CS 3630!

Lecture 15:
Differential Drive Robots: Kinematics

Rounding up Logistics:

Recap:

- 2D continuous space
- Omnidirectional motion
- Gaussian motion model
- Complex sensors
- Sophisticated perception

One approach to path planning:

- Place high reward at the goal configuration.
- Place negative reward along obstacle edges.
- Compute the value function.
- Follow the gradient of the value function from the current configuration to the goal.

Reminder about Value Iteration

Recall the Value Iteration Equation:

$$\boldsymbol{V^{k+1}}(\boldsymbol{x}) \leftarrow \max_{\boldsymbol{a}} \left\{ \overline{\boldsymbol{R}}(\boldsymbol{x}, \boldsymbol{a}) + \gamma \sum_{\boldsymbol{x}'} \boldsymbol{P}(\boldsymbol{x}' | \boldsymbol{x}, \boldsymbol{a})) \boldsymbol{V^{k}}(\boldsymbol{x}') \right\}$$

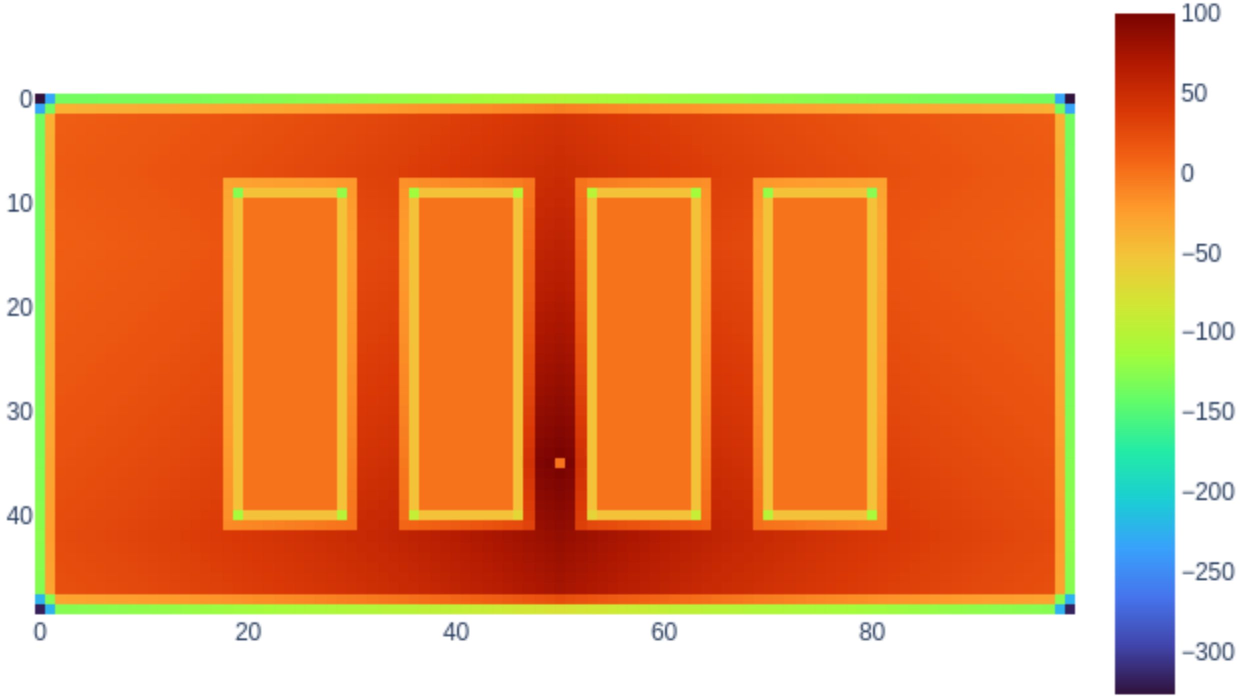
- $\overline{\boldsymbol{R}}(\boldsymbol{x}, \boldsymbol{a})$ is the expected reward for applying action $\boldsymbol{a}$ in state $\boldsymbol{x}$.
- $\boldsymbol{P}(\boldsymbol{x}' | \boldsymbol{x}, \boldsymbol{a}))$ is the motion model.
- $\boldsymbol{V^{k}}(\boldsymbol{x}')$ is the approximation to the optimal value function at iteration $\boldsymbol{k}$.

➢ For classical path planning, we ignore uncertainty, so that $\boldsymbol{P}(\boldsymbol{x}' | \boldsymbol{x}, \boldsymbol{a})) = \mathbf{1}$ for the deterministic outcome, and $\boldsymbol{P}(\boldsymbol{x}' | \boldsymbol{x}, \boldsymbol{a})) = \mathbf{0}$ for all other outcomes.

➢ If we add a bit of uncertainty, we increase robustness of the plan (e.g., decrease the probability that the robot might accidentally collide with an obstacle).

Value Iteration in the Warehouse

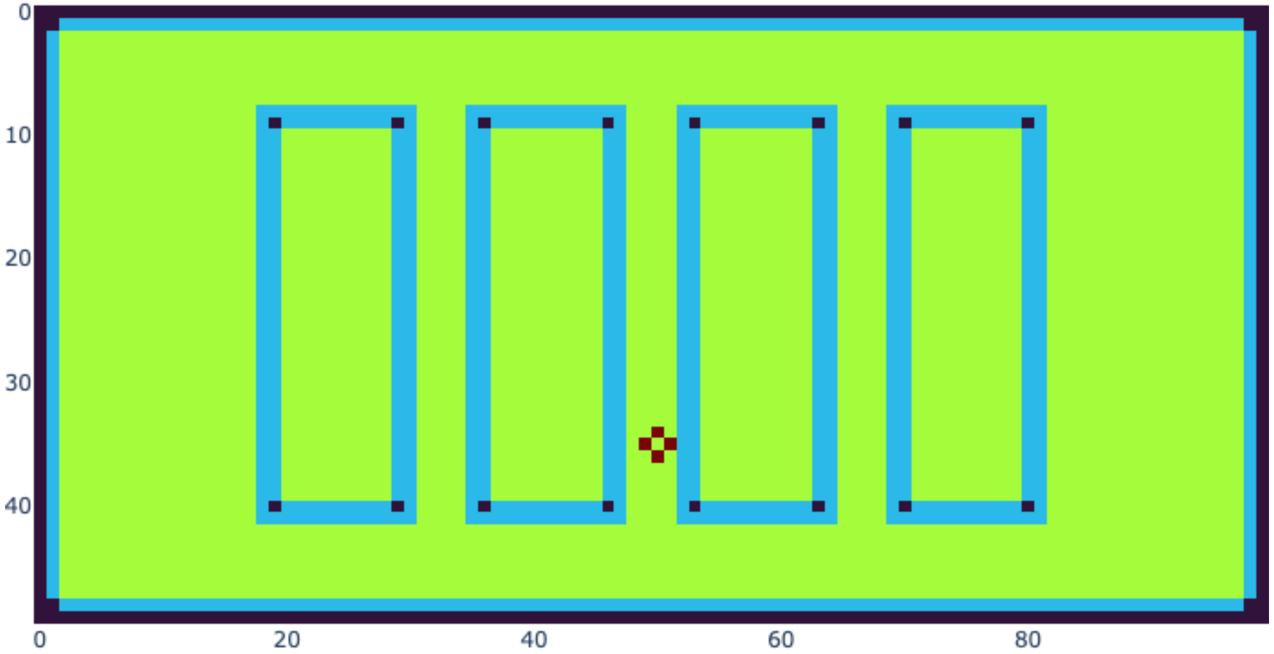
- In our warehouse example, we set the reward to 100 for the goal, and to -50 for obstacle edges, and used value iteration to compute the value function:



- To move from ***any*** point in the warehouse toward the goal, simply follow the gradient of the value function!

