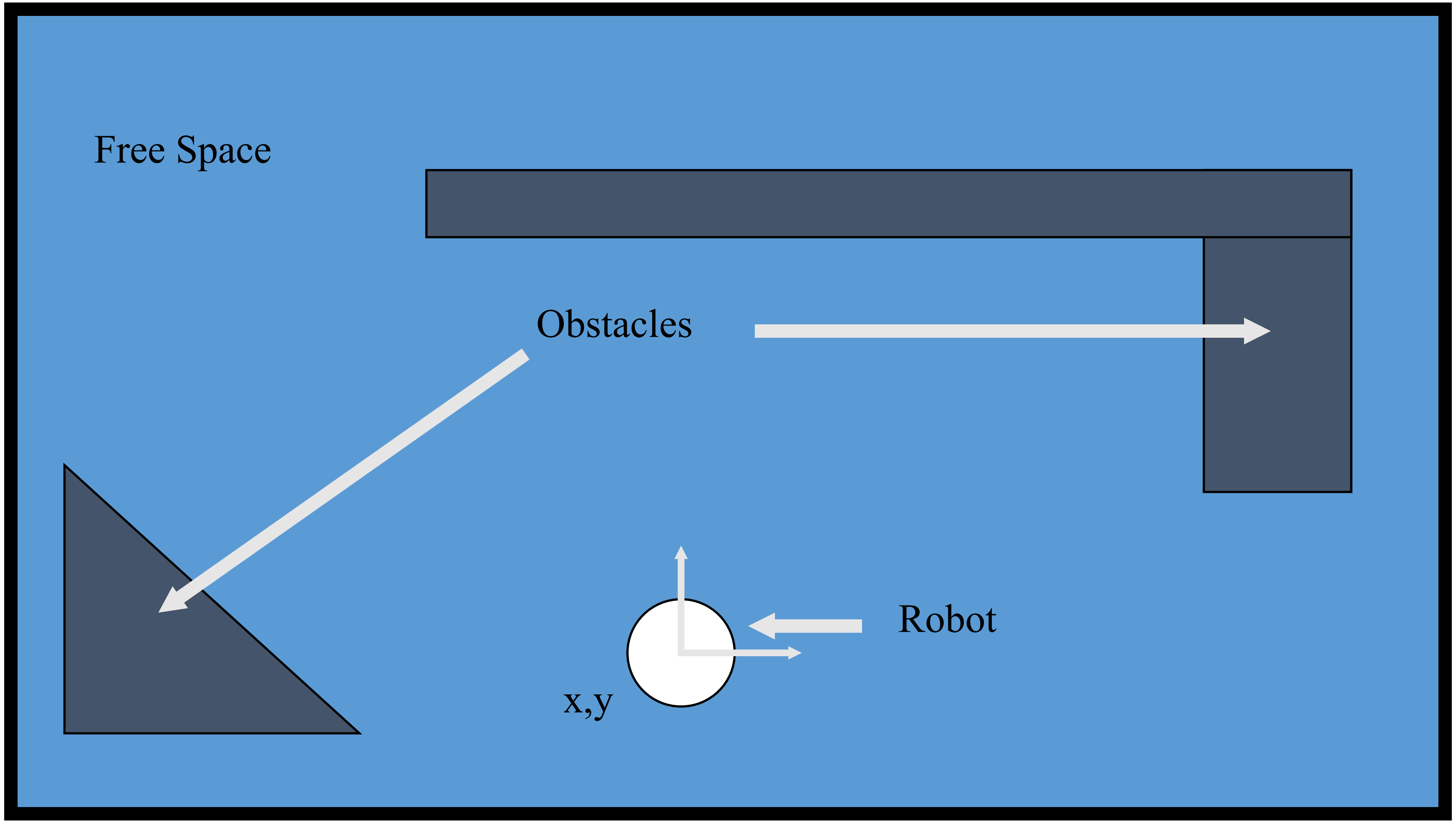
# Disc in 2-D workspace

workspace

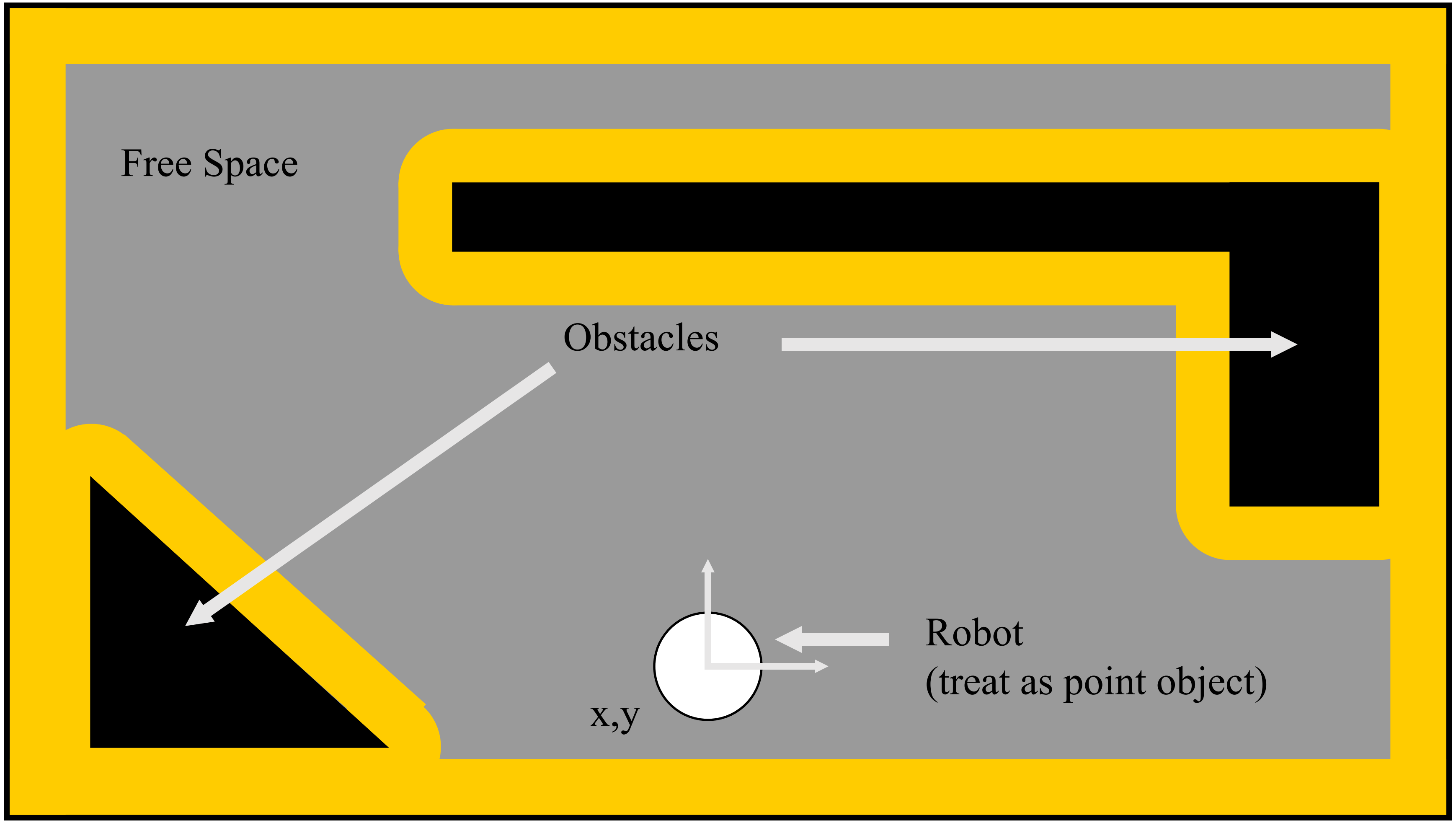



configuration
space

# Example of a World (and Robot)

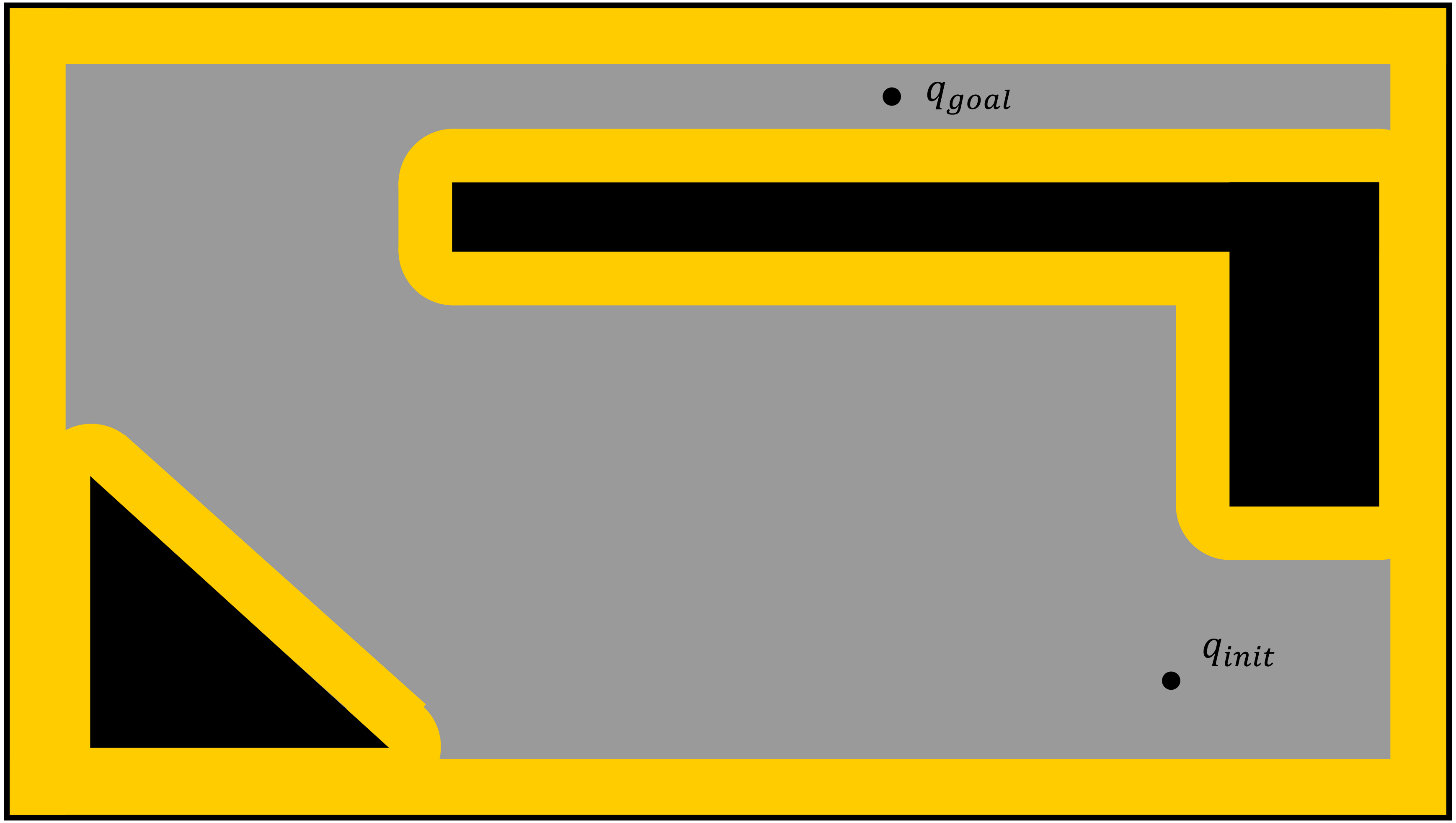


# Configuration Space: Accommodate Robot Size



# Planning a Collision-Free Path

***Find a collision-free path from $q_{init}$ to $q_{goal}$***



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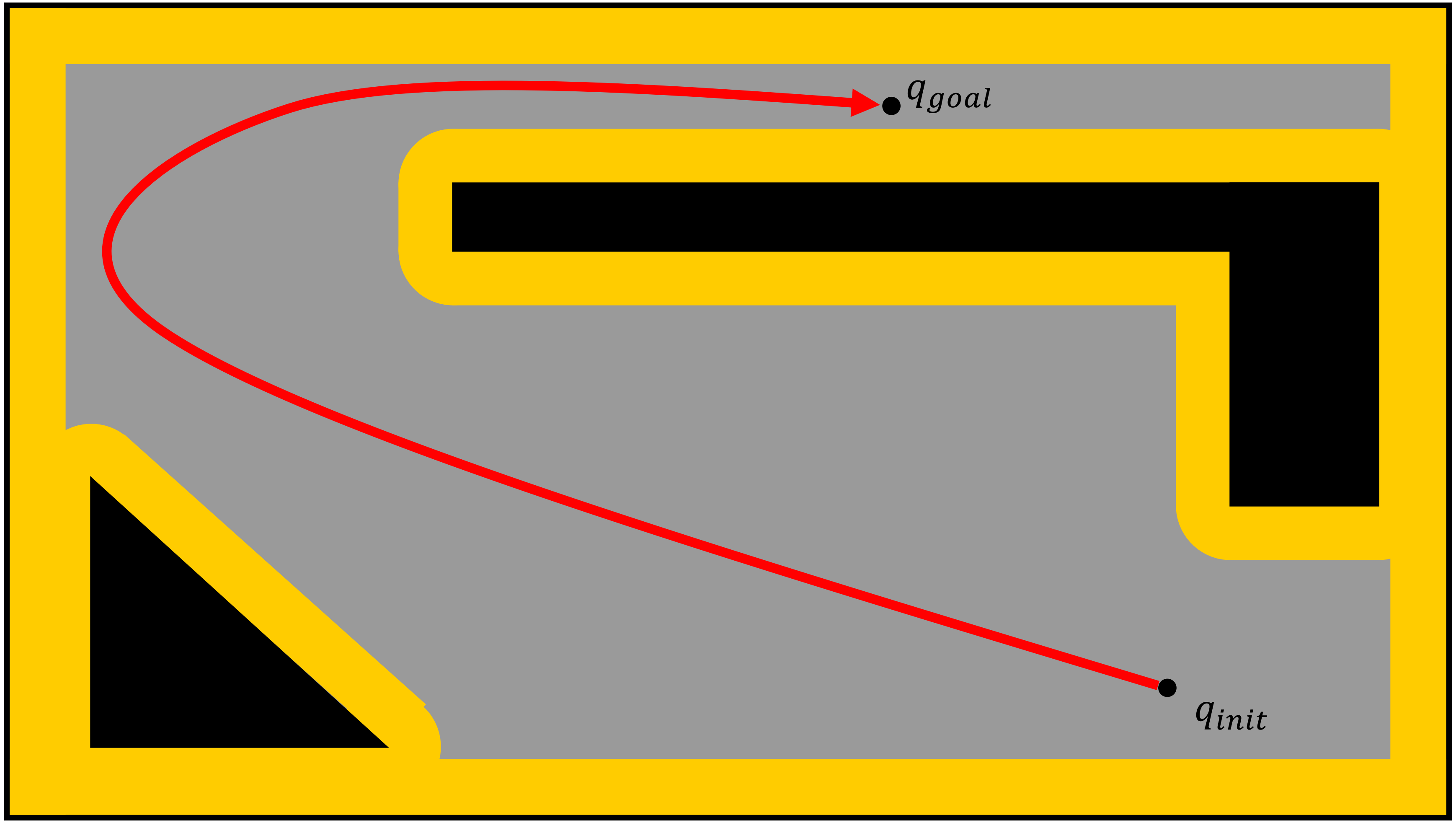

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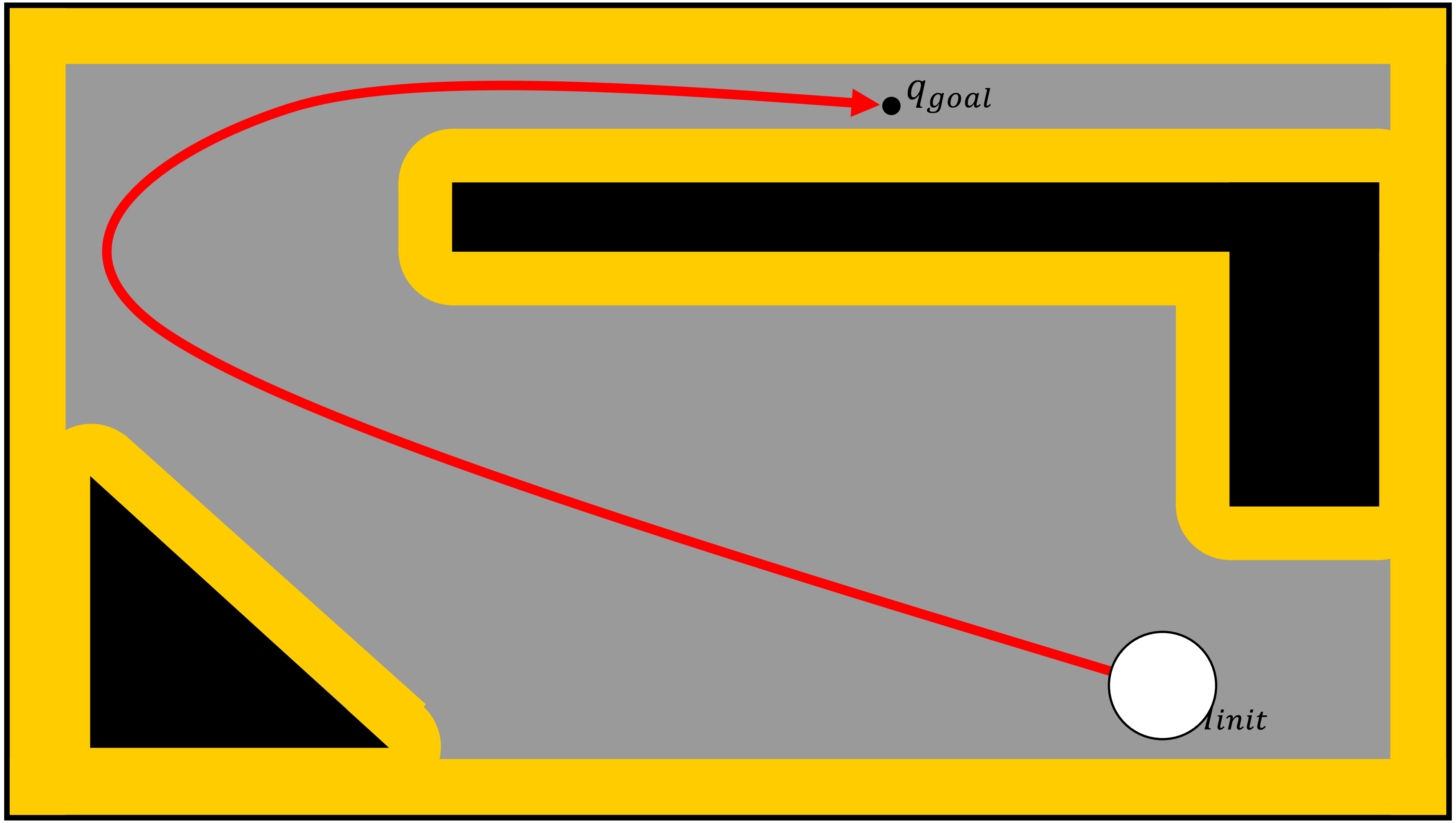

# Probabilistic Roadmaps

With so many slides and ideas from so many people, including:

Howie Choset, Nancy Amato, David Hsu,

Sonia Chernova, Steve LaValle, James Kuffner,

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# Completeness

- Complete algorithm → Slow
  - A **complete** algorithm finds a path if one exists and reports no otherwise in finite time.
  - Heuristic algorithm → Unreliable (e.g., potential fields)

- **Probabilistic completeness**
  - Intuition: If there is a solution path, the algorithm will find it with high probability.

# The Rise of Monte Carlo Techniques

- KEY IDEA:
  Rather than exhaustively explore ALL possibilities, randomly explore a smaller subset of possibilities while keeping track of progress

- Facilities "probing" deeper in a search tree much earlier than any exhaustive algorithm can

- What's the catch?
  Typically, we must sacrifice both *completeness* and *optimality*
  Classic tradeoff between solution quality and runtime performance

*Sampling Based Planning:*

Search for collision-free path
only by sampling points.

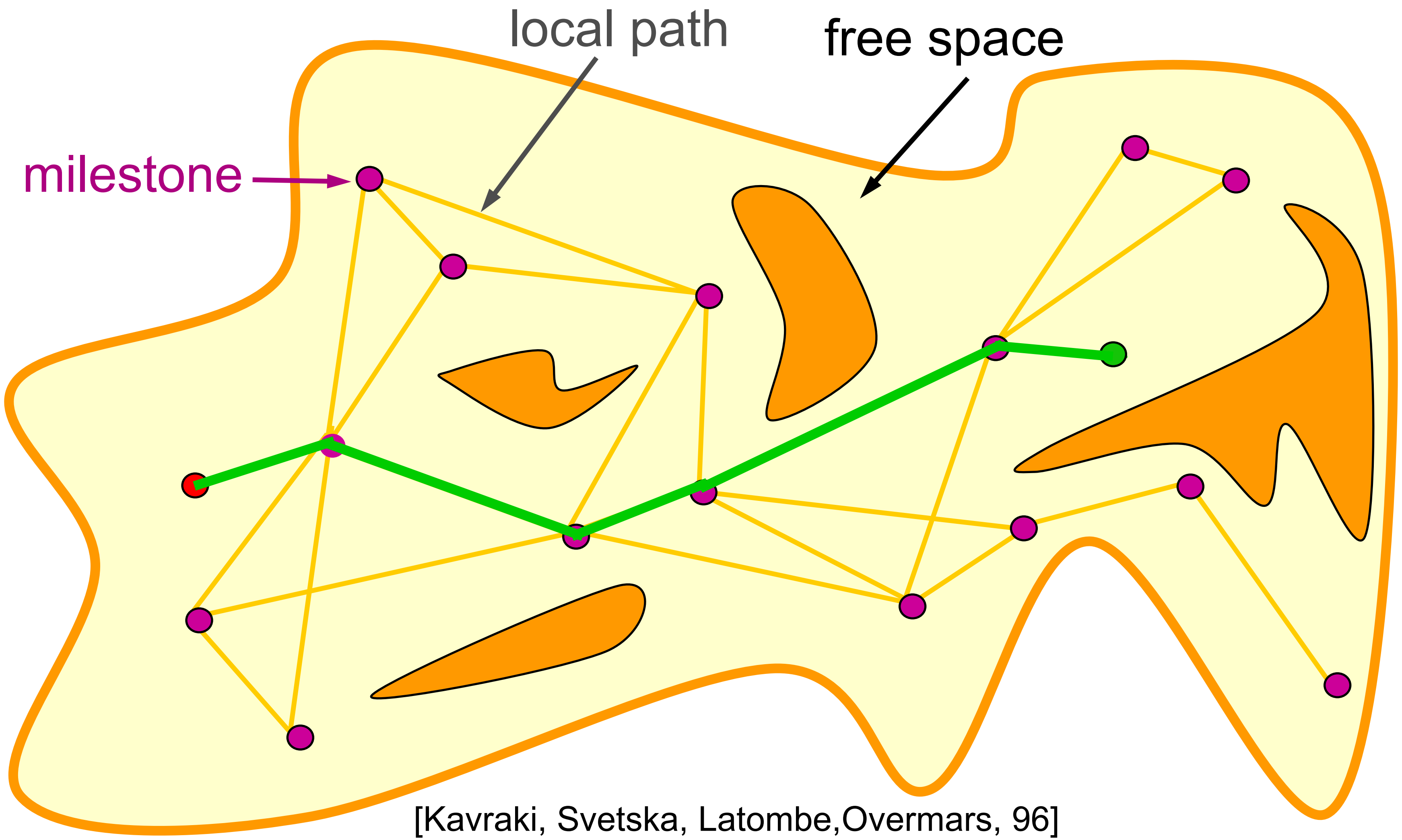
# Probabilistic Road Map (PRM)

- Probabilistic Roadmap methods proceed in two phases:

1. Preprocessing Phase – to construct the roadmap $G$
2. Query Phase – to search given $q_{init}$ and $q_{goal}$

The roadmap is an undirected graph $G = (N, E)$. The nodes in N are a set of configurations of the robot chosen over C-free. The edges in E correspond to feasible straight-line paths.

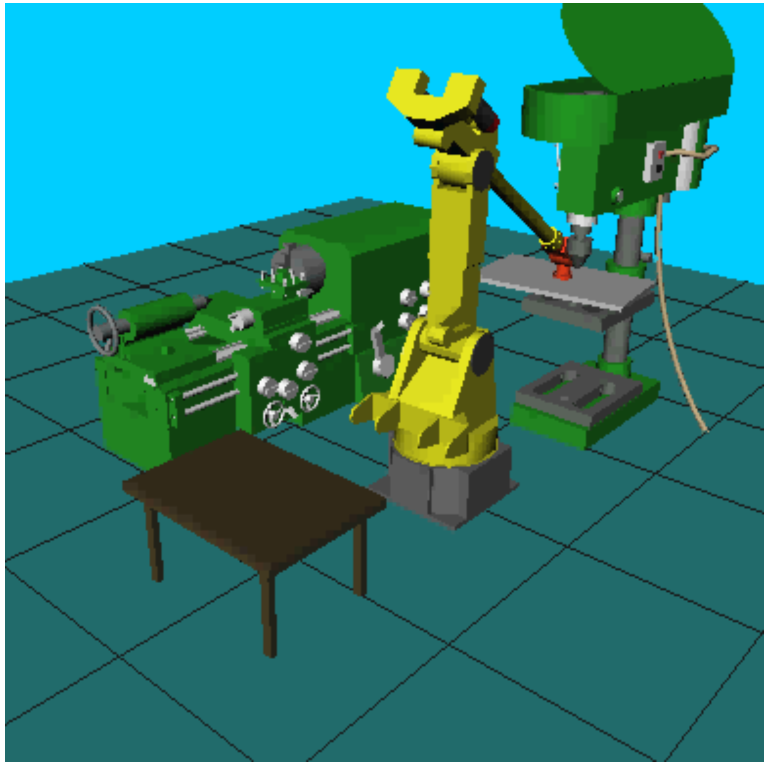
# Probabilistic Roadmap (PRM): multiple queries




[Kavraki, Svetska, Latombe,Overmars, 96]

# Assumptions

- Static obstacles
- Many queries to be processed in the same environment
- Examples
  - Navigation in static virtual environments
  - Robot manipulator arm in a workcell
- Advantages:
  - Amortize the cost of planning over many problems
  - Probabilistically complete



# Uniform sampling

**Input:** geometry of the moving object & obstacles
**Output:** roadmap G = (V, E)

```
V ← ∅ and E ← ∅.
repeat
  q ← a configuration sampled uniformly at random from C.
   if CLEAR(q) then
     Add q to V.
     N_q ← a set of nodes in V that are close to q.
     for each q' ∈ N_q, in order of increasing d(q,q')
       if LINK(q',q) then
         Add an edge between q and q' to E.
```

# Some terminology

- The graph G is called a **probabilistic roadmap**.
- The nodes in G are called **milestones**.

# How do we determine a *random free* configuration?

- We want the nodes of V to be a ***uniform*** sampling of $Q_{free}$

  - Draw each of its coordinates from the interval of values of the corresponding degrees of freedom. (Use the uniform probability distribution over the interval)

  - Check for collision both with robot itself and with obstacles

  - If collision free, add to V, otherwise discard

  - What about rotations? Strategies for sampling orientation are beyond the scope of this class. Since DDRs live in the plane, we could merely sample uniformly in the interval $[0, 2\pi]$.

# What's the local path planner: Link(q',q) ?

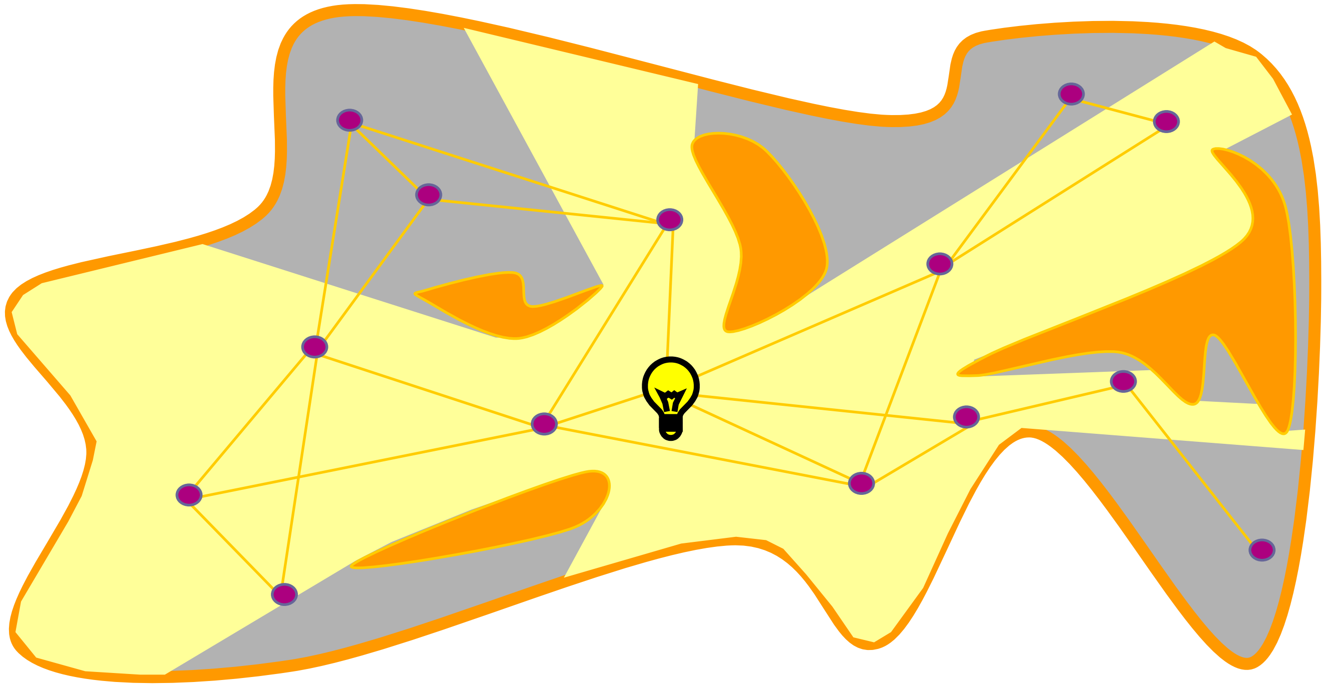
- There are plenty of possibilities
  - Nondeterministic (include a randomized "wandering" component)
    - We'll have to store local paths in roadmap
  - Powerful
    - Slower but maybe we'll need fewer nodes if we do some hard work during roadmap construction?
  - Fast and simple
    - Less powerful, Roadmap will need more nodes

# Go with the fast local planner

- Need to make sure start and goal configurations can connect to graph, which requires a somewhat dense roadmap
- Can reuse local planner at query time to connect start and goal configurations
- Don’t need to memorize local paths

# Why does it work? Intuition

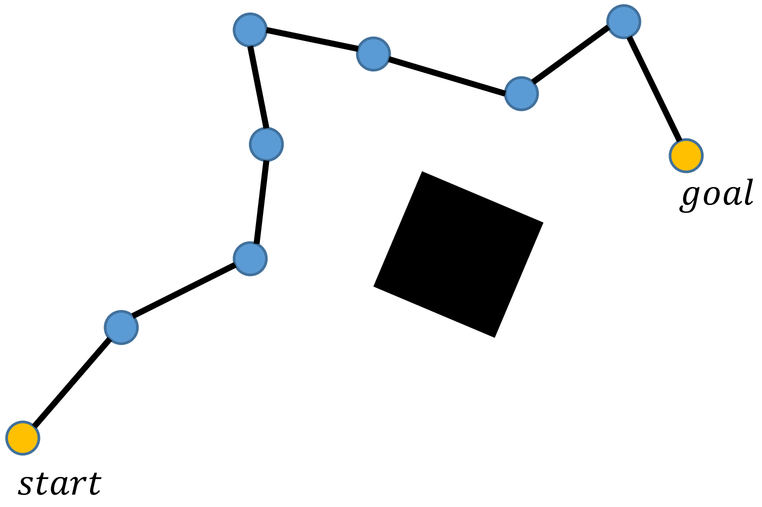
- A small number of milestones **almost** “cover” the **entire** configuration space.



- Rigorous definitions exist (of course!)

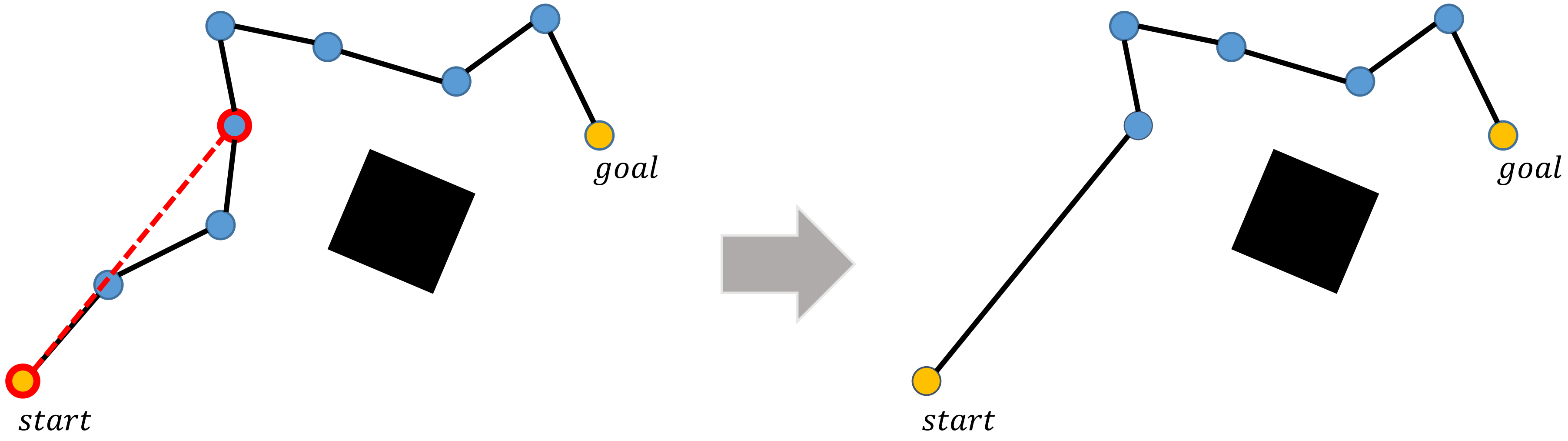
# Optimizing the path

- Milestone-based paths are far from optimal and require additional refinement before they are usable
- A typical solution can look like this:



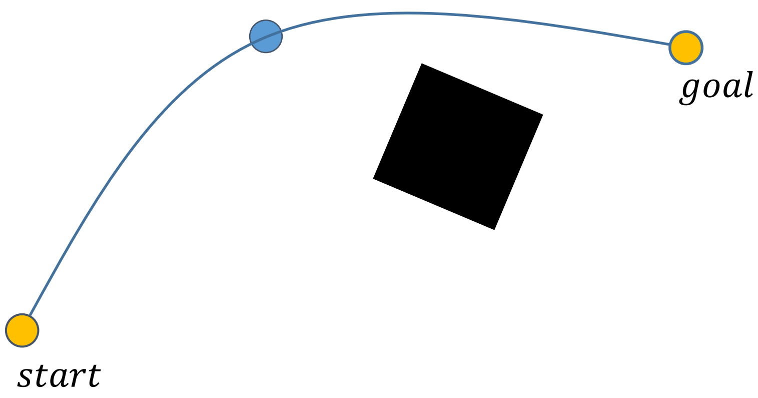
# Optimizing the path

- A simple way to improve the path, is to repeatedly pick two nodes at random, and check whether they can be connected by a straight line without collision.  If so, use the line to shorten the path.

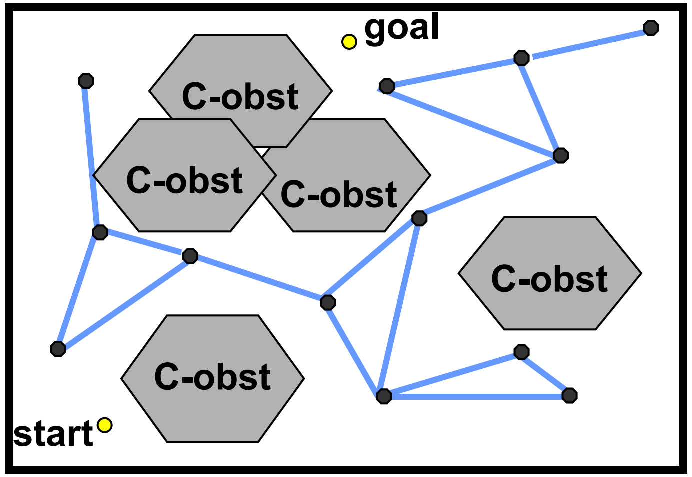


# Smoothing the path

- Optionally, the shortened path can then be smoothed to allow for continuous robot motion



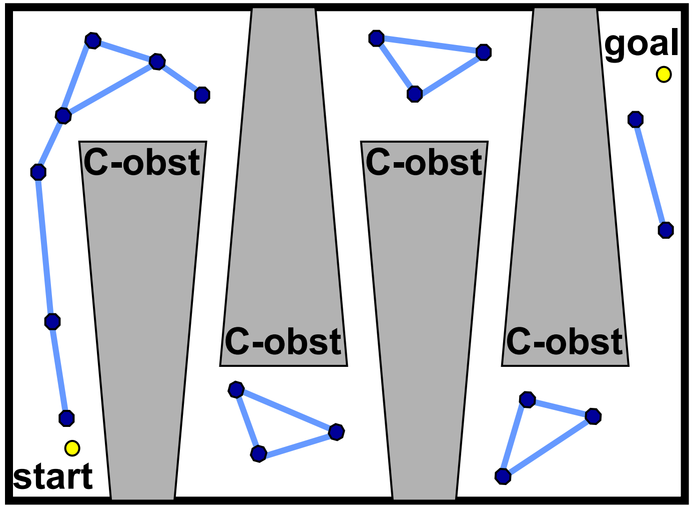
# Good news, but bad news too




## Sample-based: The Good News

1. *probabilistically complete*
2. Do not construct the C-space
3. apply easily to high-dimensional C-space
4. support fast queries w/ enough preprocessing

Many success stories where PRMs solve previously unsolved problems




## Sample-Based: The Bad News

1. don’t work as well for some problems:
– unlikely to sample nodes in *narrow passages*
– hard to sample/connect nodes on constraint surfaces
2. No optimality or completeness

# Rapidly-Exploring Random Trees (RRTs)

With so many slides and ideas from so many people, including:

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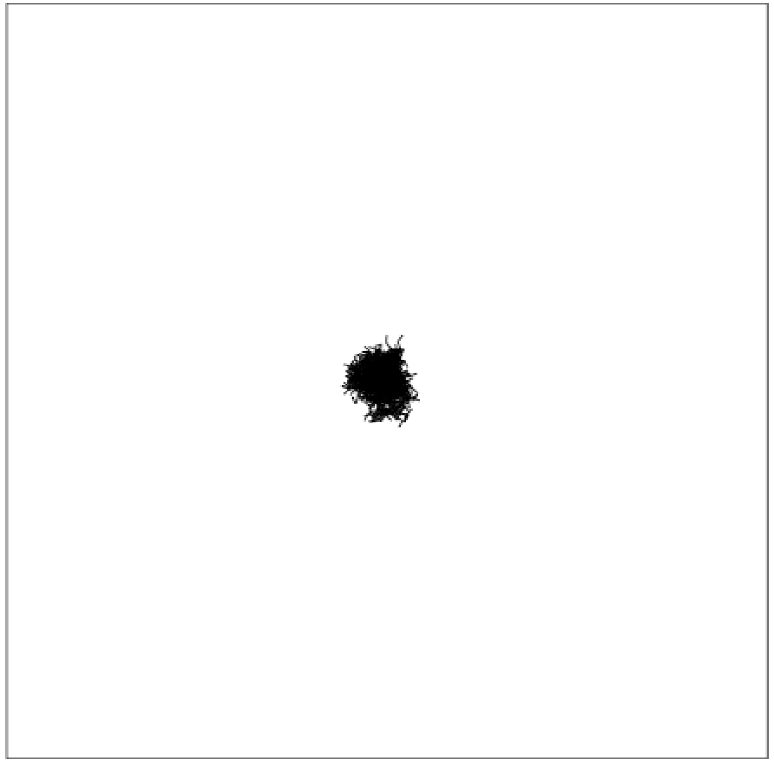
# Rapidly-Exploring Random Tree (RRT)

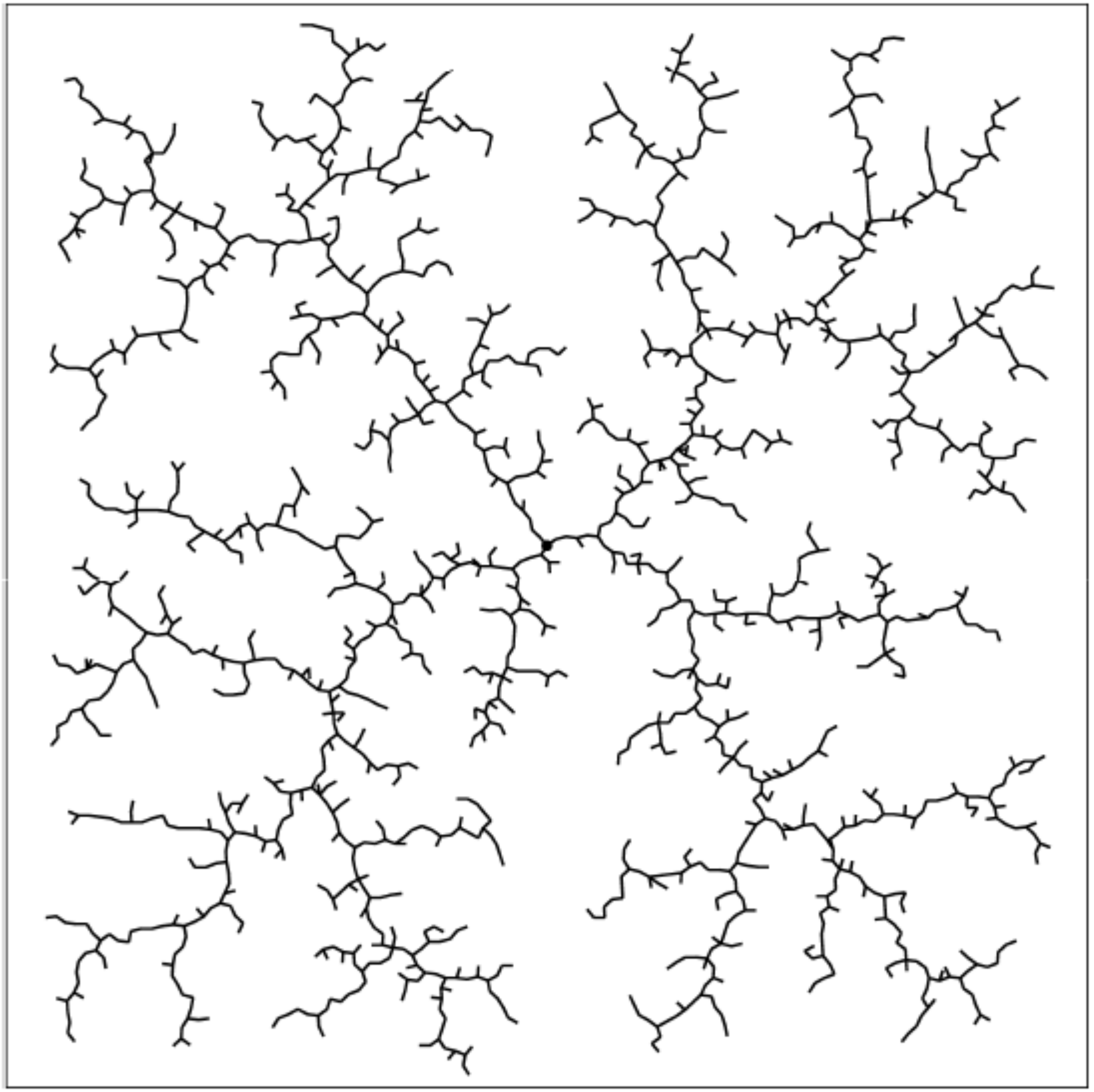
- Searches for a path from the initial configuration to the goal configuration by expanding a search tree
- For each step,
  - The algorithm samples a target configuration and expands the tree towards it.
  - The sample can either be a random configuration or the goal configuration itself, depends on the probability value defined by the user.