Value Iteration at Work

- *Value iteration is expensive.*
- At each iteration, we must compute N updates (one for each possible state).
- For the warehouse, we have $N = 50\times100 = 5000$.
- Many problems have much larger state spaces, which makes value iteration impractical (and possibly intractable).



➢ ***At each iteration, every grid cell updates its estimate of the optimal value function.***

Kinematics of Differential Drive Robots

Our logistics robot had super simple kinematics:

- Thanks to omni-wheels, the logistics robot could roll in any direction at any time.
- Because of this, there was no need to pay attention to the orientation of the robot.
- We didn’t really worry about a body-attached coordinate frame, since the robot frame was always parallel to the world frame.

Differential Drive Robots don’t have omni-wheels…

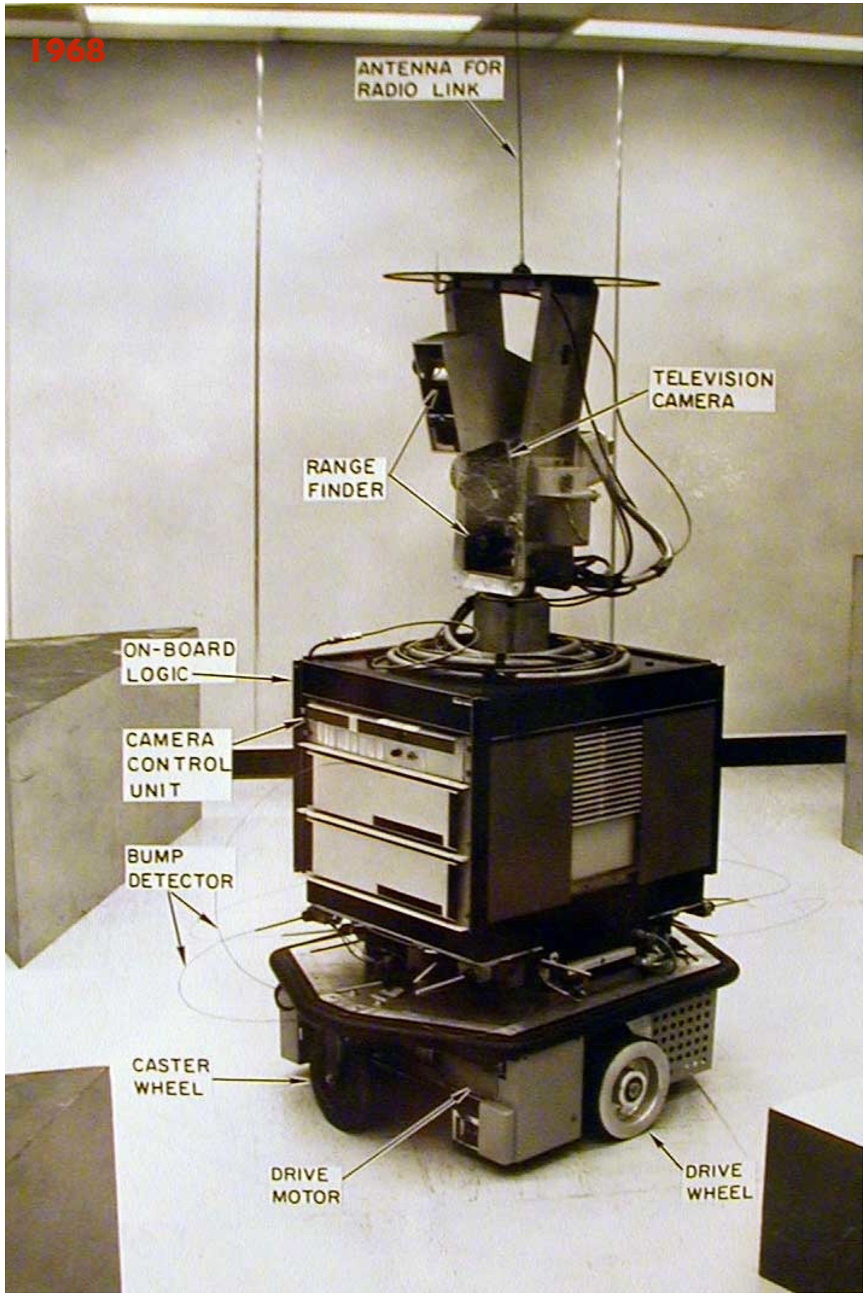
- The kinematics (relationship between input commands and robot motion) are more interesting.
- We need to explicitly pay attention to the orientation.

Mobile Robots

- There are many kinds of wheeled mobile robots.
- We have seen omni-directional robots.
- Now we’ll study *differential drive robots.*