# Naïve random tree

- Pick a vertex at random
- Move in a random direction to generate a new vertex
- Repeat…



# Rapidly-Exploring Random Tree



# The Basic Idea: Iteratively expand the tree

- Denote by $T_k$ the tree at iteration $k$

- Randomly choose a configuration $q_{rand}$

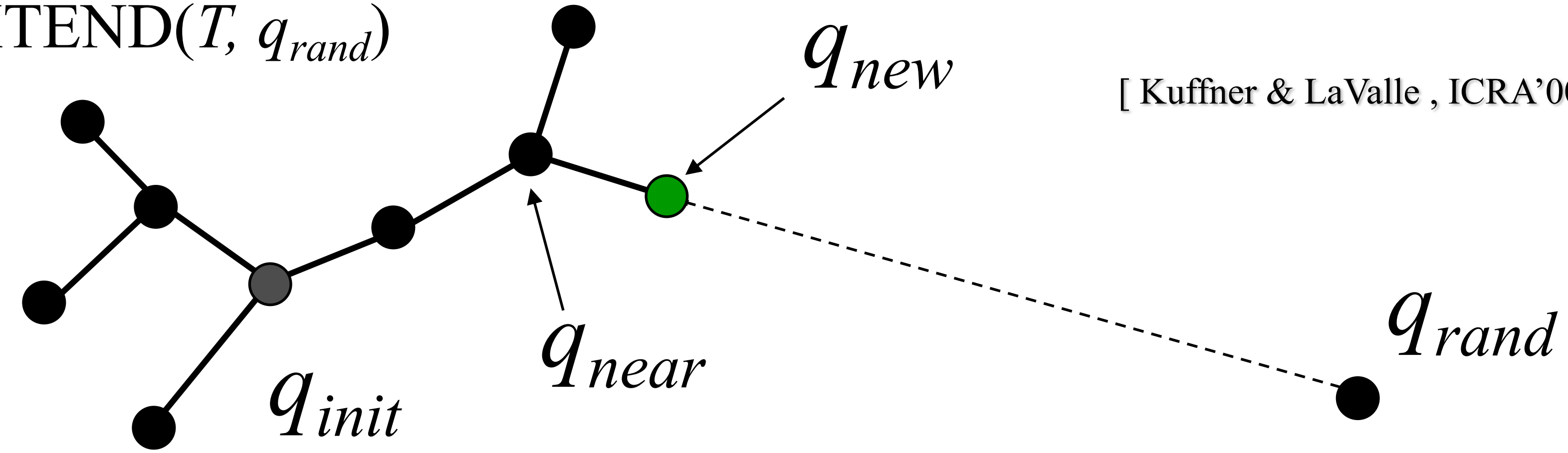
- Choose $q_{near} = \arg\min_{q \in T_k} d(q, q_{rand})$
  - ➢ $q_{near}$ is the nearest existing node in the tree to $q_{rand}$

- Create a new node, $q_{new}$ by taking a small step from $q_{near}$ toward $q_{rand}$

# Path Planning with RRTs

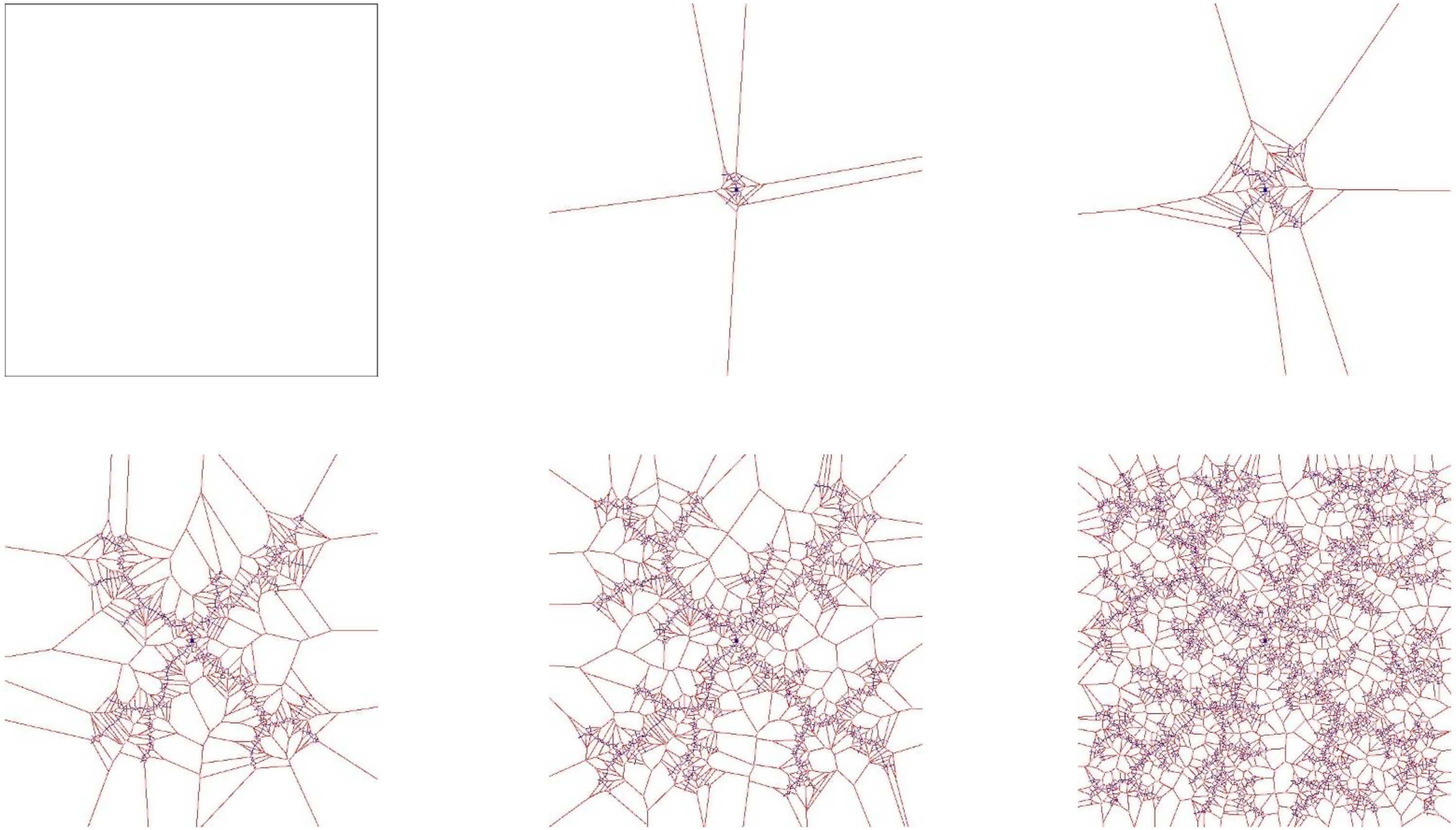
BUILD_RRT ($q_{init}$) {
    *T.init*($q_{init}$);
    for $k$ = 1 to K do
        $q_{rand}$ = RANDOM_CONFIG();
        EXTEND($T$, $q_{rand}$)
}

EXTEND($T$, $q_{rand}$)



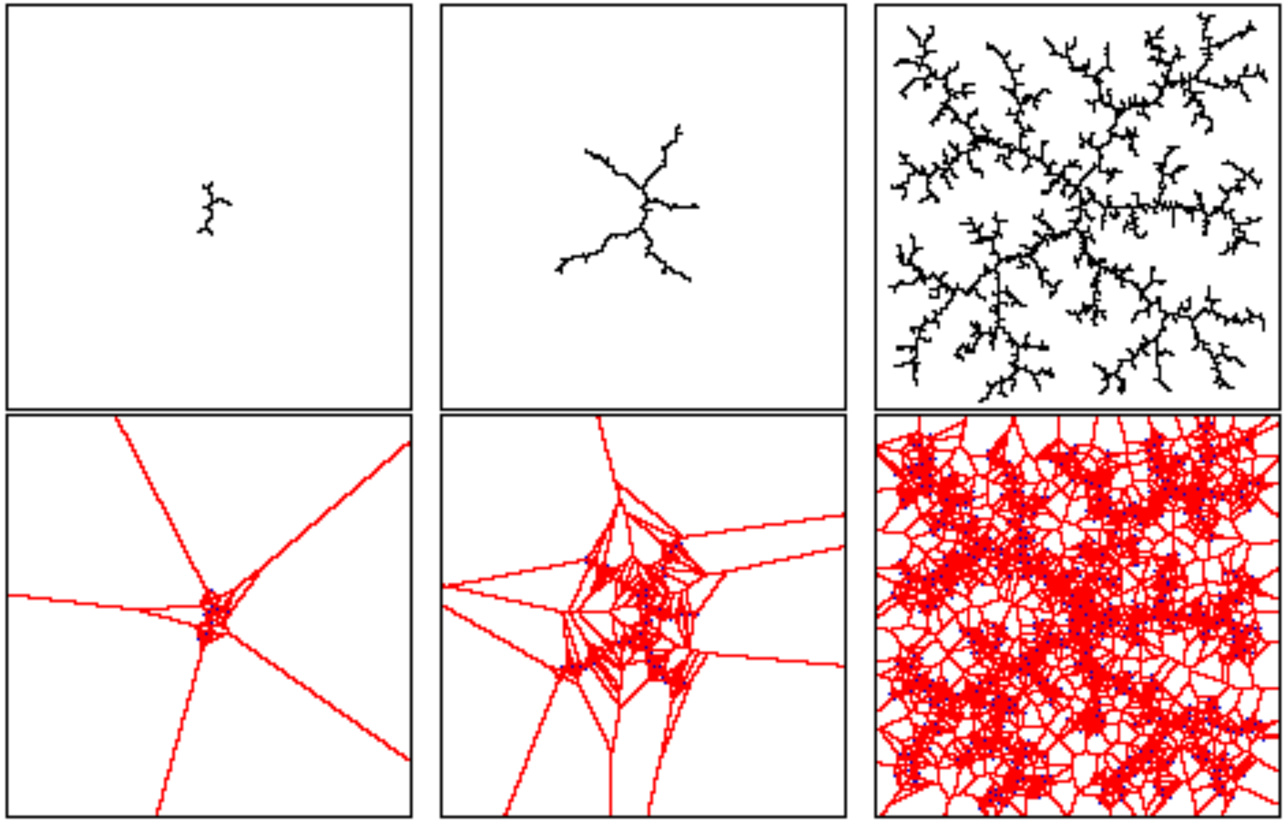

[ Kuffner & LaValle , ICRA'00]

# RRTs and
# Bias toward large Voronoi regions



http://msl.cs.uiuc.edu/rrt/gallery.html

# Why are RRT's rapidly exploring?



The probability of a node being selected for expansion (i.e. being a nearest neighbor to a new randomly picked point) is proportional to the area of its Voronoi region.

# Biases

- Bias toward larger spaces
- Bias toward goal
  - When generating a random sample, with some probability pick the goal instead of a random node when expanding
  - This introduces another parameter
  - James' experience is that 5-10% is the right choice
  - If you do this 100%, then this is a RPP

# RRT

Requires the following functions:

- p = RandomSample()
  - ➢ Uniform random sampling of free configuration space
- v = Nearest(p)
  - ➢ Given point in Cspace, find vertex on tree that is closest to that point
- p’ = Steer(p, goal)
  - ➢ For a point p and a goal point, find p’ that is closer to the goal than p
- ObstacleFree(p)

➢ Check if a given Cspace point is in the free space

# RRT in Action…

# RRT

$V \leftarrow \{x_{init}\}; \quad E \leftarrow \emptyset$

**for** $i = 1$ to $N$

$\quad G \leftarrow (V, E)$

$\quad x_{rand} \leftarrow RandomSample()$

$\quad x_{nearest} \leftarrow Nearest(G, x_{rand})$

$\quad x_{new} \leftarrow Steer(x_{nearest}, x_{rand})$

$\quad$ **if** $ObstacleFree(x_{nearest}, x_{new})$

$\quad\quad V \leftarrow V \cup \{x_{new}\}$

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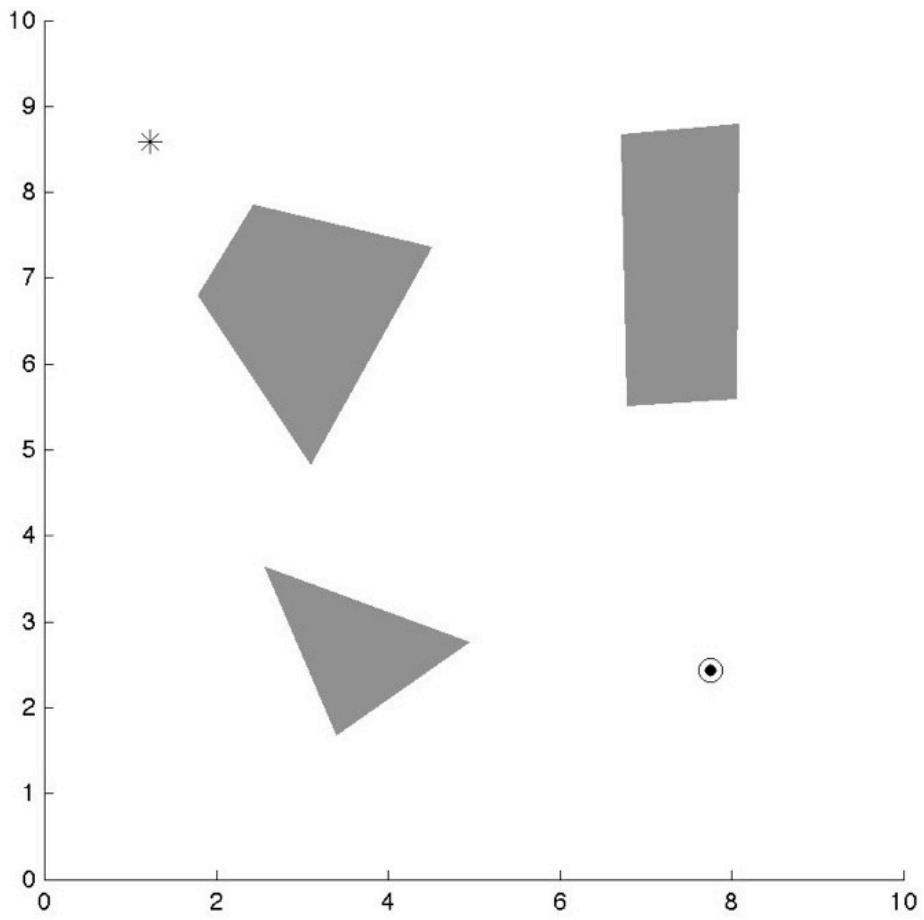

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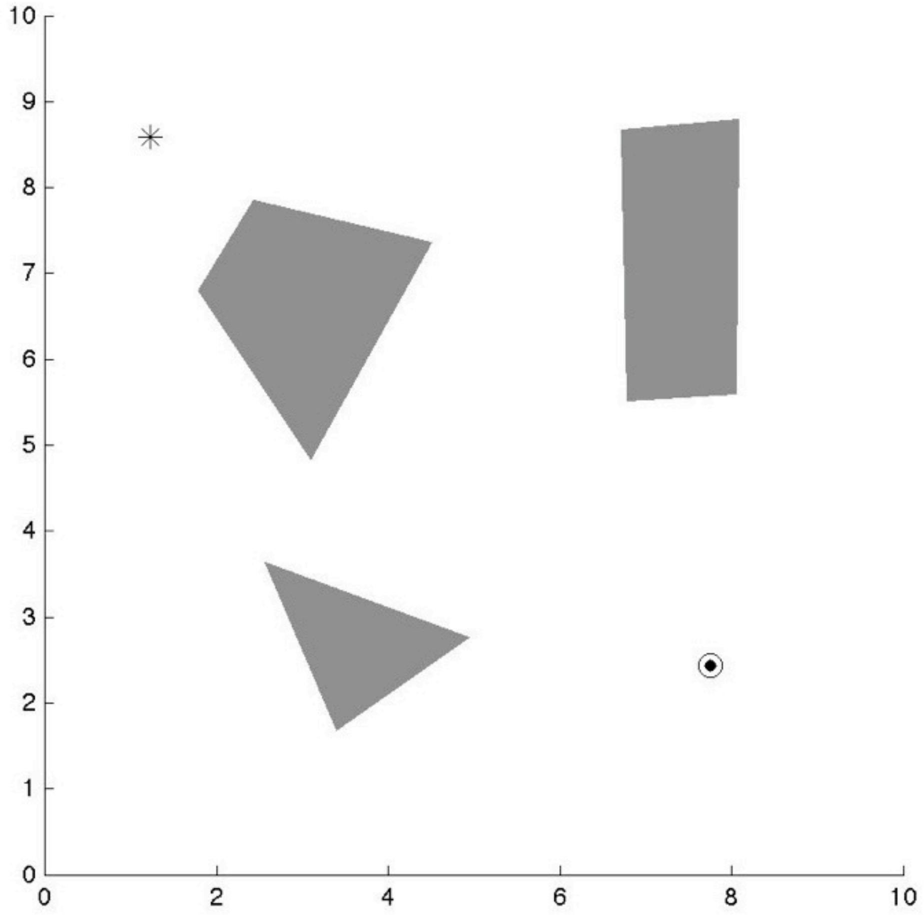

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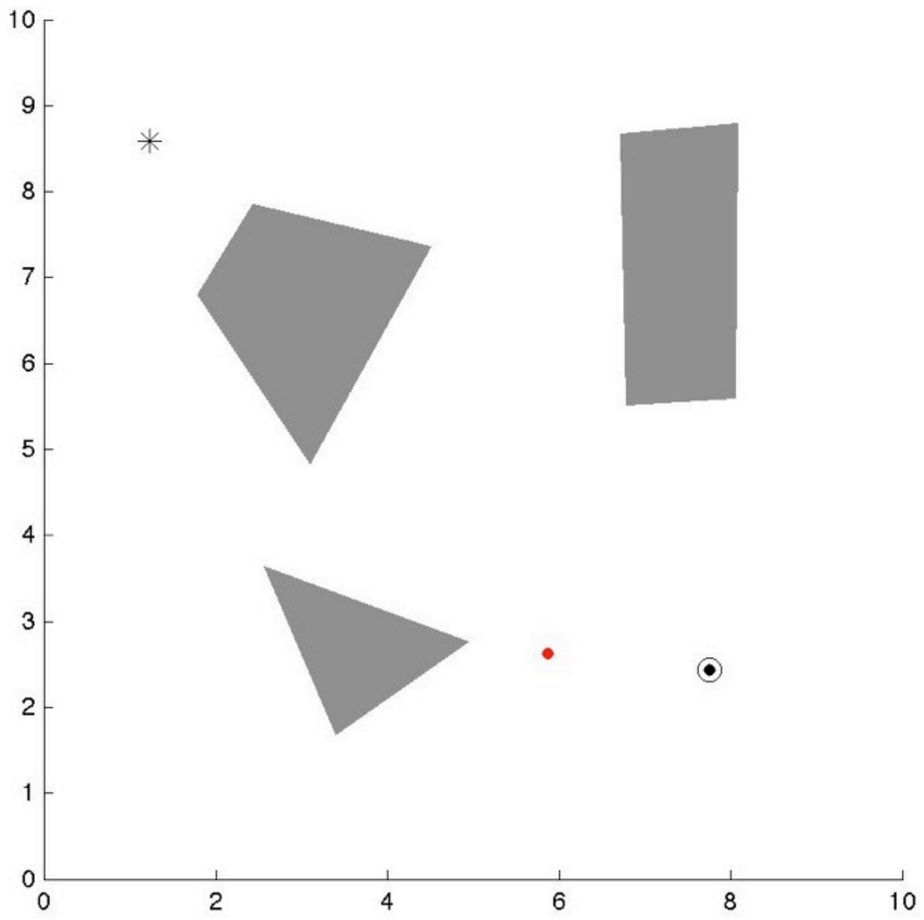

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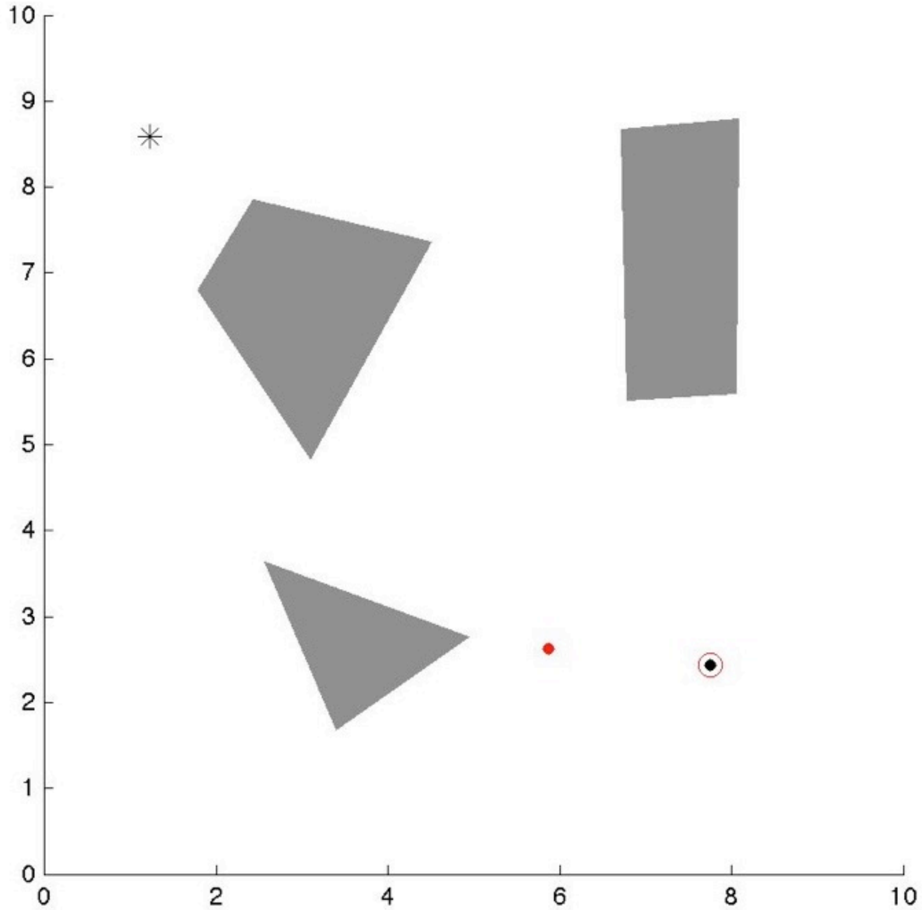

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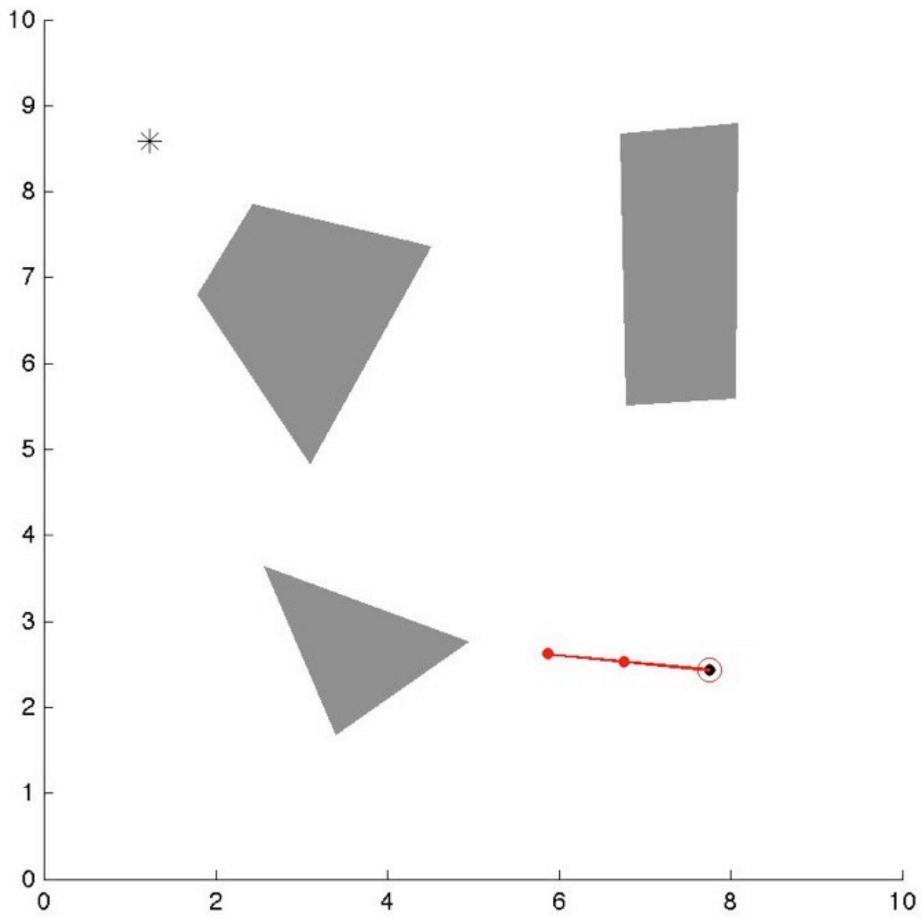

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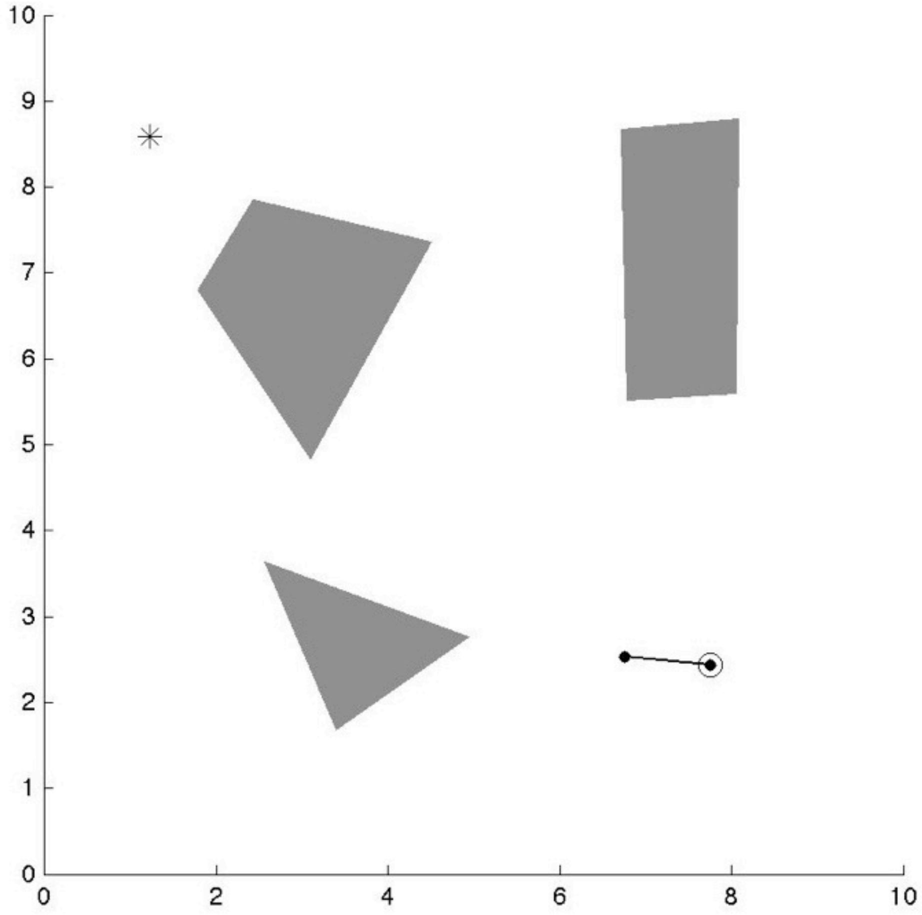

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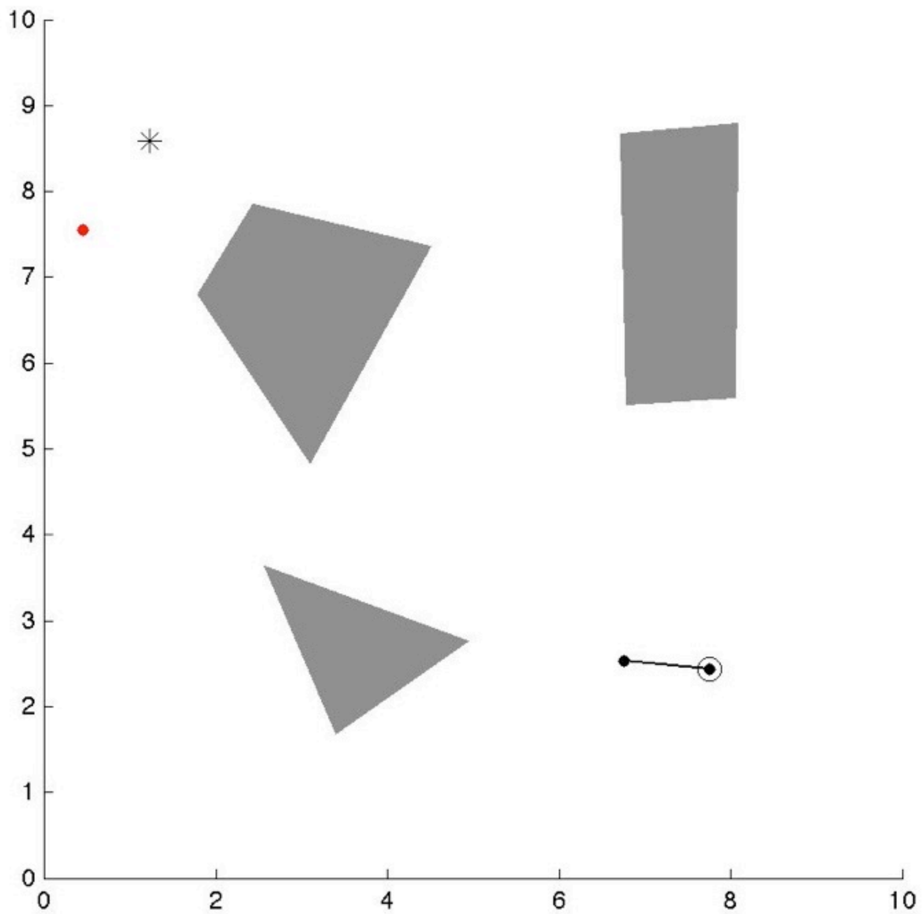

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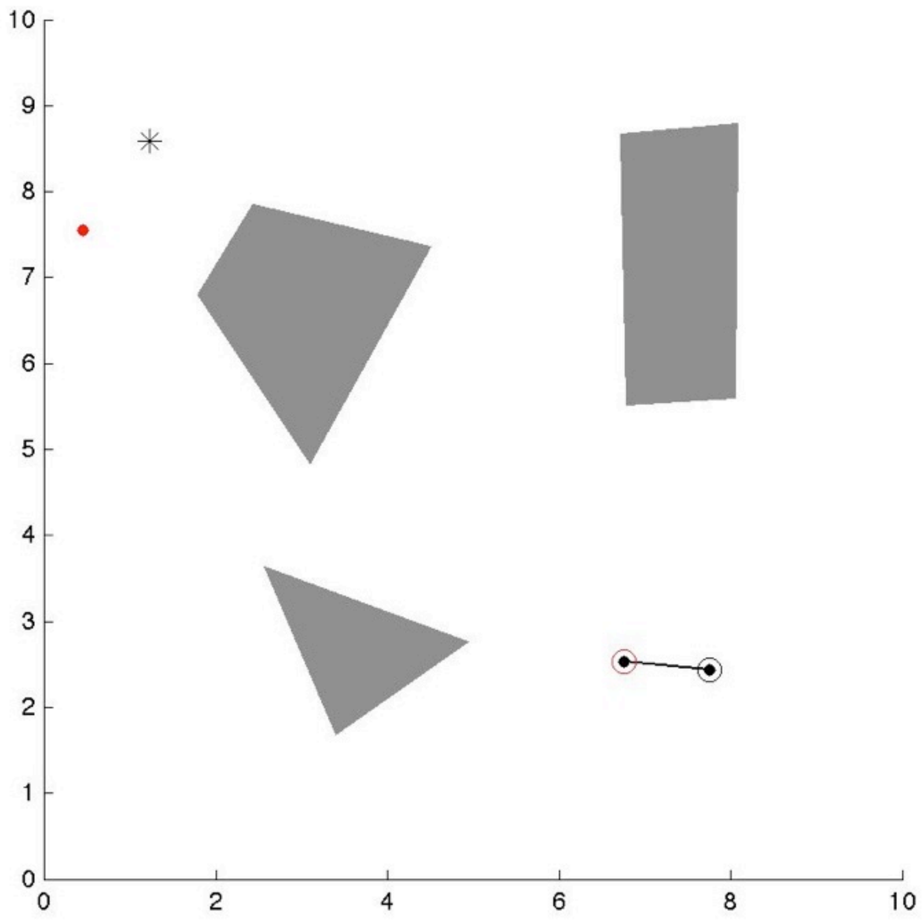

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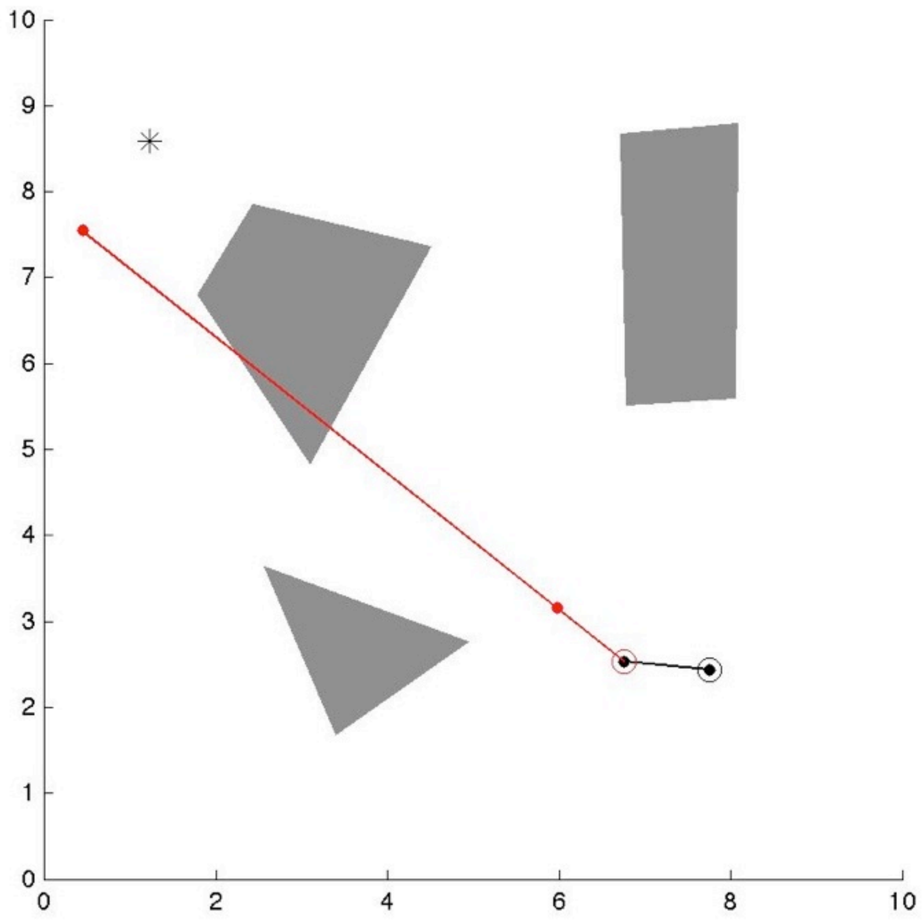

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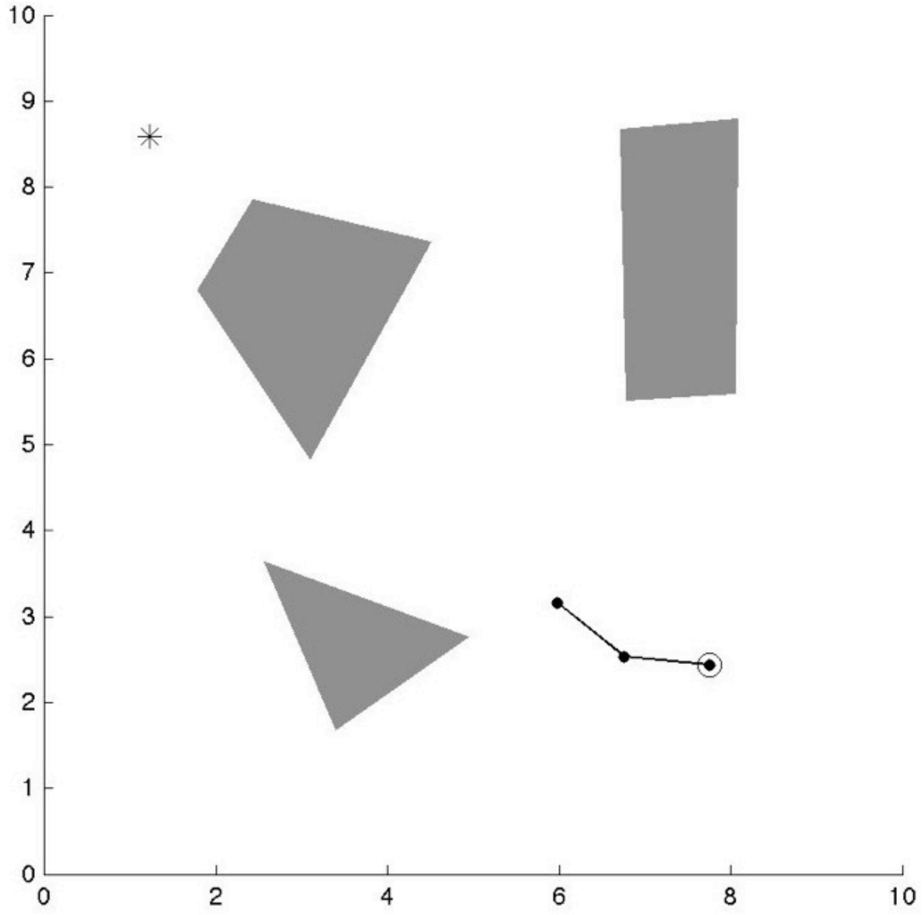

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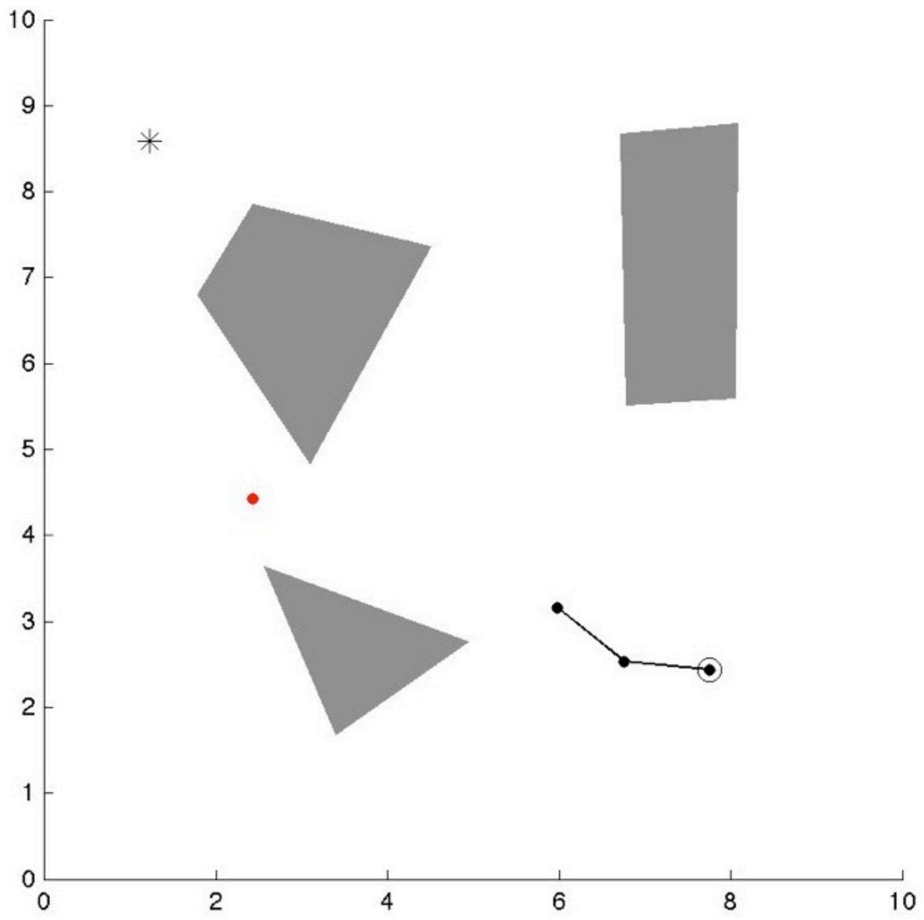