Mobile Robot Kinematics

- Relationship between input commands (e.g., wheel velocity) and pose of the robot, not considering forces. *If the wheels turn at a certain rate, what is the resulting robot motion?*
- No direct way to measure pose (unless we sensorize the environment), but we can integrate velocity (odometry) to obtain a good estimate.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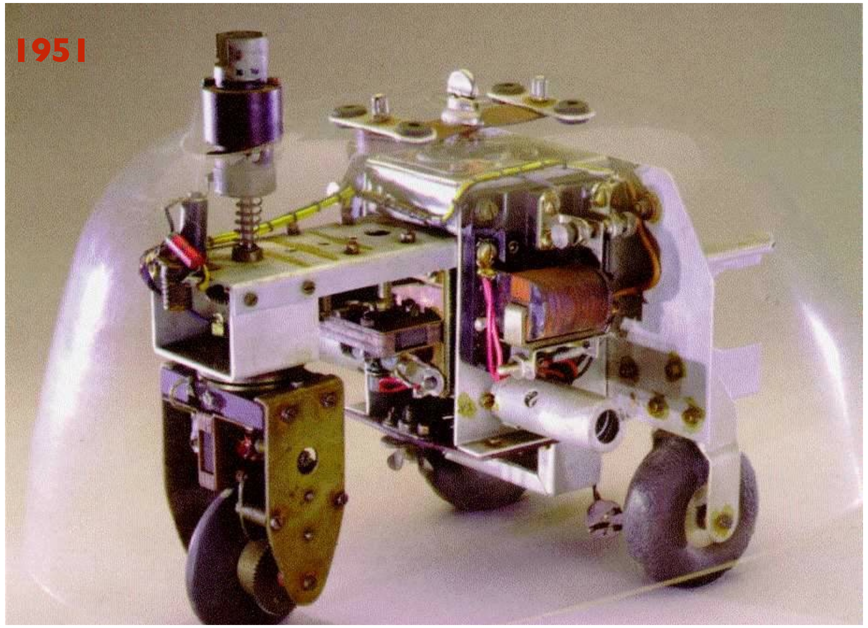
ANTENNA FOR RADIO LINK
TELEVISION CAMERA
RANGE FINDER
ON-BOARD LOGIC
CAMERA CONTROL UNIT
BUMP DETECTOR
CASTER WHEEL
DRIVE MOTOR
DRIVE WHEEL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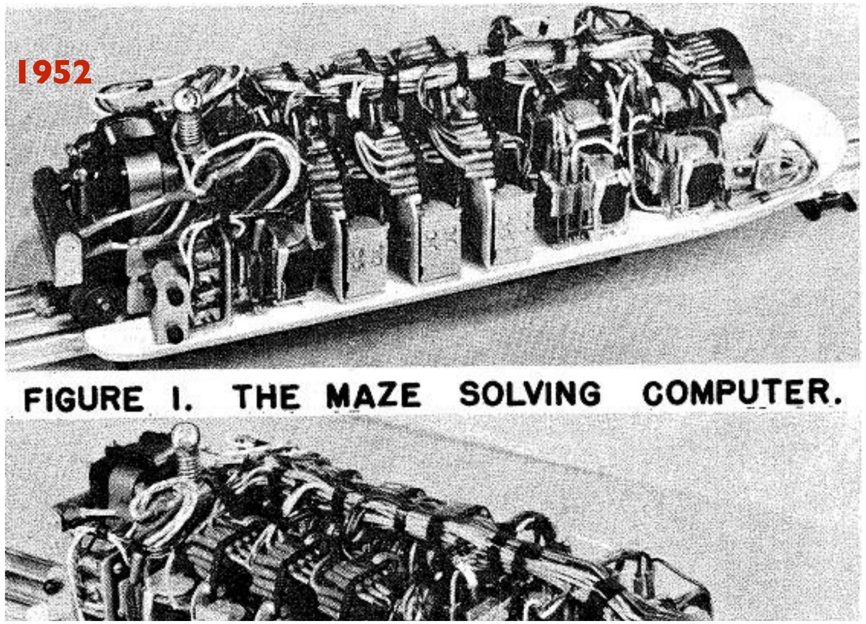
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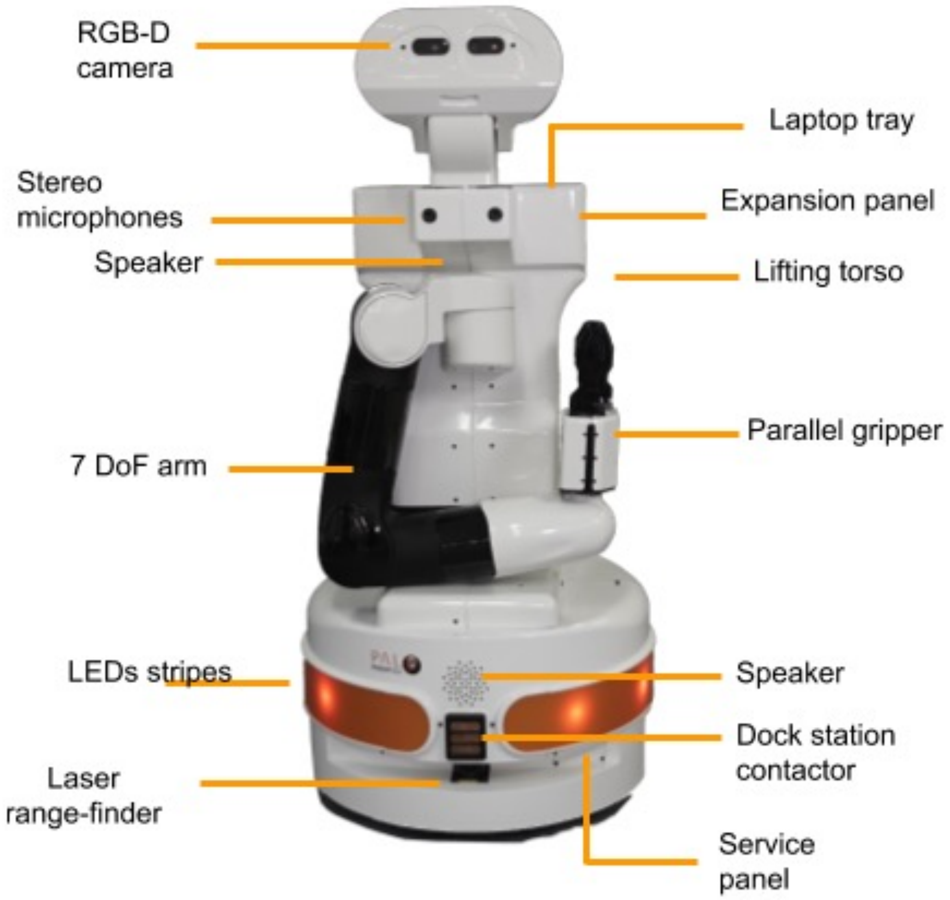
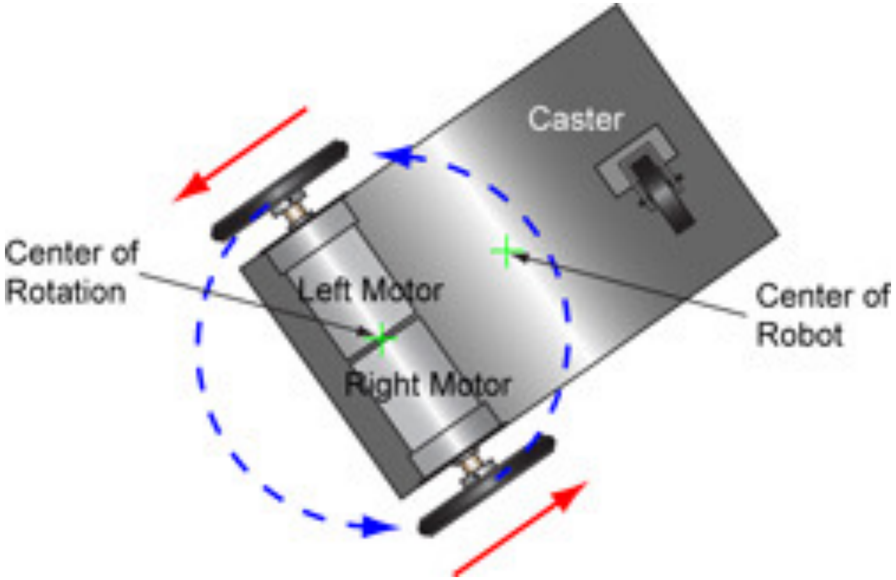
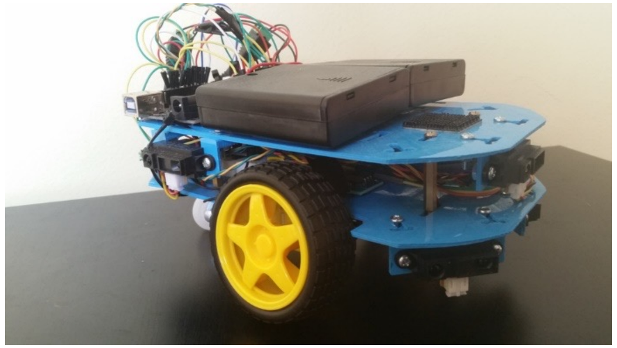
FIGURE I. THE MAZE SOLVING COMPUTER.