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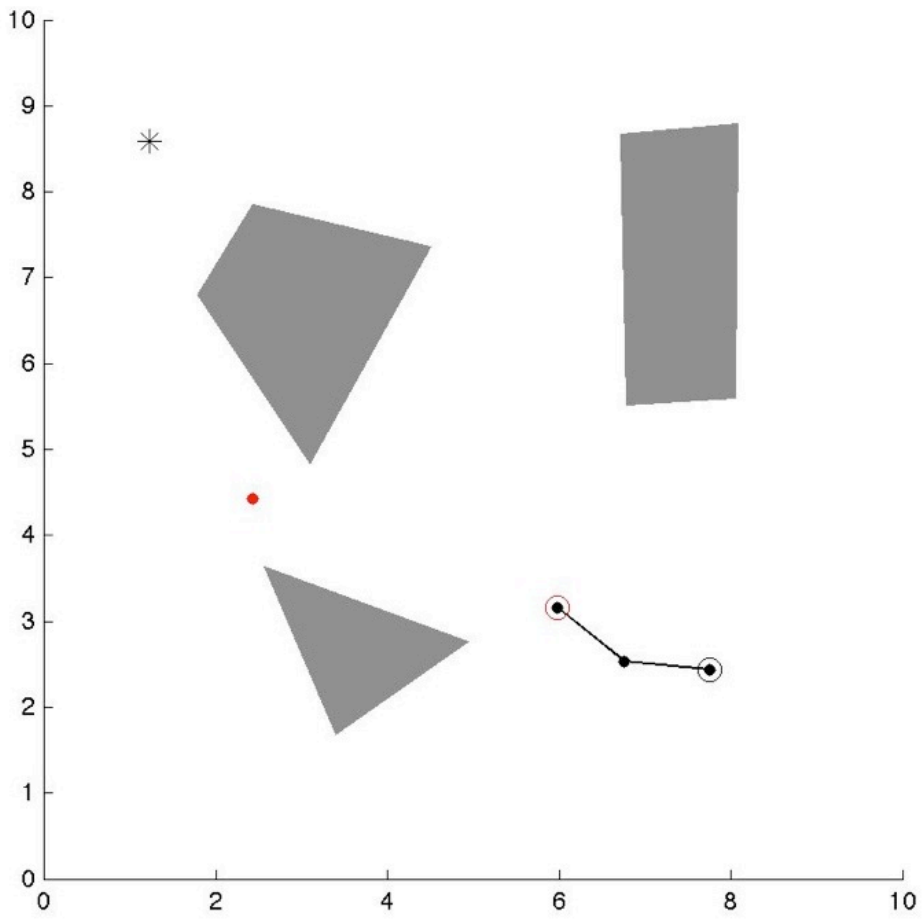

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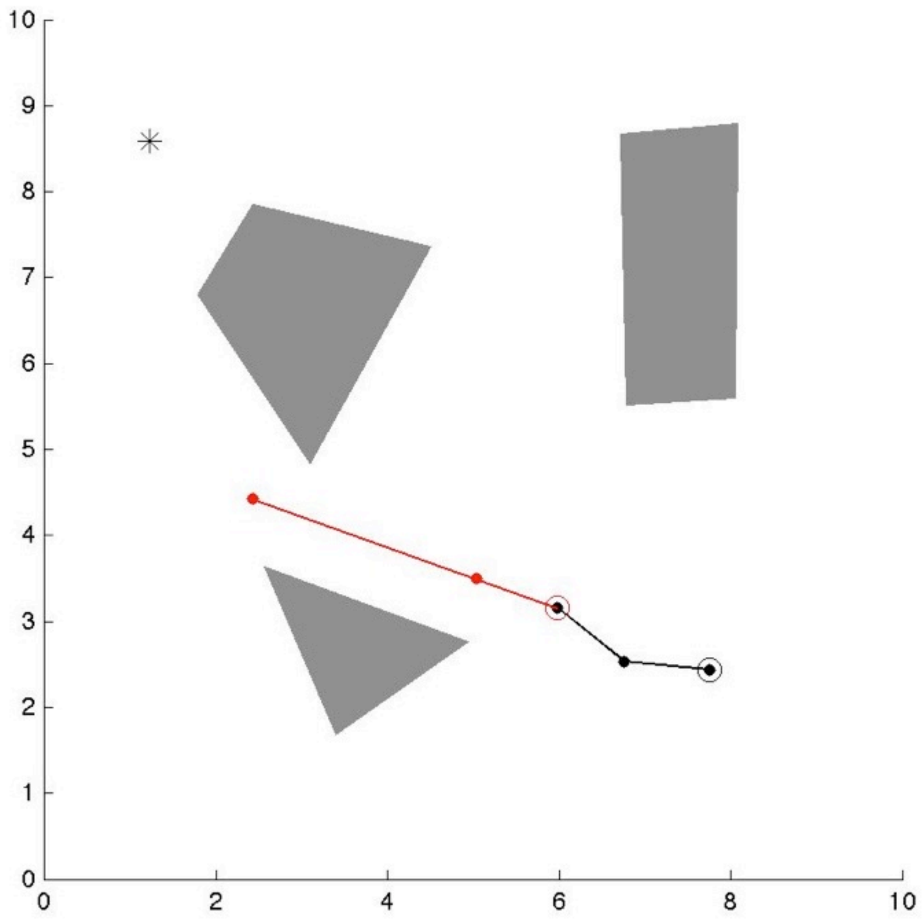

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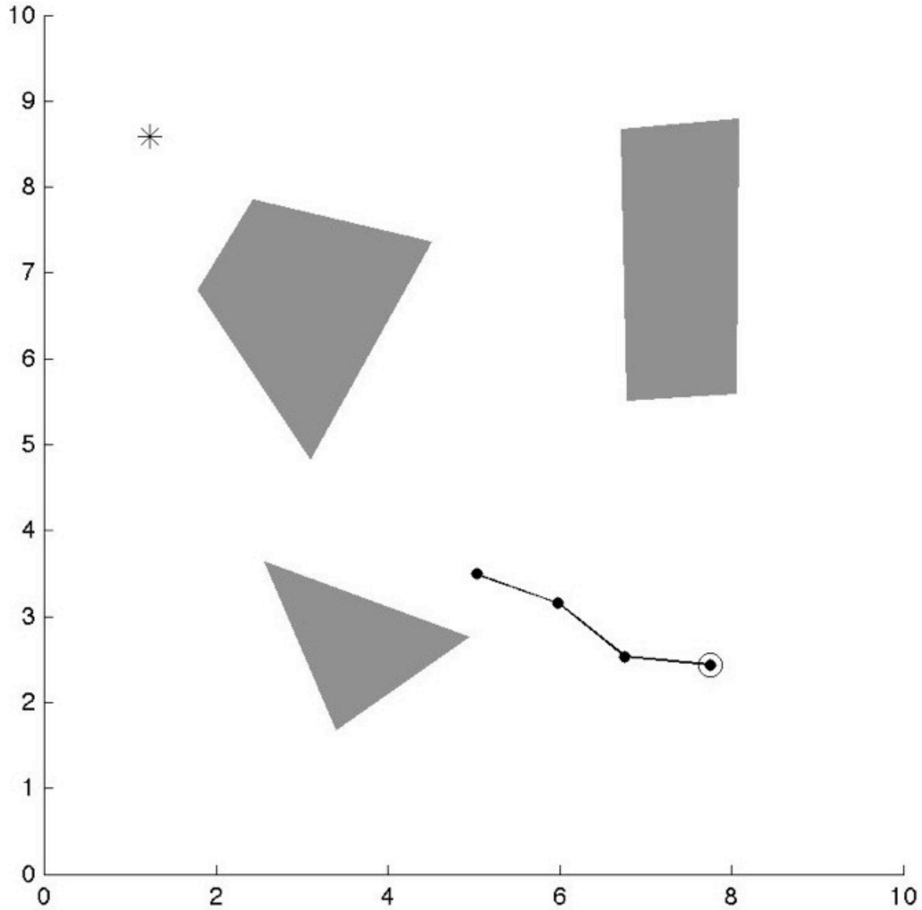

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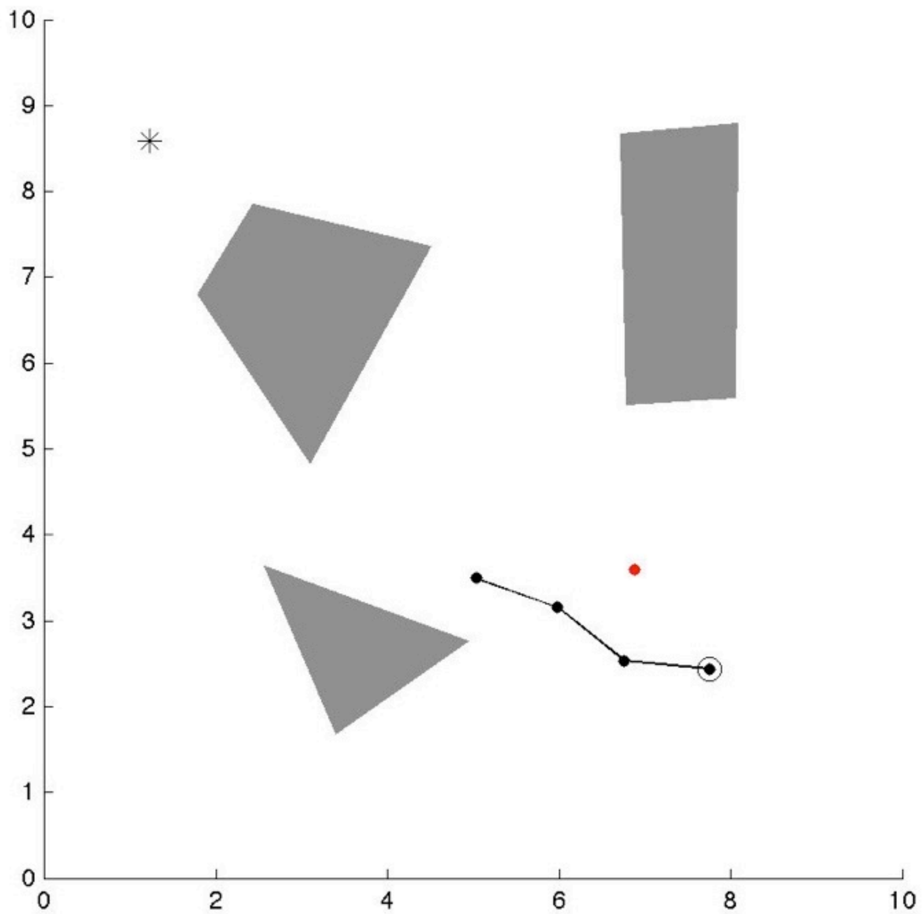

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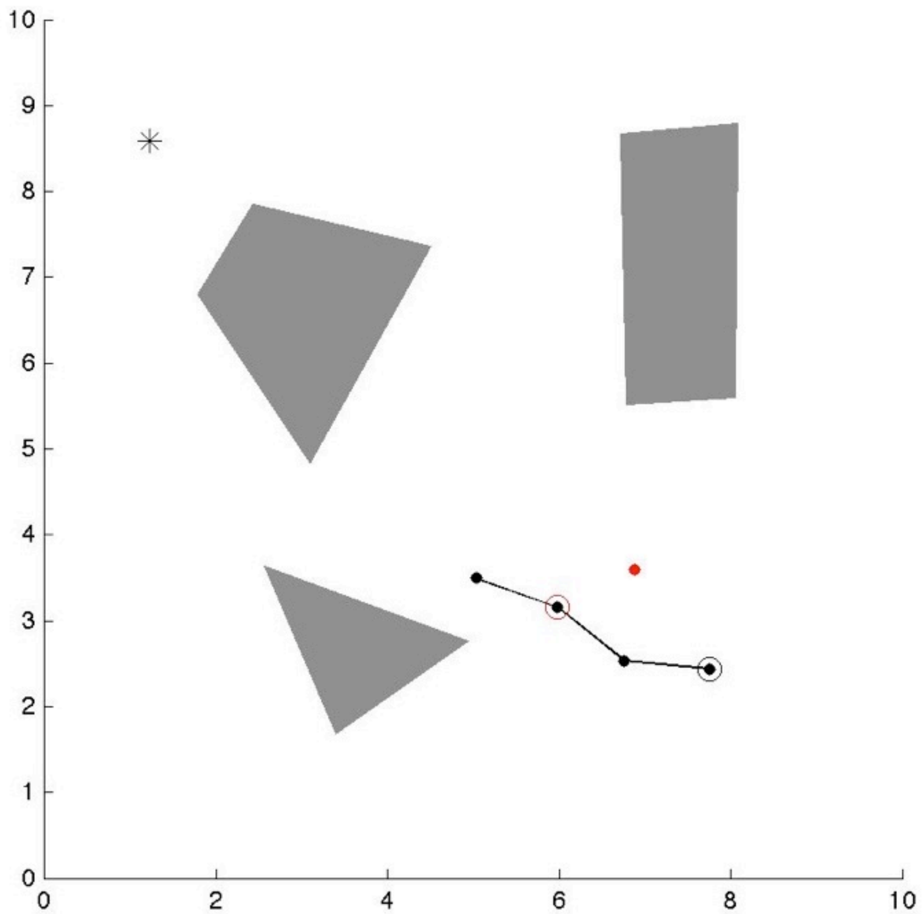

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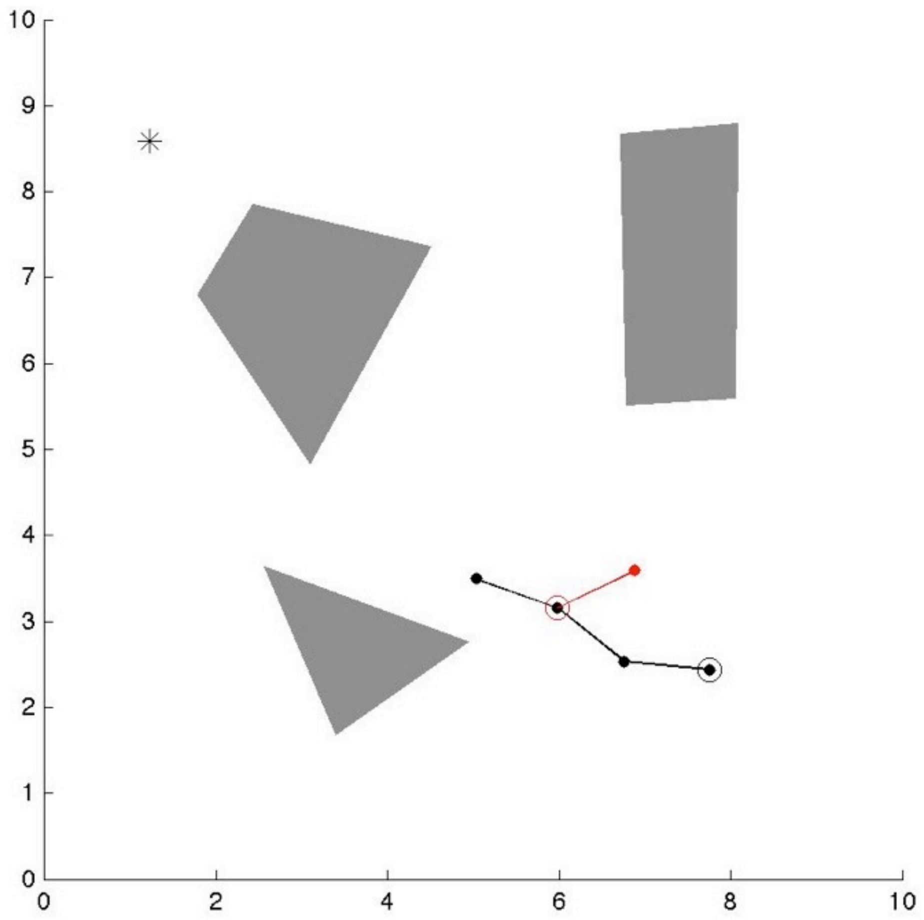

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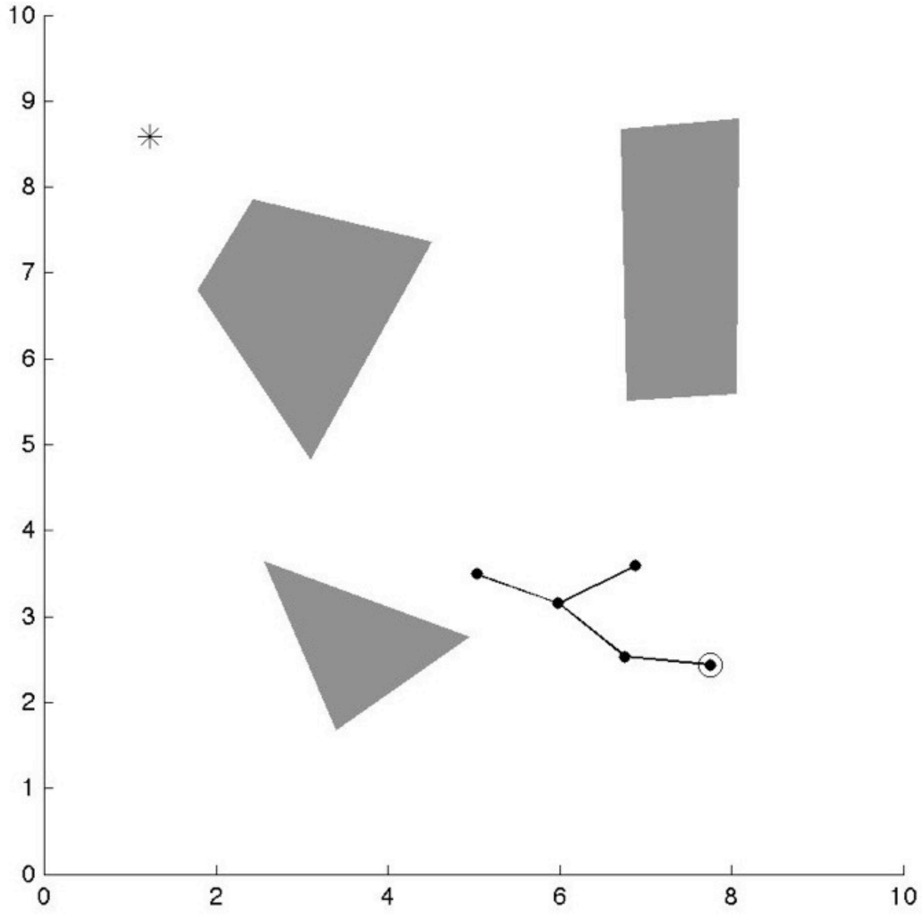

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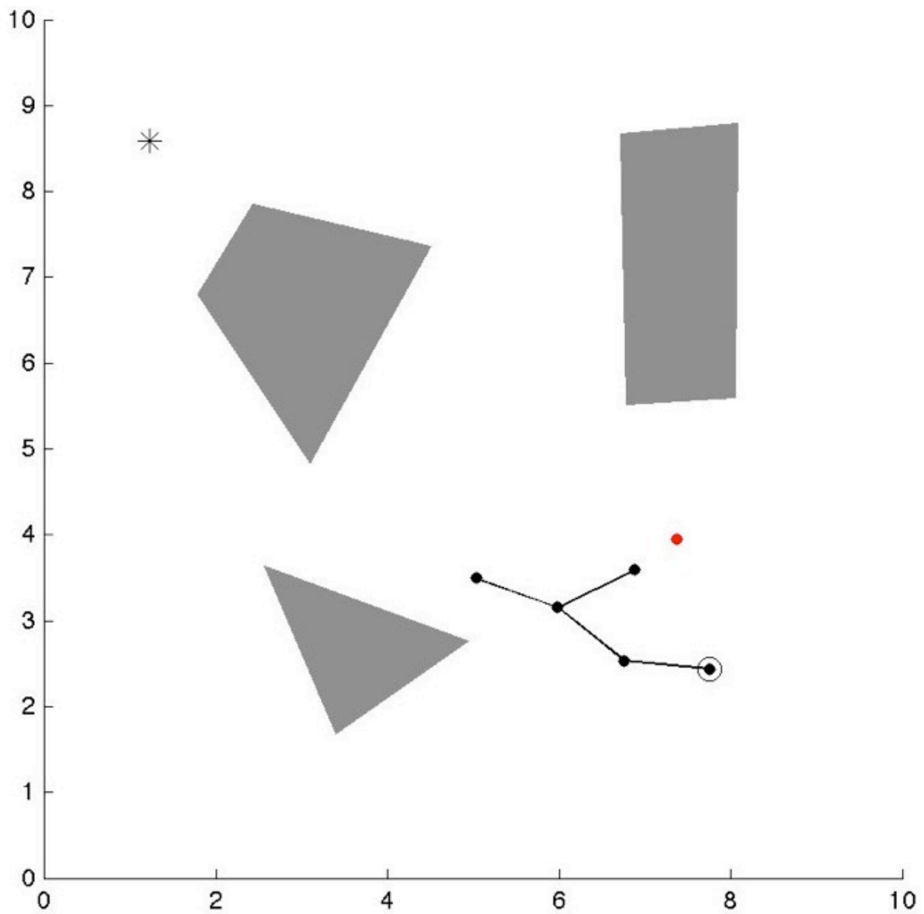

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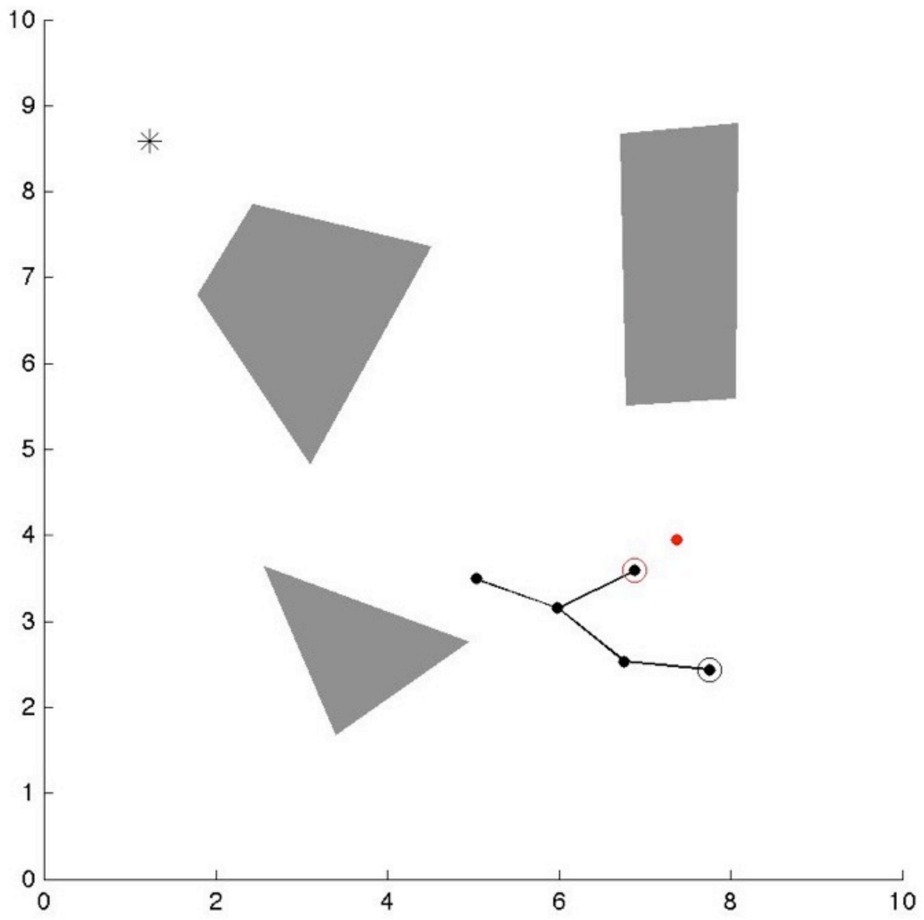

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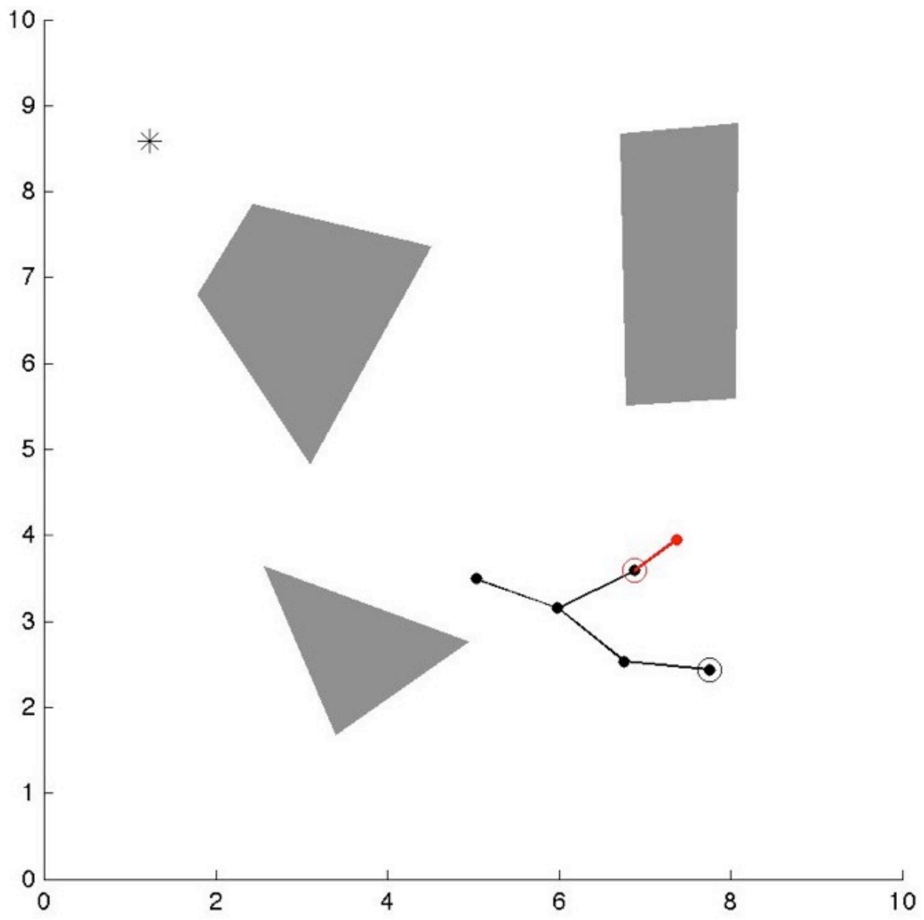

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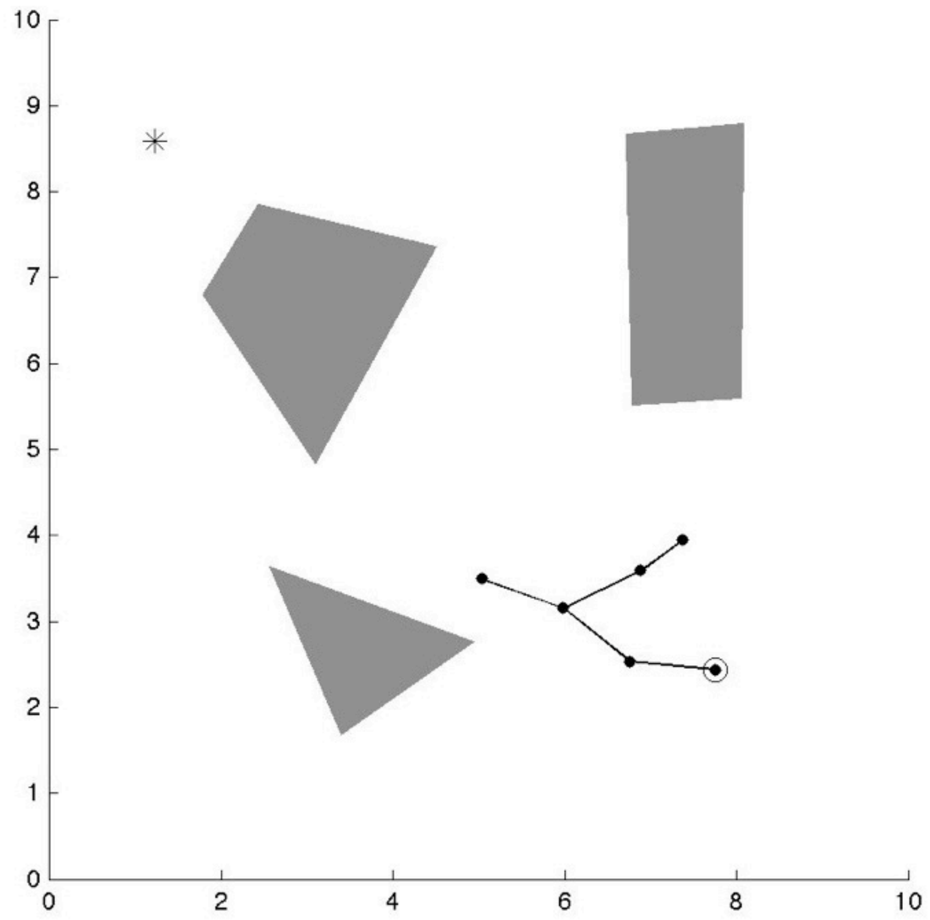

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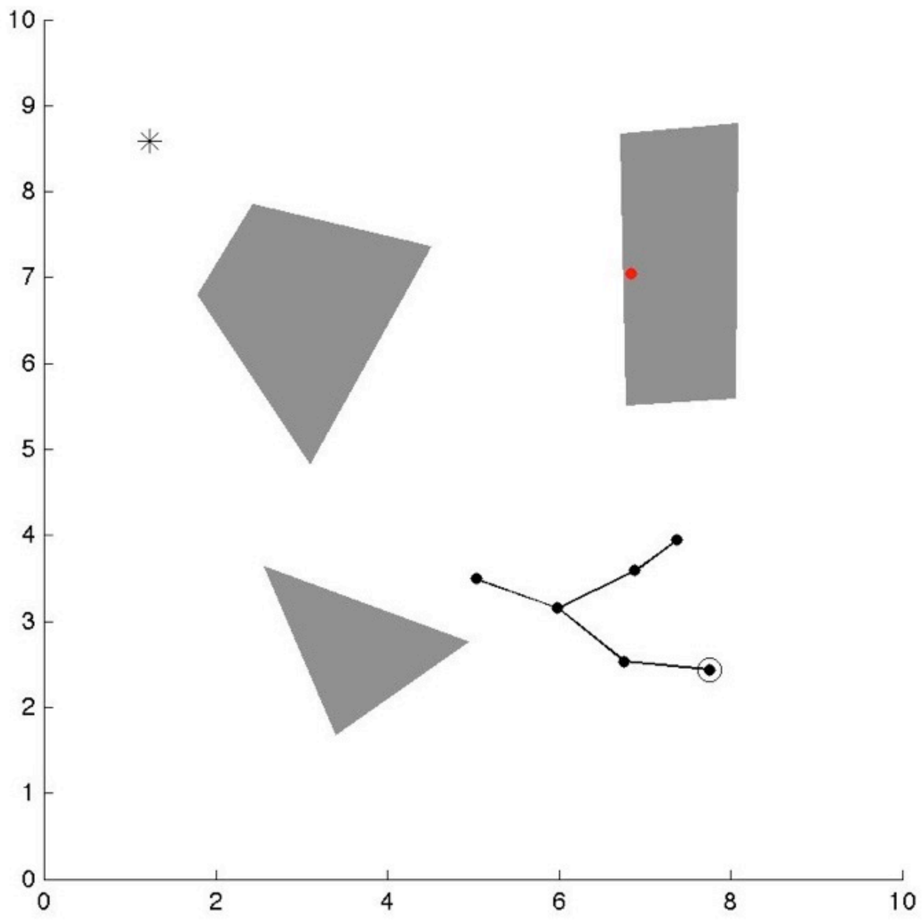

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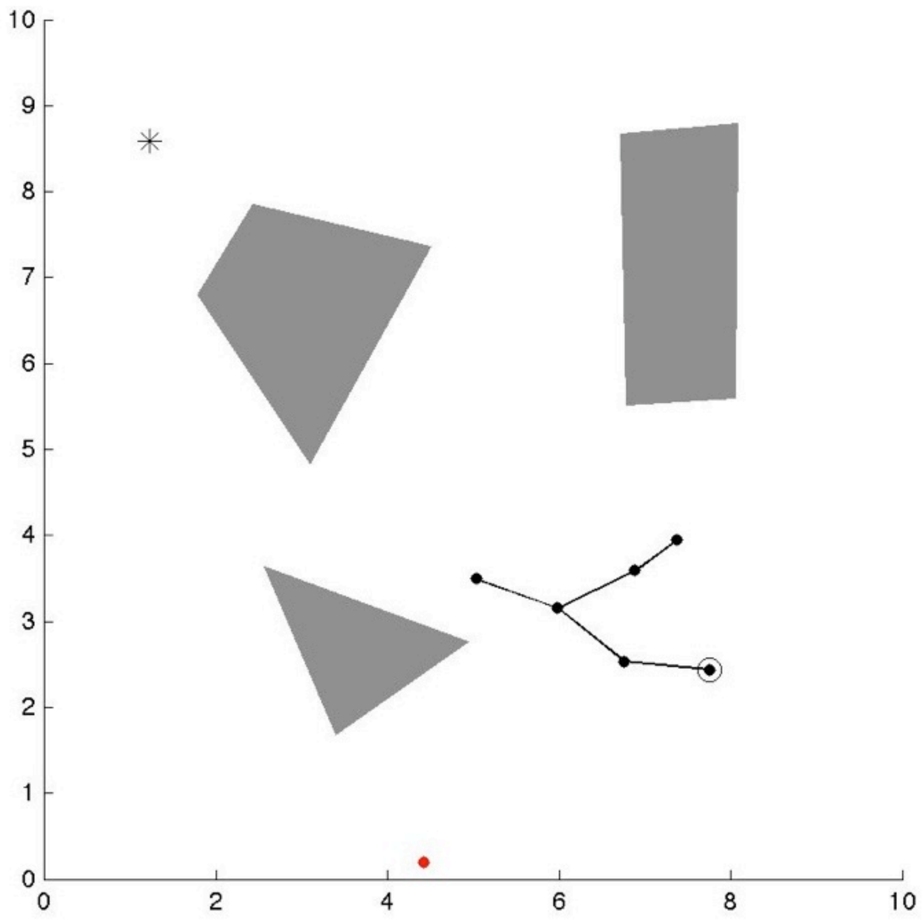

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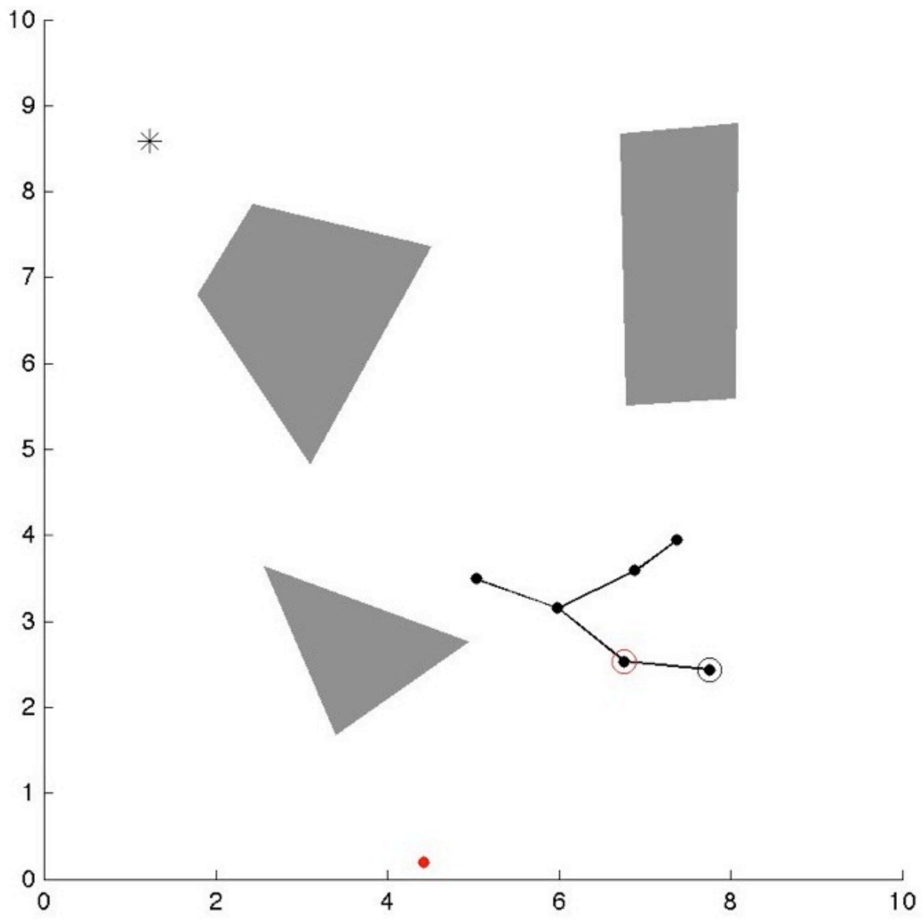

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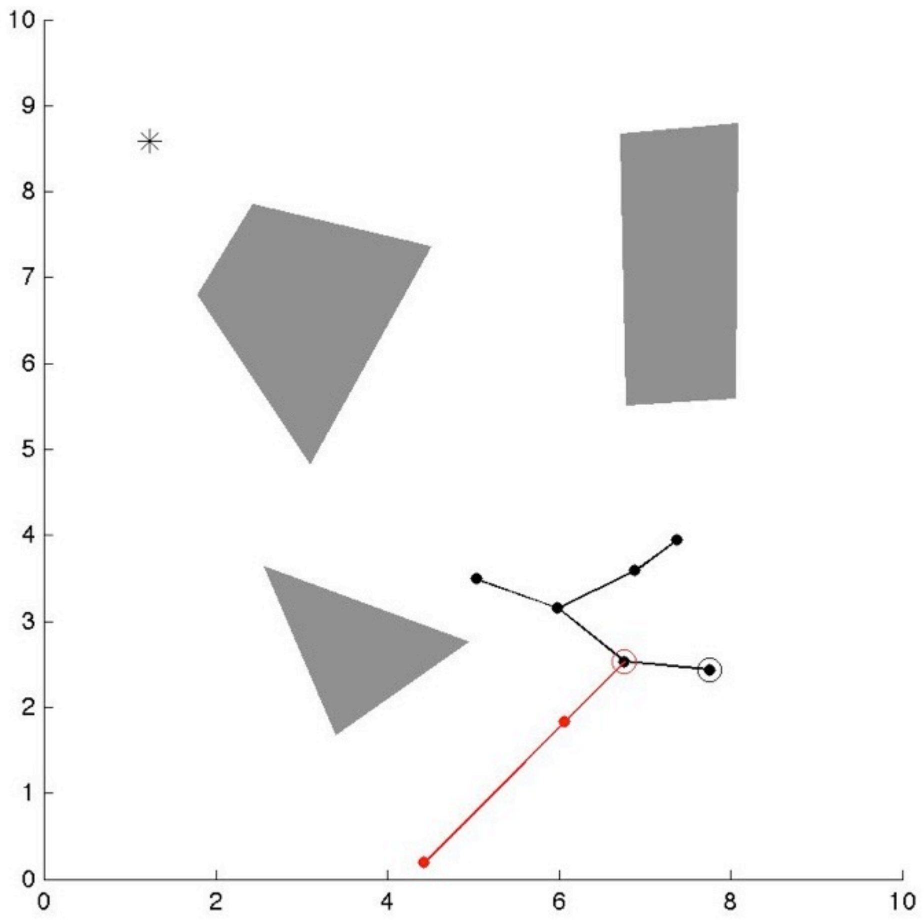

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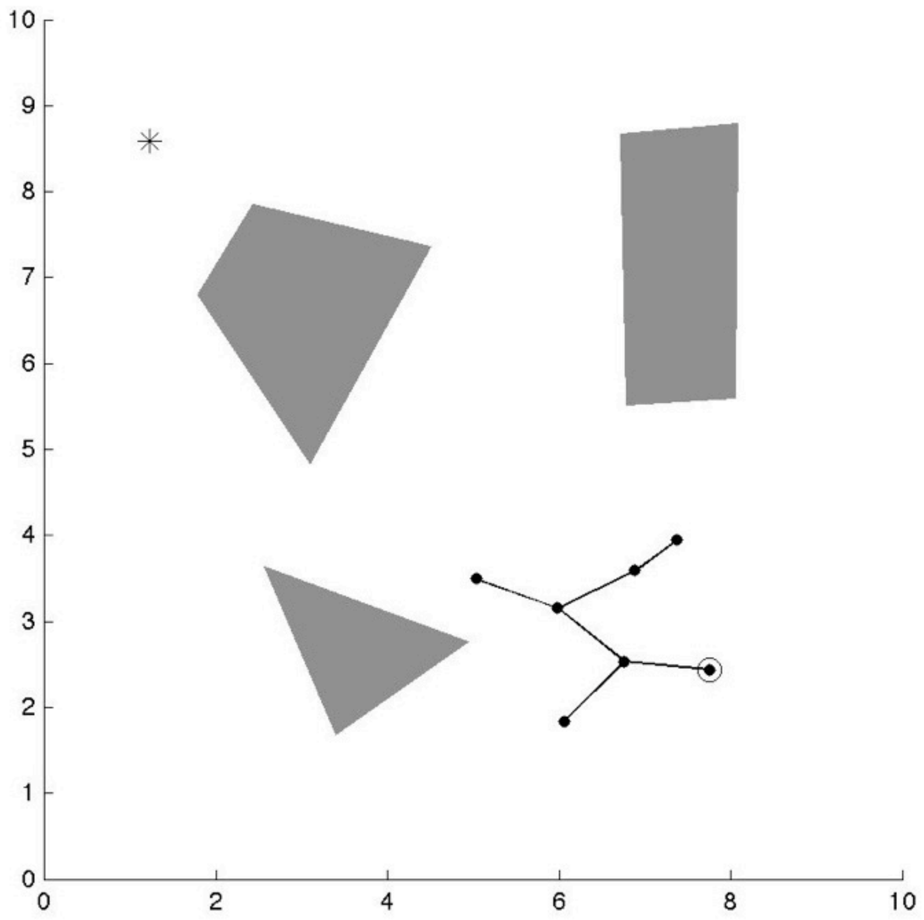