More Modern AGVs

Differential Drive Robots

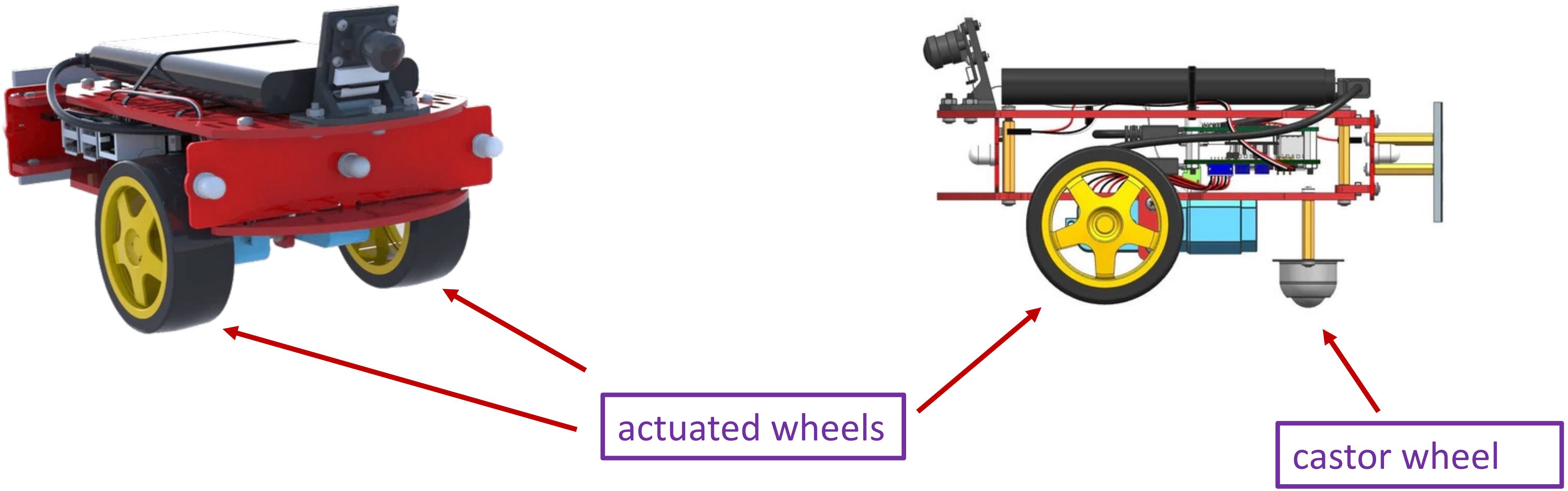






Two wheels with a common axis, and that can spin independently

The Duckiebot Platform

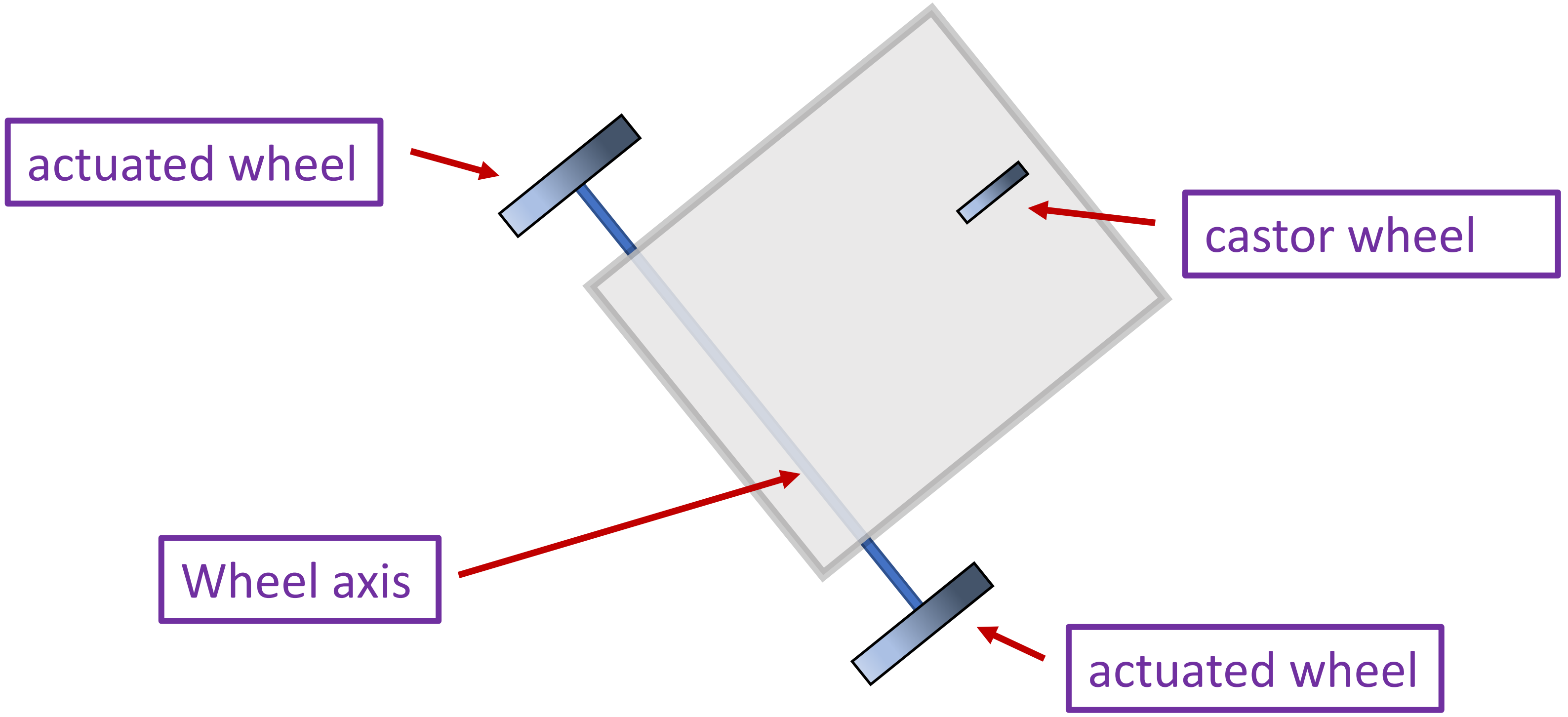


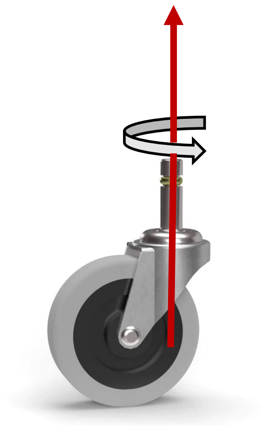
A typical DDR, with two actuated wheels in front, and a passive castor wheel in the back.

Differential Drive Robots

Differential drive robots (aka DDRs):

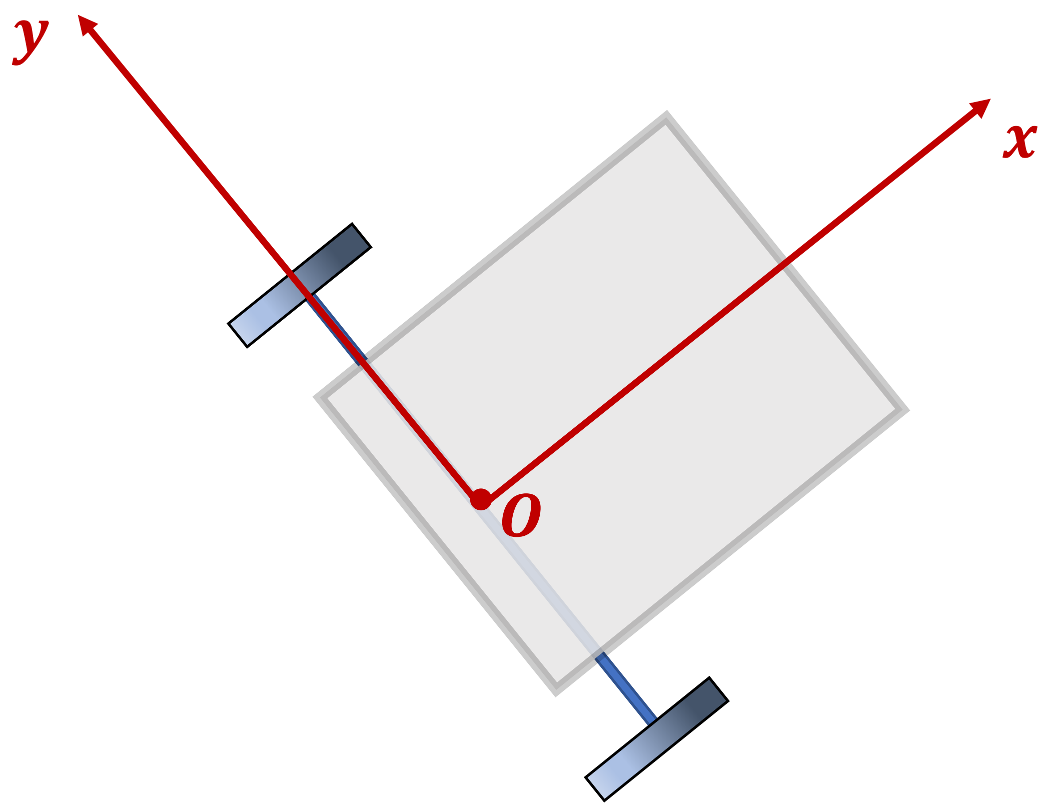
- Two actuated wheels that share an axis
- A castor wheel that rotates freely, mainly to stabilize the robot (three points define a plane – castor wheel keeps the robot from tipping over).





A castor wheel is able to spin freely about the vertical axis.

Differential Drive Robots



To specify the position and orientation of the DDR, we attach a coordinate frame to the robot.

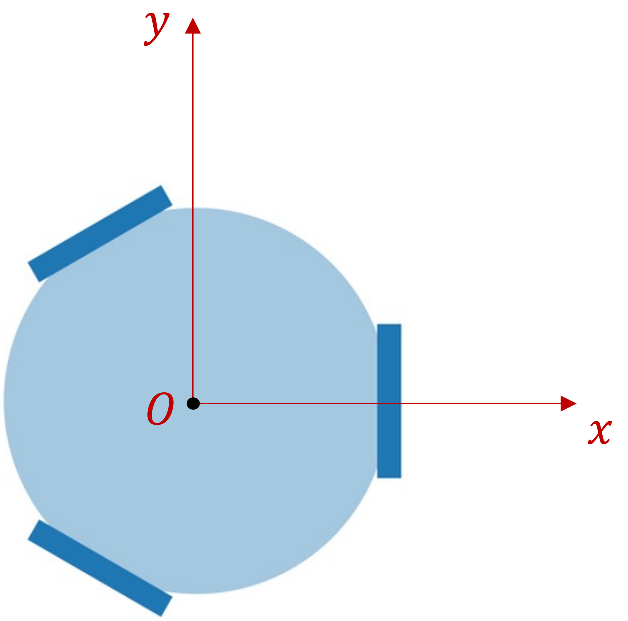
- This frame is called the ***body-attached frame***, or the robot frame.
- The body attached frame is ***rigidly*** attached to the robot: it translates and rotates with the robot.
- The origin of the body-attached frame is located at the midpoint between the two wheels along their axis of rotation.
- The x-axis is the steering (or driving) direction of the robot.
- The y -axis is coincident with the common axis.

Configuration Space

- A ***configuration*** is a complete specification of the position of every point in a robot system.
- The ***configuration space*** is the set of all configurations.
- We use q to denote a point in a configuration space Q.

Example:

- Our logistics robot was able to translate in the plane.
- It's orientation never changed (i.e., it could not rotate).
- We can attach a coordinate frame with origin at the center of the robot, and axes parallel to the world x- and y-axes.
- If we know the x, y coordinates of this coordinate frame, we can easily determine the location of any desired point on the robot w.r.t. the world coordinate frame (in the x- y plane).



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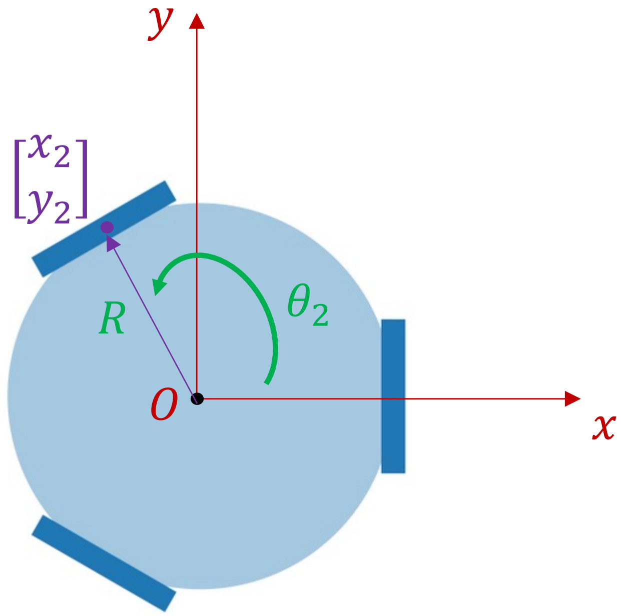
- If we know the x, y coordinates of this coordinate frame, we can easily determine the location of any desired point on the robot w.r.t. the world coordinate frame (in the x- y plane).
- For example, if the robot has radius R, then the center of wheel 2 is:

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_{robot} + R\cos\theta_2 \\ y_{robot} + R\sin\theta_2 \end{bmatrix}$$



Configuration Space

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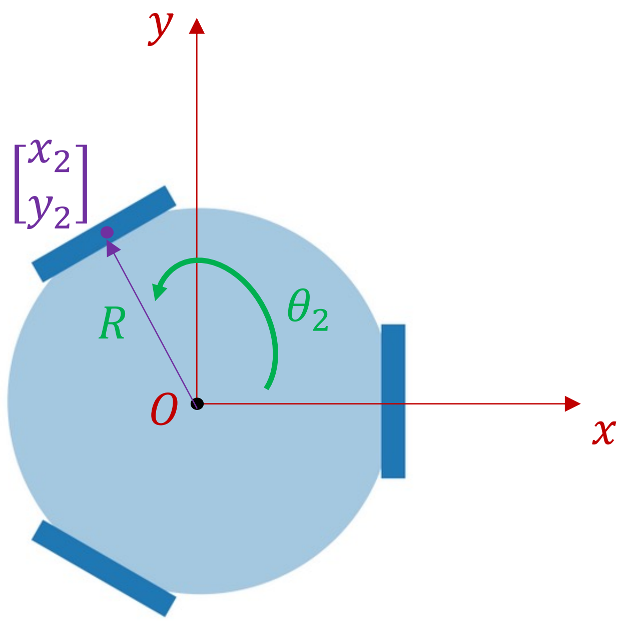
In this example, the configuration space is easy to characterize:

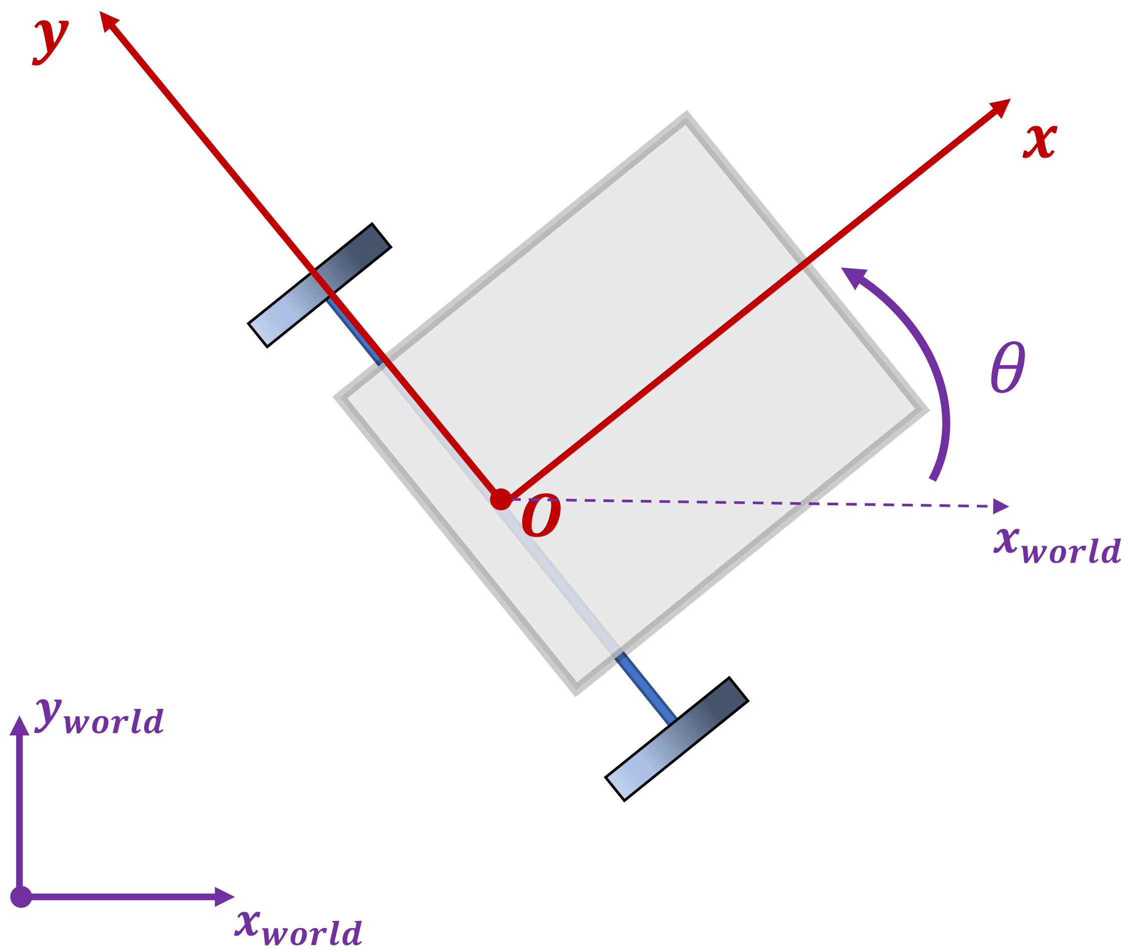
$$Q = \mathfrak{D} \subset \mathbb{R}^2$$
$$q = (x, y) \in \mathfrak{D}$$

where $\mathfrak{D}$ is the floor space of the warehouse.