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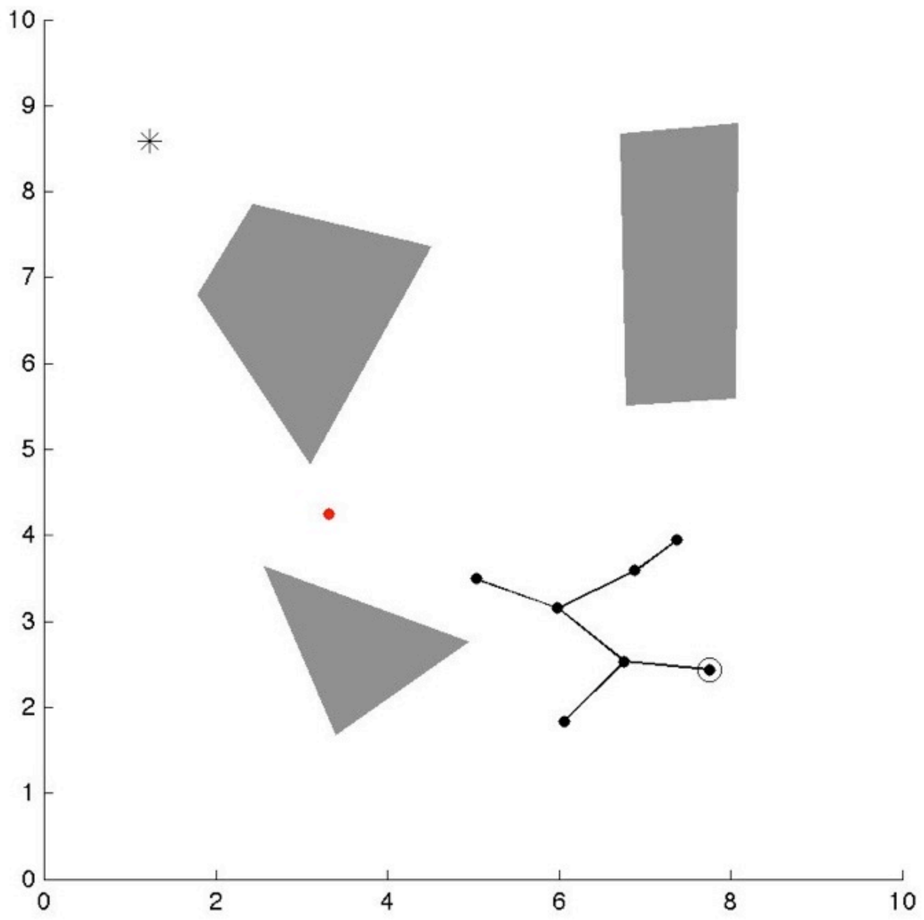

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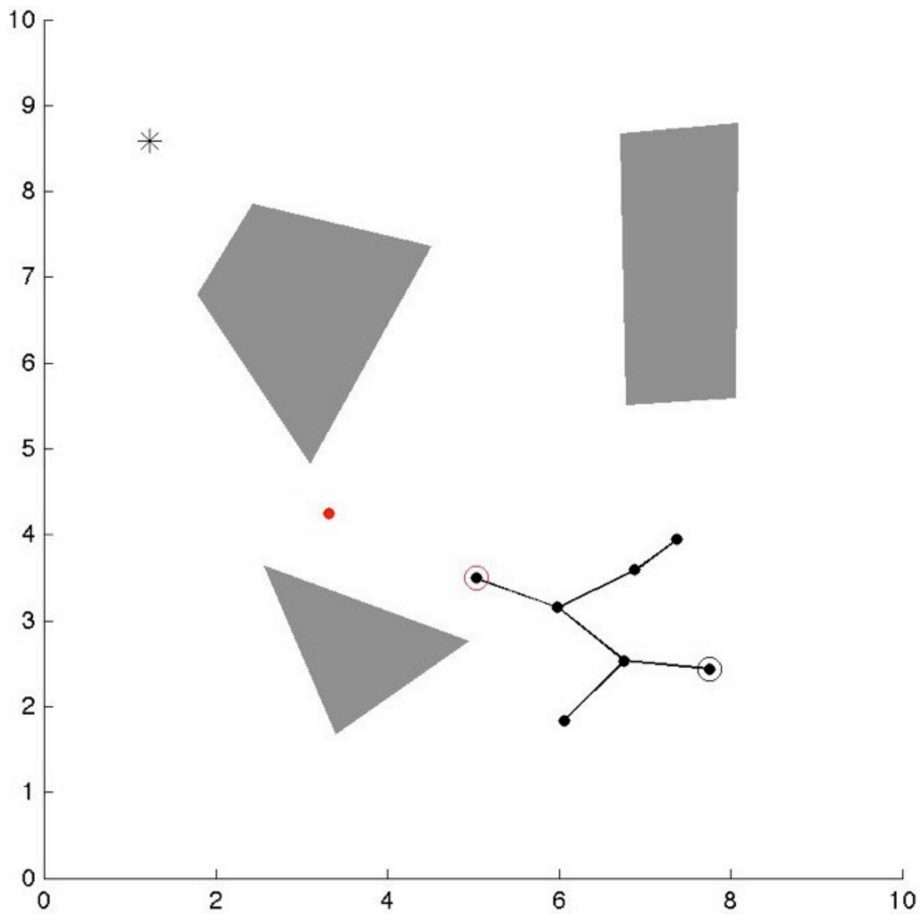

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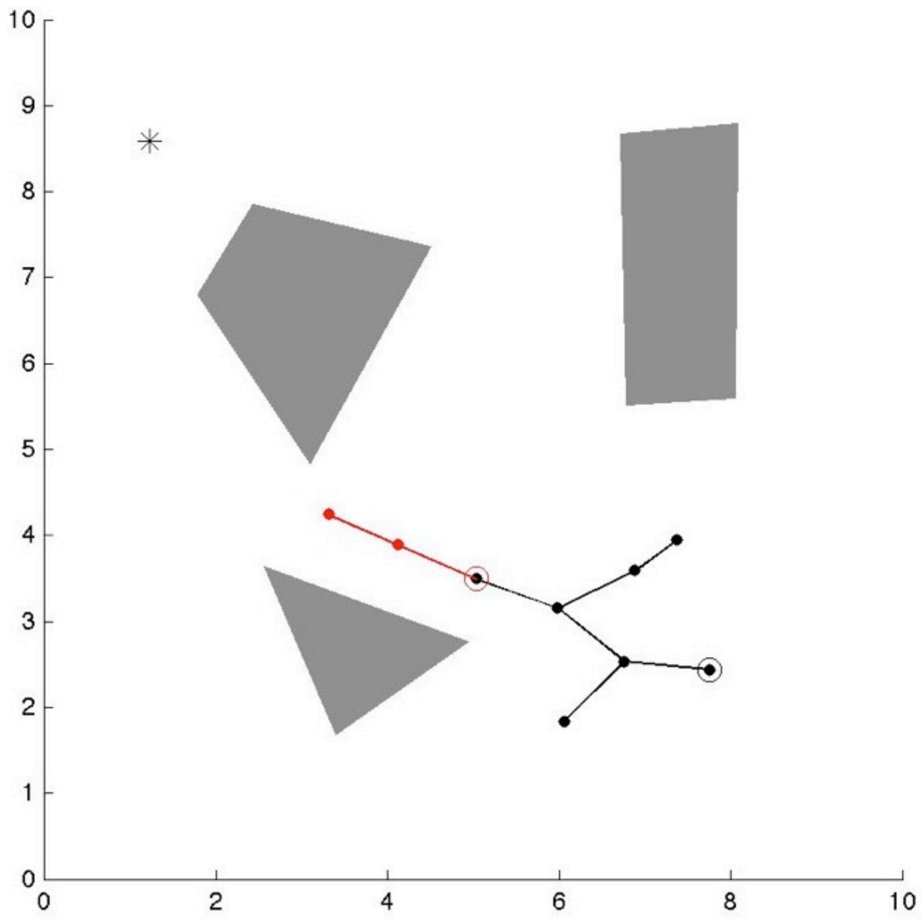

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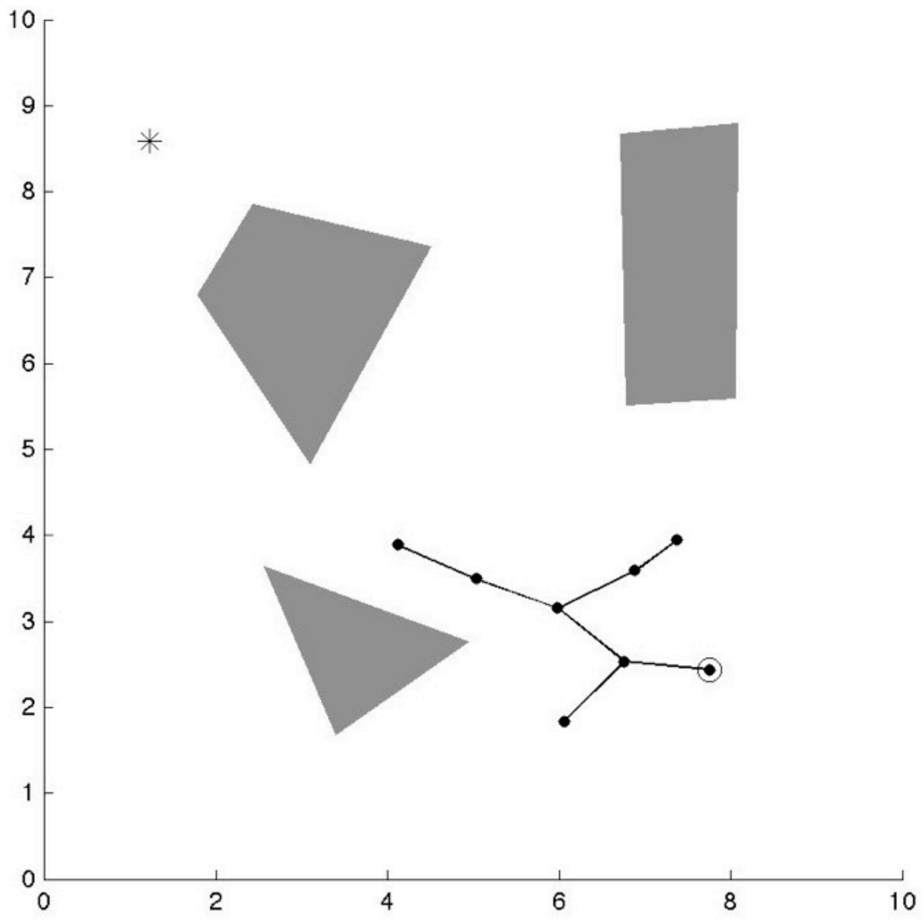

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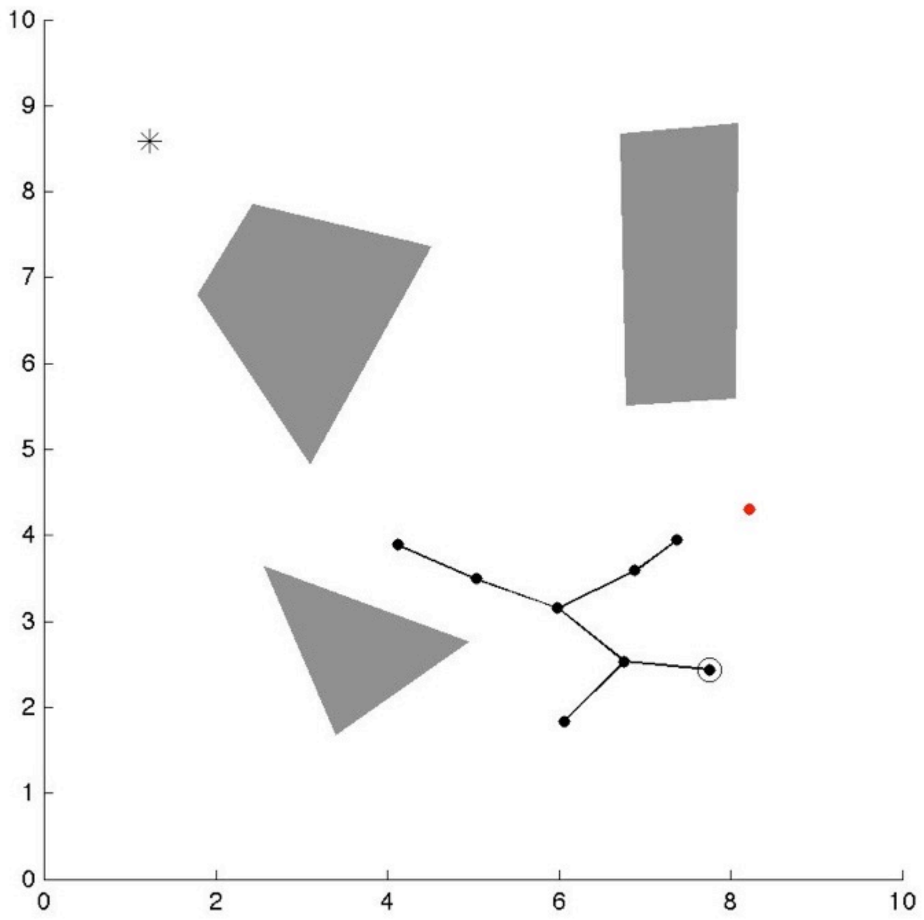

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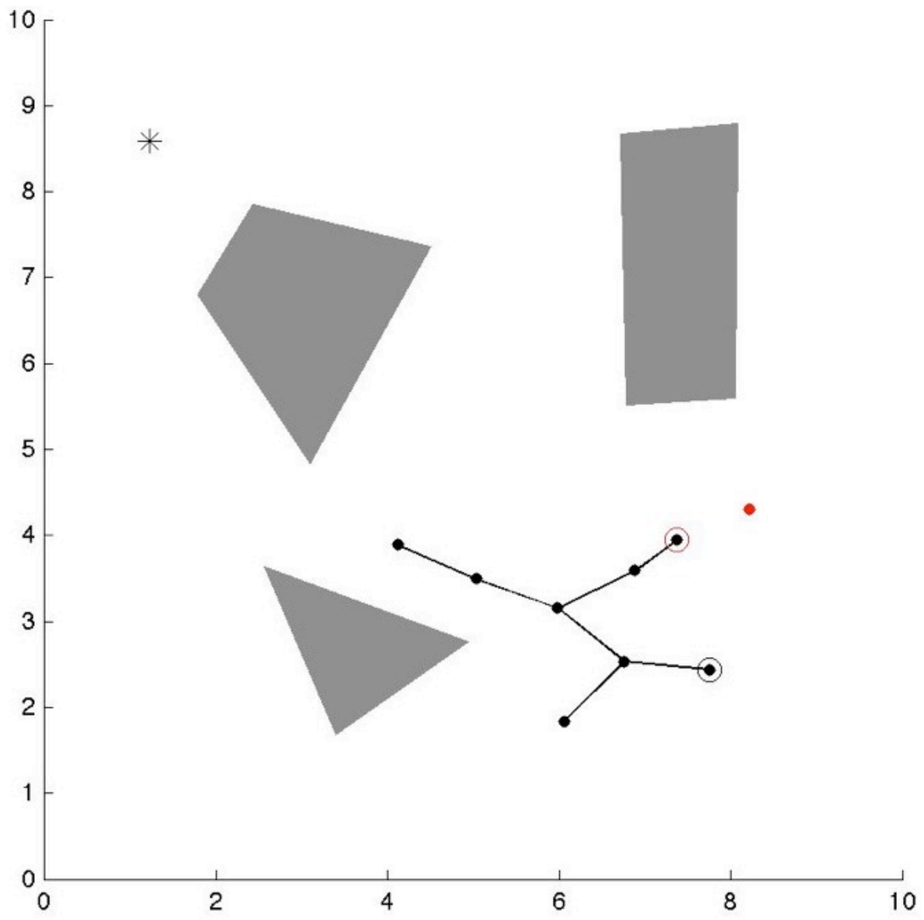

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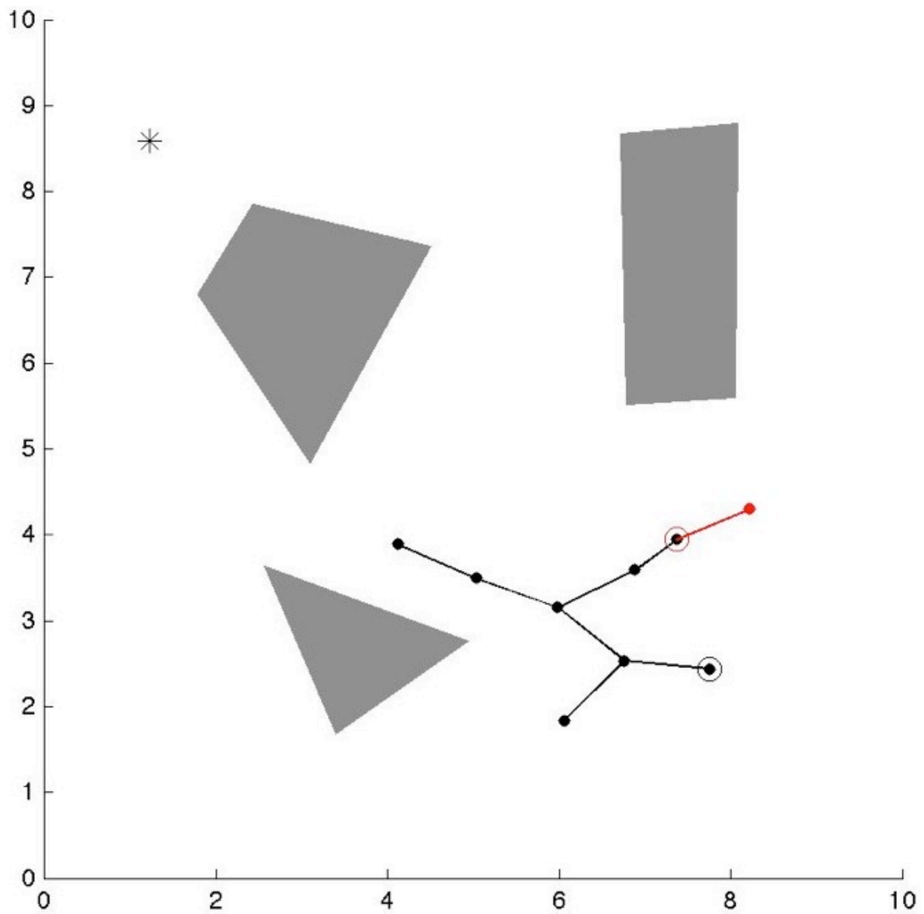

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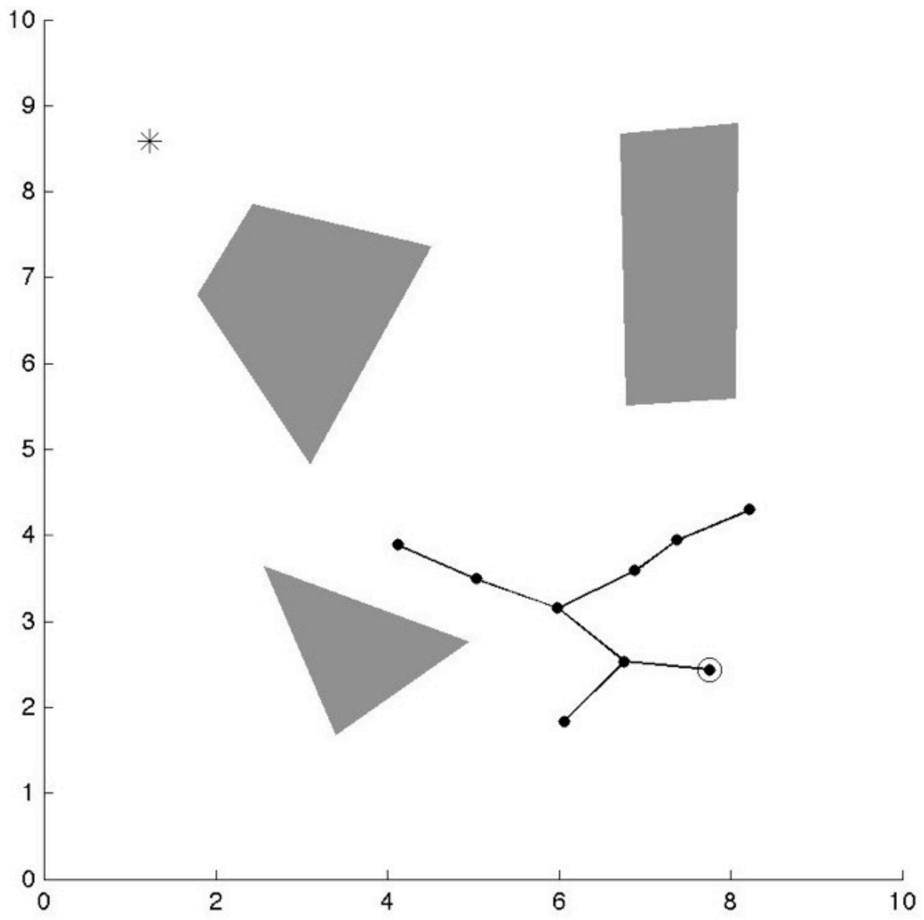

# RRT

$$
\begin{aligned}
& V \leftarrow \{x_{init}\}; \quad E \leftarrow \emptyset \\
& \textbf{for } i = 1 \text{ to } N \\
& \quad G \leftarrow (V, E) \\
& \quad x_{rand} \leftarrow RandomSample() \\
& \quad x_{nearest} \leftarrow Nearest(G, x_{rand}) \\
& \quad x_{new} \leftarrow Steer(x_{nearest}, x_{rand}) \\
& \quad \textbf{if } ObstacleFree(x_{nearest}, x_{new}) \\
& \quad\quad V \leftarrow V \cup \{x_{new}\} \\
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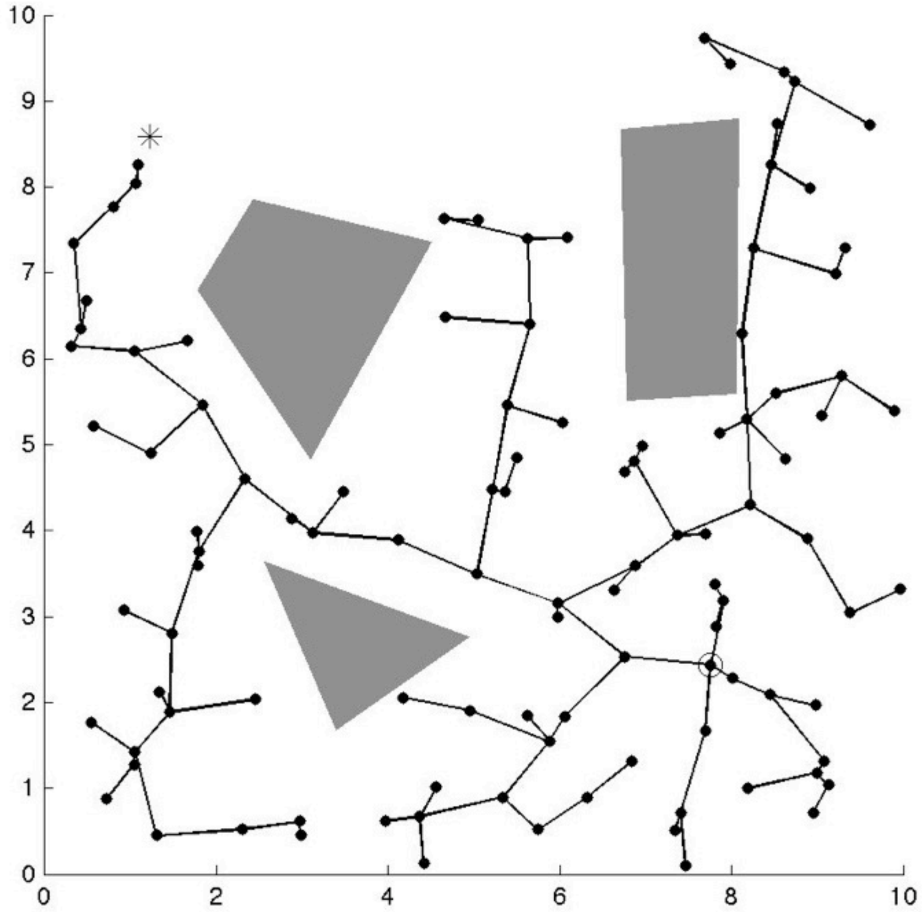
\end{aligned}
$$