- ➢ Given $q = (x, y)$, we can calculate the position of ***any*** point on the robot.
- ➢ Note: This assumes, of course, that we have a model of our robot, which we do.

Configuration Space for a DDR



Because our DDR can rotate in the plane, it is necessary to know both the position and the orientation of the body-attached frame to specify a configuration:

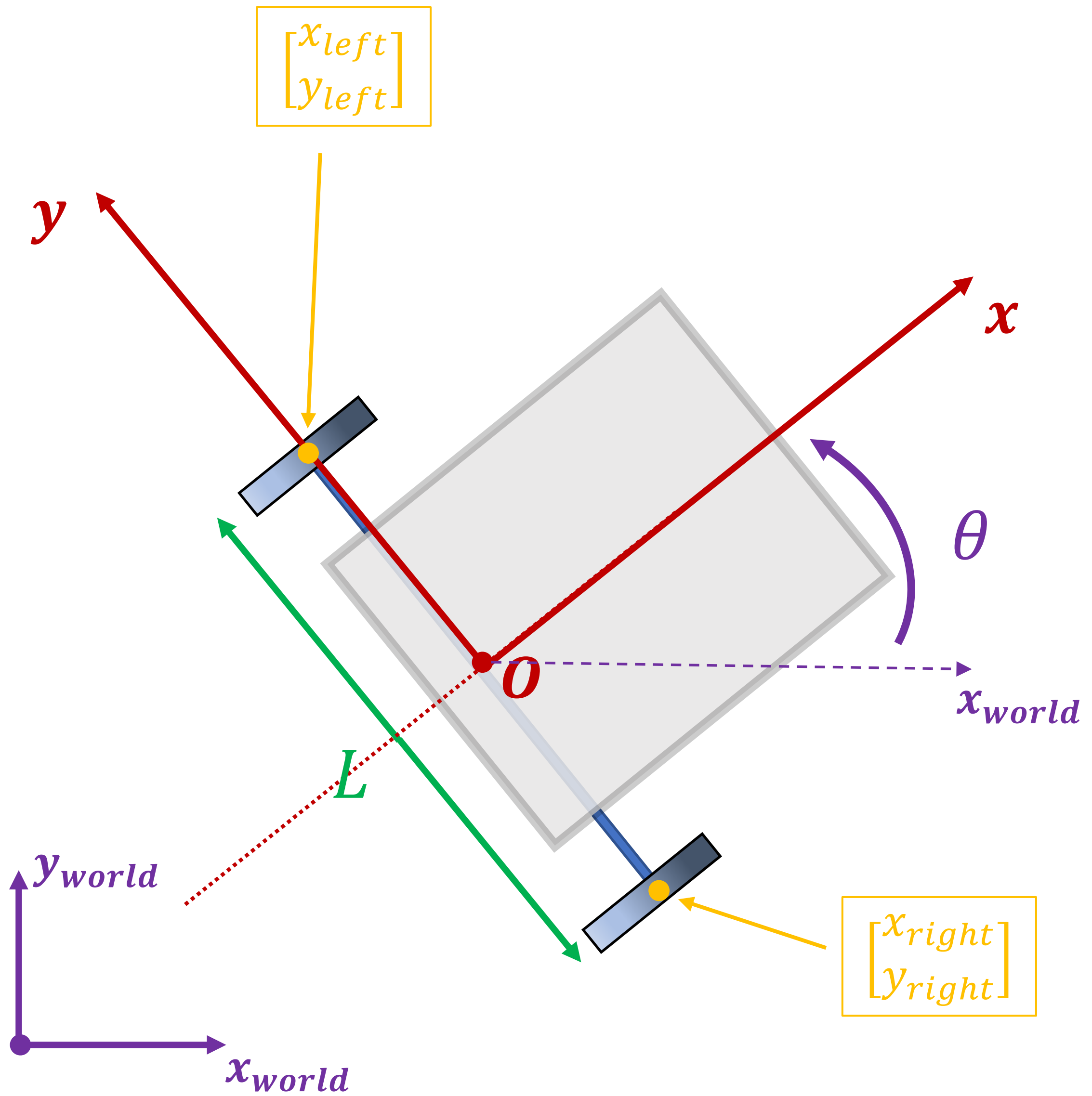
$$Q = \mathbb{R}^2 \times [0,2\pi)$$

$$q = (x, y, \theta) \in Q$$

If we know the configuration, $q = (x, y, \theta)$, we can compute the location of any point on the robot.

Let's start with the wheel centers.

Configuration Space for a DDR



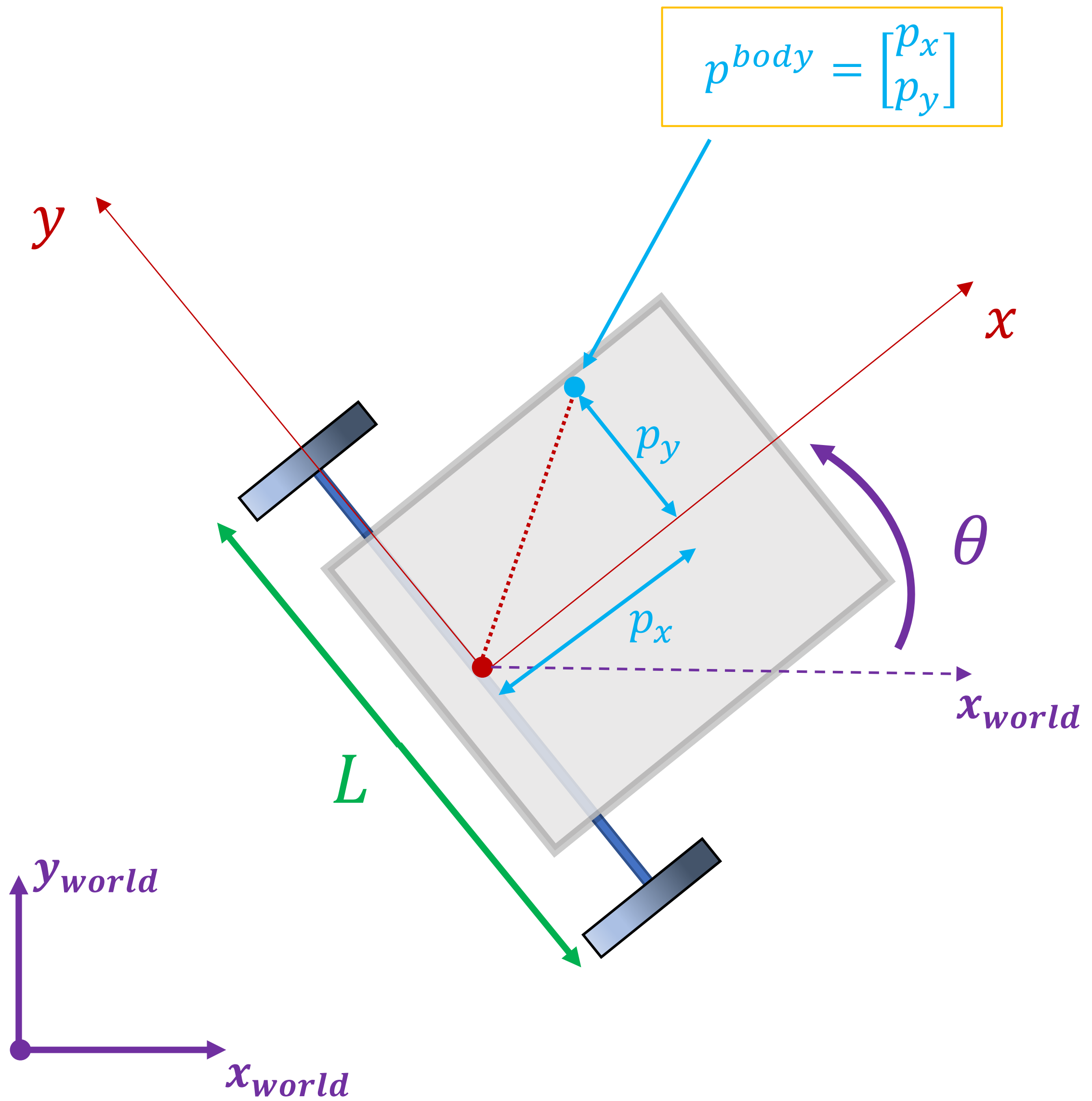
If the robot is in configuration $q = (x, y, \theta)$, the left and right wheel centers are located at:

$$\begin{bmatrix} x_{left} \\ y_{left} \end{bmatrix} = \begin{bmatrix} x - \frac{L}{2}\sin\theta \\ y + \frac{L}{2}\cos\theta \end{bmatrix}$$

and

$$\begin{bmatrix} x_{right} \\ y_{right} \end{bmatrix} = \begin{bmatrix} x + \frac{L}{2}\sin\theta \\ y - \frac{L}{2}\cos\theta \end{bmatrix}$$

Kinematics of DDRs



- We can generalize this to any point p on the DDR.
- Suppose the coordinates of p in the body frame are given by

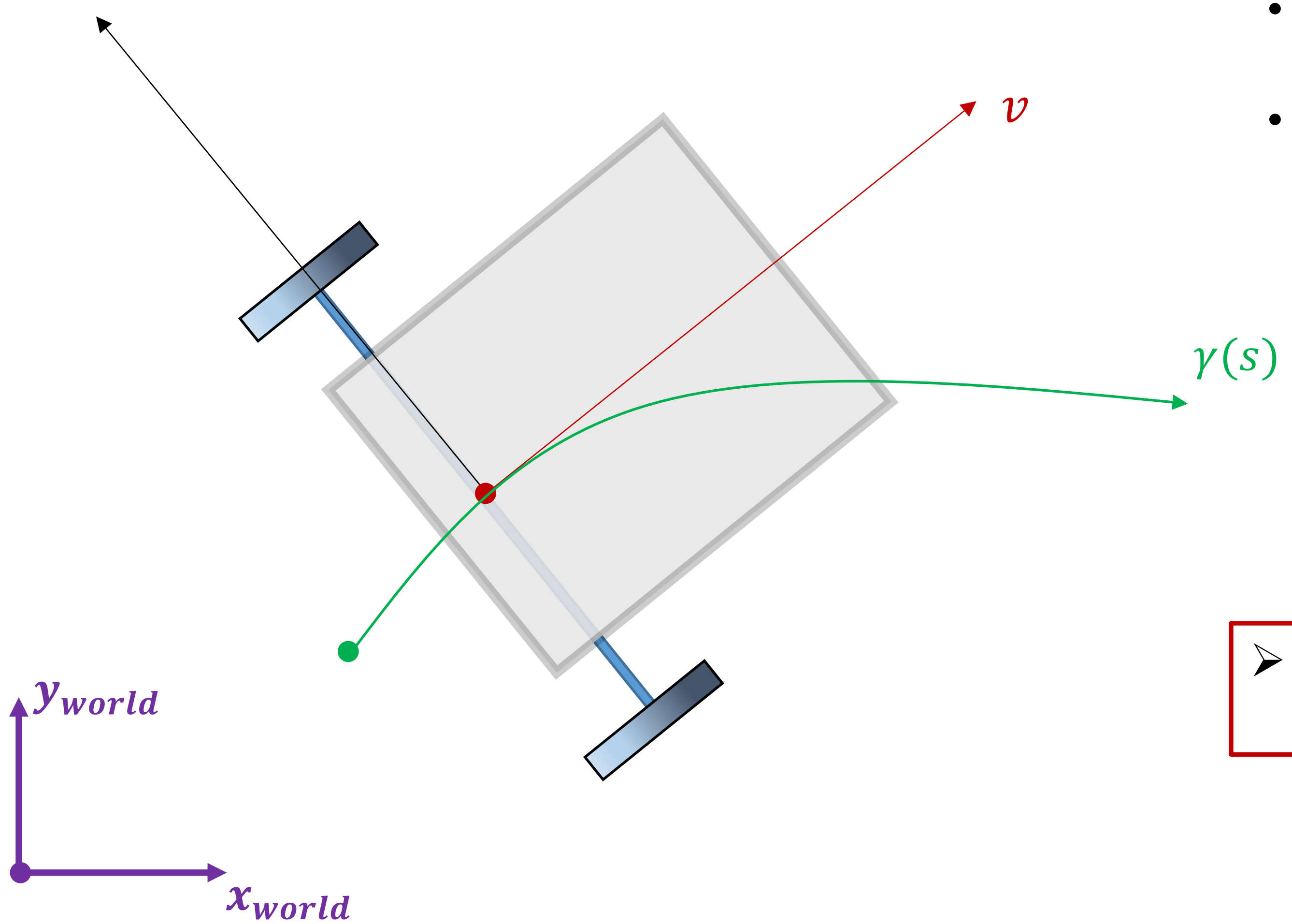
$$p^{body} = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

- If the robot is in configuration $q = (x, y, \theta)$, the coordinates of p in the world frame are given by:

$$p^{world} = \begin{bmatrix} x + p_x \cos\theta - p_y \sin\theta \\ y + p_x \sin\theta + p_y \cos\theta \end{bmatrix}$$

Soon, we will generalize this technique using homogeneous coordinate transformations… but not today.