# RRT - Bias to Goal

$V \leftarrow \{x_{init}\}; \quad E \leftarrow \emptyset$

**for** $i = 1$ to $N$

$\quad G \leftarrow (V, E)$

$\quad$**with** probability $p$

$\quad\quad x_{rand} \leftarrow RandomSample()$

$\quad$**otherwise**

$\quad\quad x_{rand} \leftarrow x_{goal}$

$\quad x_{nearest} \leftarrow Nearest(G, x_{rand})$

$\quad x_{new} \leftarrow Steer(x_{nearest}, x_{rand})$

$\quad$**if** $ObstacleFree(x_{nearest}, x_{new})$

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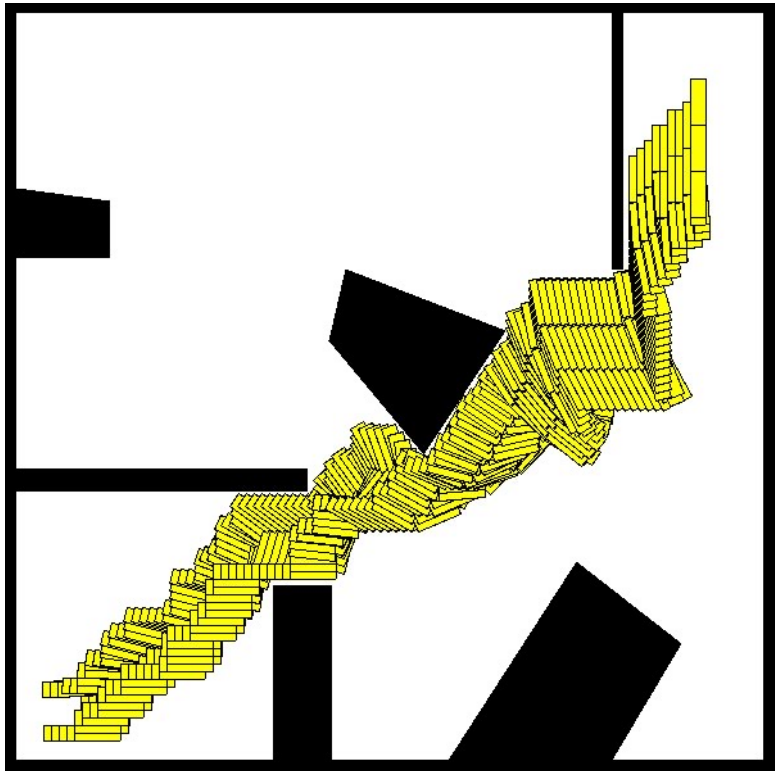
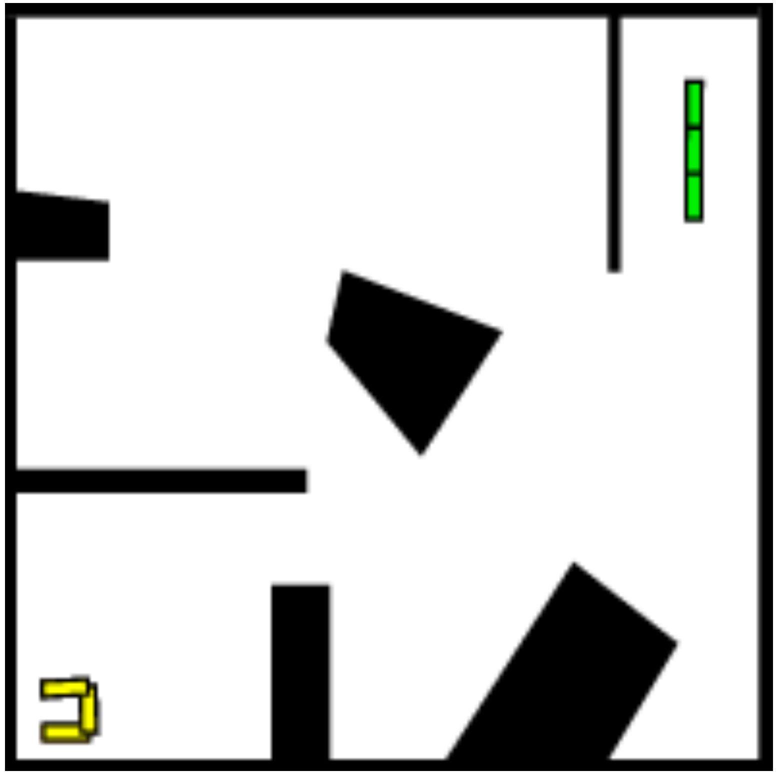
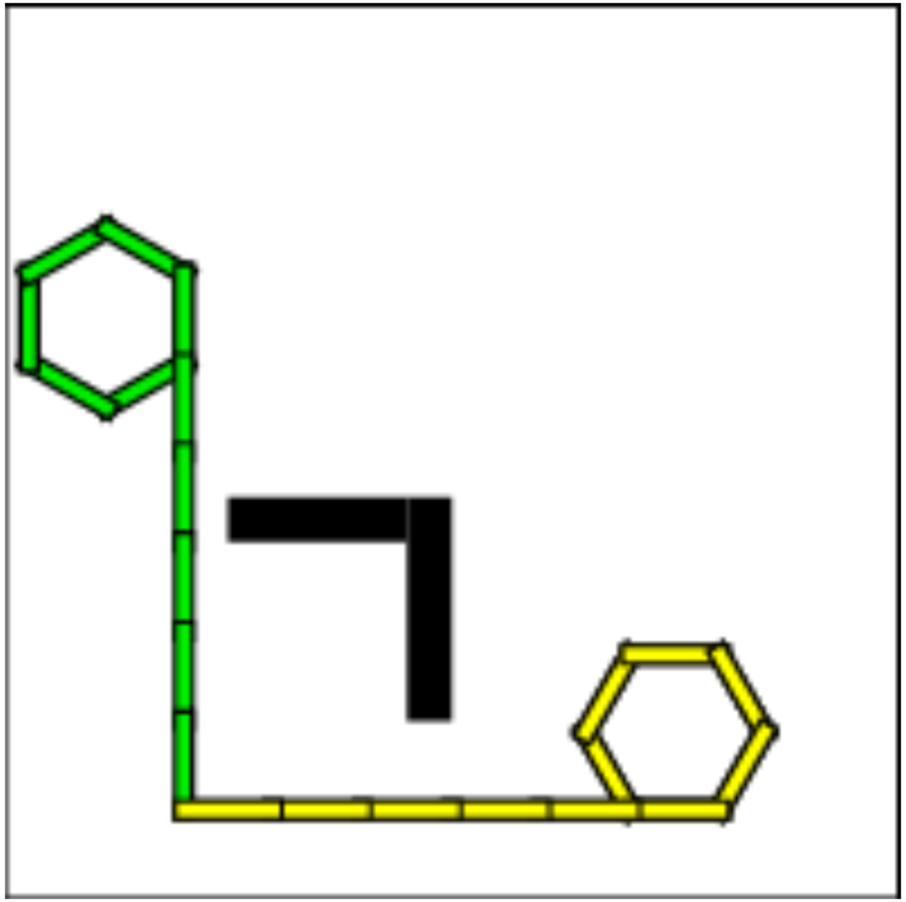
# Articulated Robot

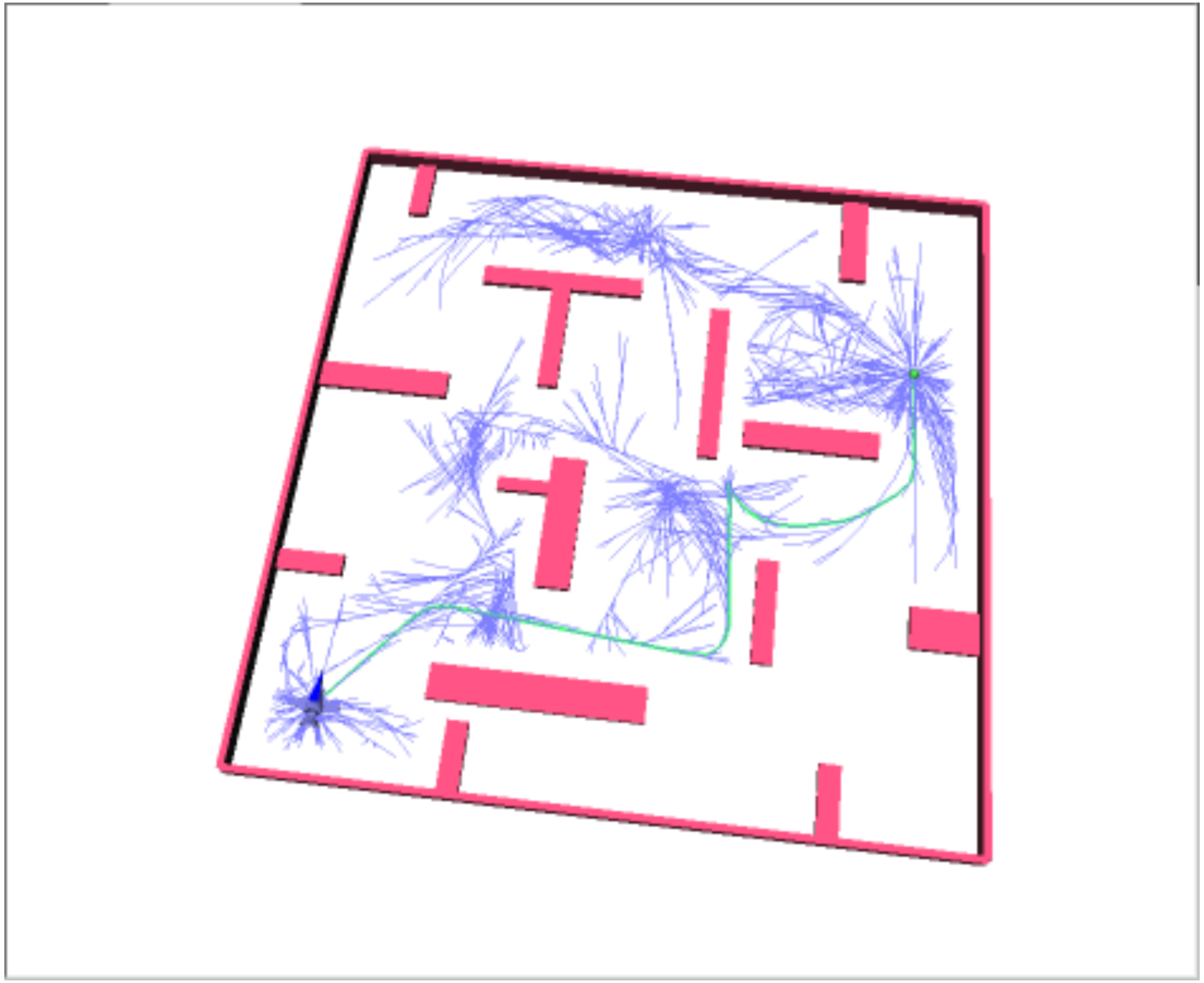




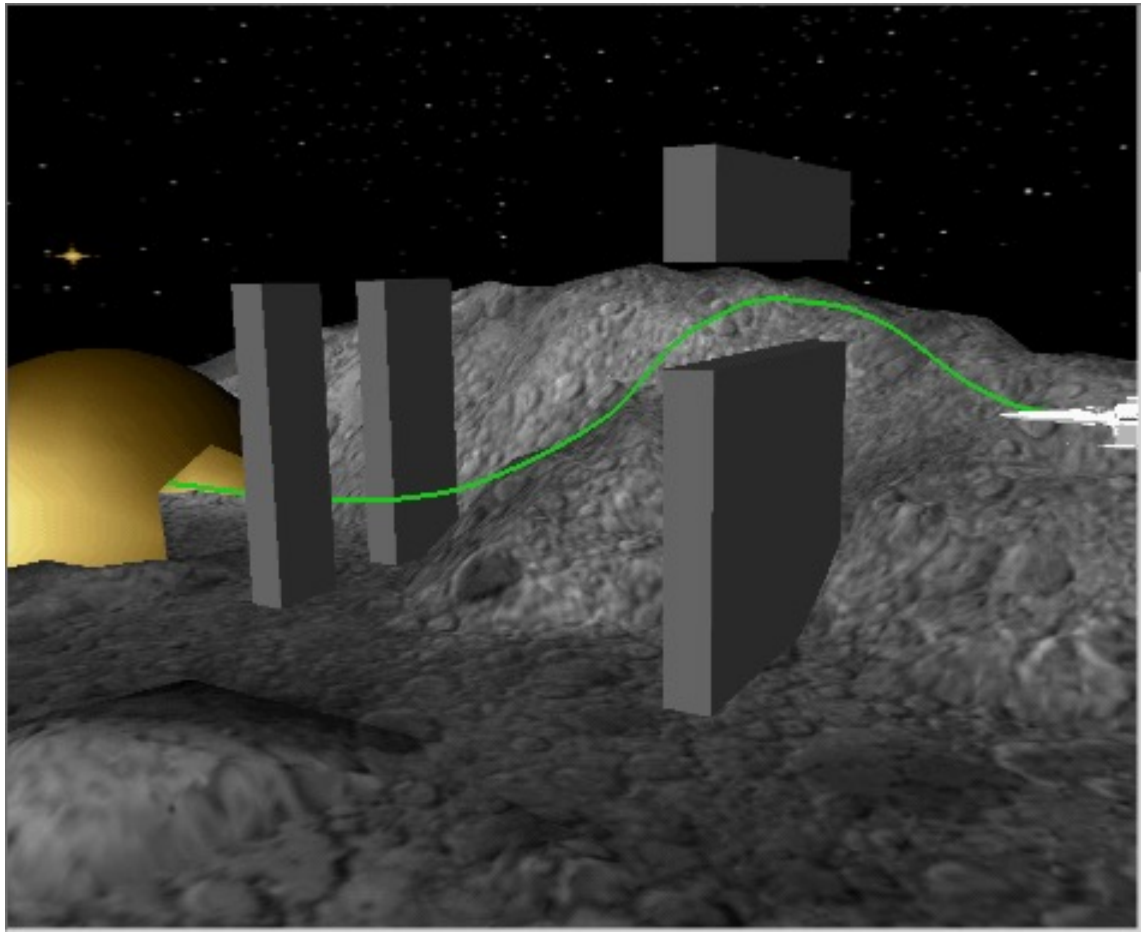
# Highly Articulated Robot



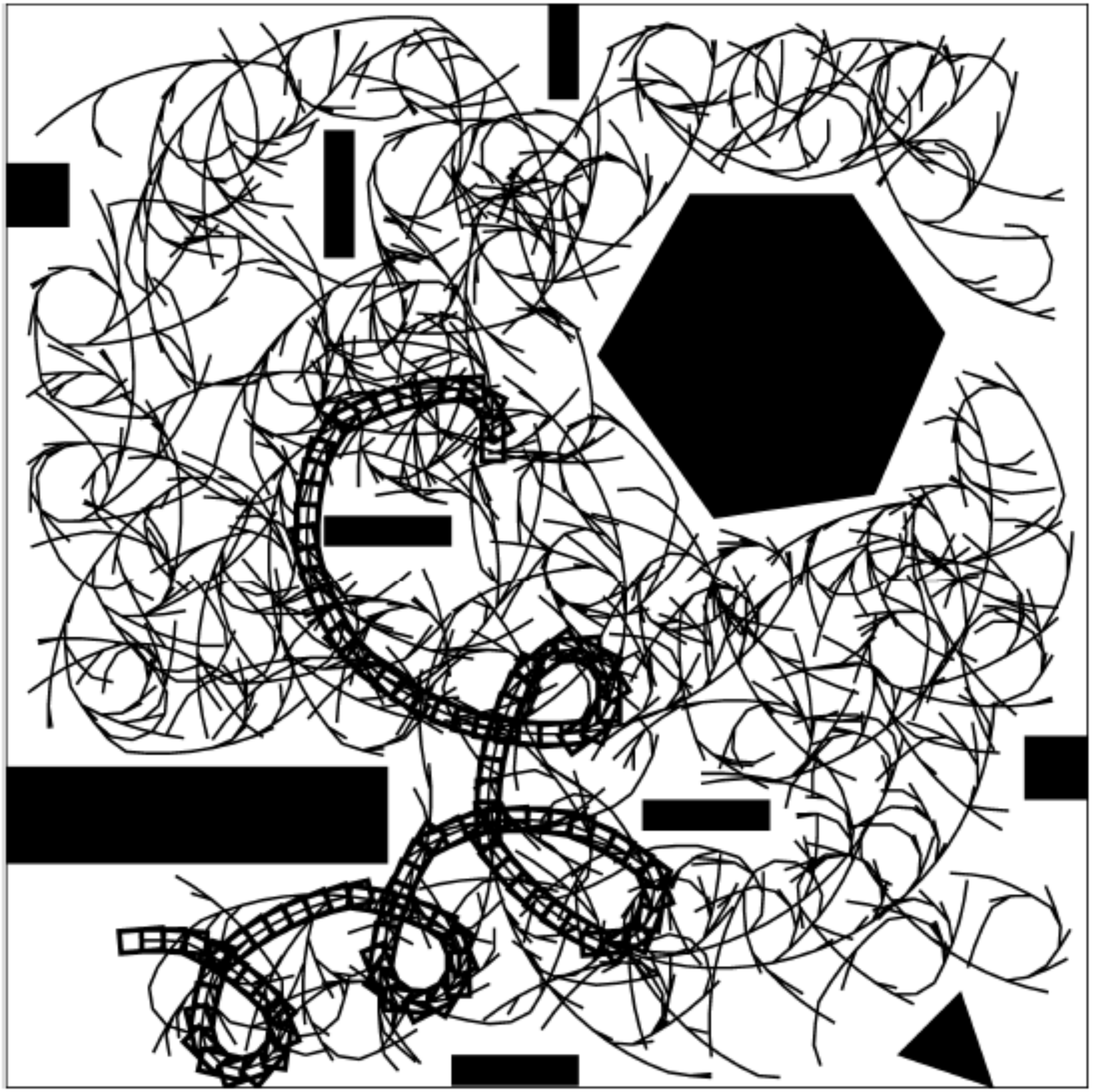
# Hovercraft with 2 Thusters



# Out of This World Demo



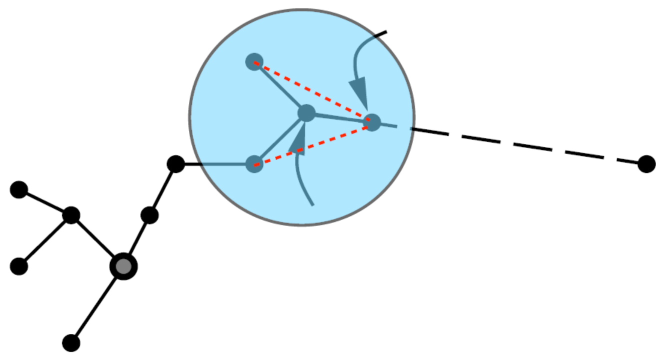
# Left-turn only forward car



# Rapidly-Exploring Random Tree (RRT)

- Advantages of RRT: very fast, works well for dynamic environments
- Disadvantages: Not optimal
    - in fact, it has been proven by Karaman & Frazzoli that the probability of RRT converging to an optimal solution is 0

# Variants of RRT



- There are (very) many…
- Rapidly-exploring Random Graph (RRG):
  - Connect all vertices within neighboring region, forming a graph
- RRT*:
  - a variant of RRG that essentially “rewires" the tree as better paths are discovered.

# Summary

- Both RRT and PRM are examples of **sampling based algorithms** that are **probabilistically complete**

- Definition: A path planner is ***probabilistically complete*** if, given a solvable problem, the probability that the planner solves the problem goes to 1 as time goes to infinity.

# Links to Further Reading

- Steve LaValle's online book:
  "Planning Algorithms" *(chapters 5 & 14)*
  http://planning.cs.uiuc.edu/

- The RRT page:
  http://msl.cs.uiuc.edu/rrt/

- Motion Planning Benchmarks
  Parasol Group, Texas A&M
  http://parasol.tamu.edu/groups/amatogroup/benchmarks/mp/