Linear Velocity of the DDR

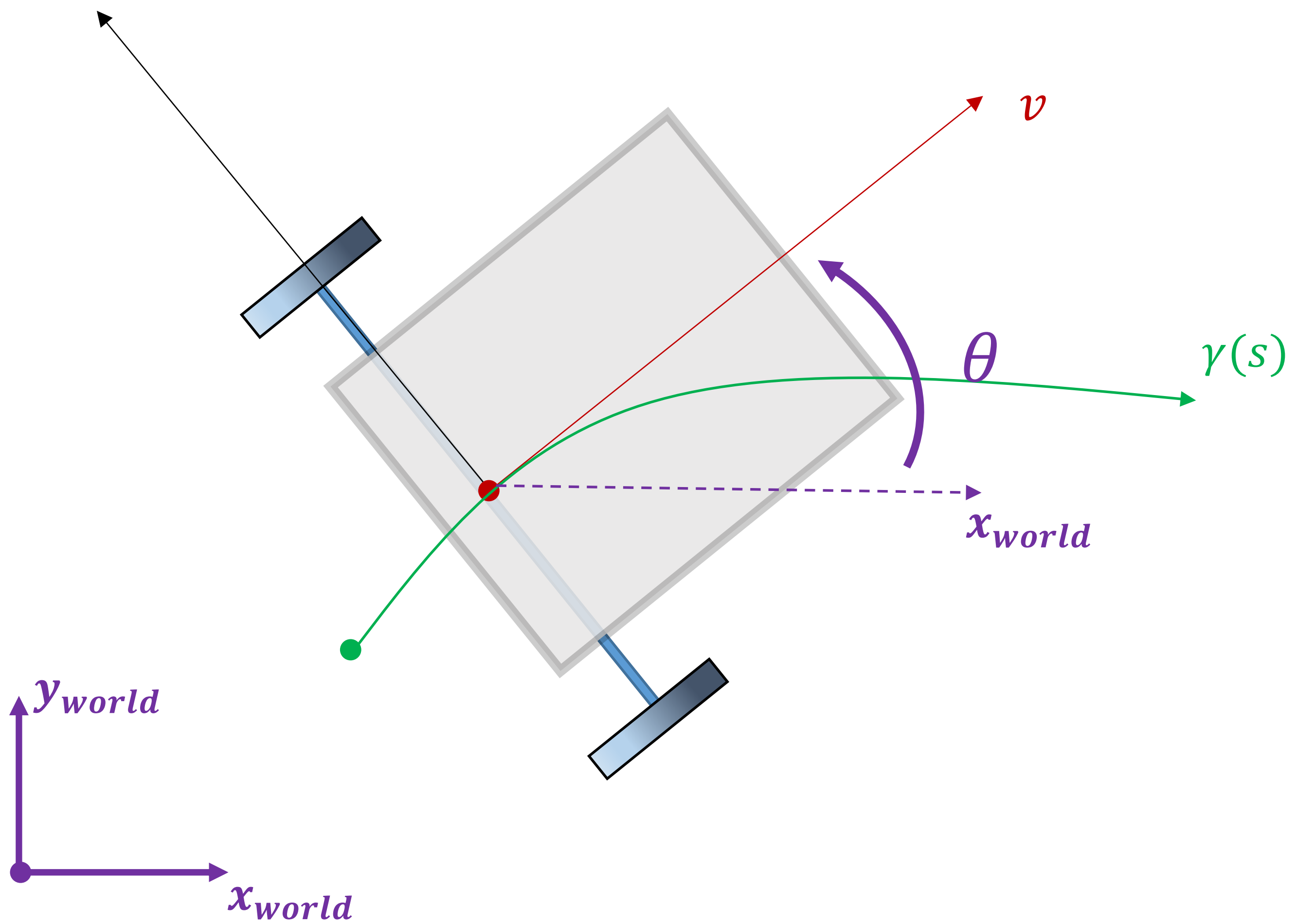


Differential Drive Robots are very different from robots with omni-wheels:

- The wheels roll without slipping – *no sideways motion*.
- The instantaneous velocity of the robot is always in the steering direction.
- The velocity perpendicular to the steering direction is always zero.

➢ If the robot follows the curve $\gamma(s)$, the instantaneous velocity v is tangent to γ.

Velocity of the DDR



- Since the robot cannot move in the direction of the body-attached y-axis, its linear velocity, when expressed with respect to the body frame is:

$$v^{body,linear} = \begin{bmatrix} v_x \\ 0 \end{bmatrix}$$

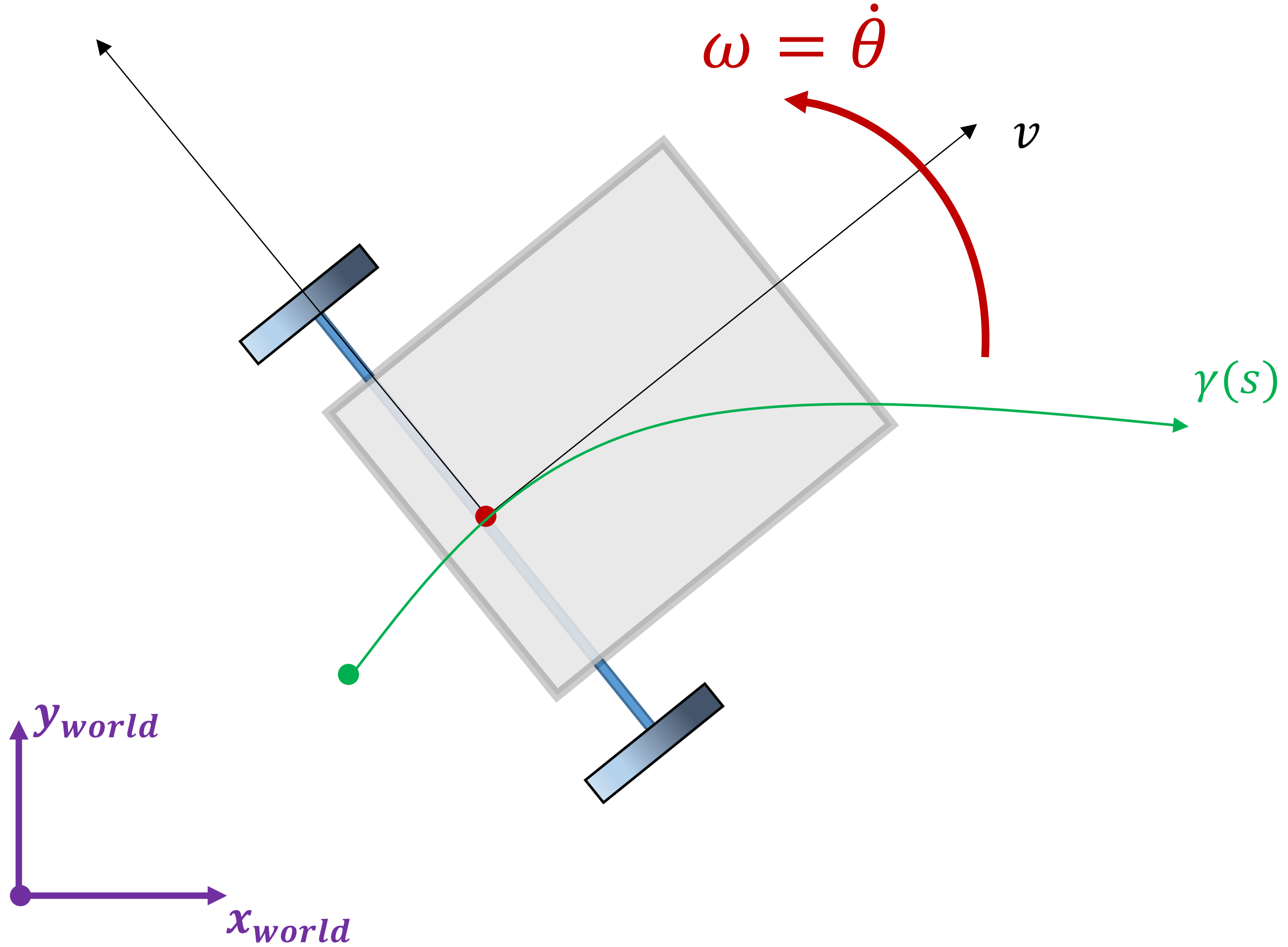
- The steering direction, expressed w.r.t. the world frame, is given by:

$$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

- Therefore, when the robot is in configuration $q(s) = (x, y, \theta)$, its linear velocity is expressed with respect to the world frame by:

$$v^{world,linear} = \begin{bmatrix} v_x \cos\theta \\ v_x \sin\theta \end{bmatrix}$$

Angular Velocity of the DDR



- The orientation of the robot is given by the angle θ.
- Since this robot is able to rotate, θ can be considered as a function of time.
- We define the robot's angular velocity as

$$\omega = \frac{d}{dt}\theta = \dot{\theta}$$

- Note that the positive sense for ω is defined using the right-hand rule: point the thumb of your right hand in the direction of the world z-axis, and your fingers will curl in the positive θ direction.

Total Velocity of the DDR

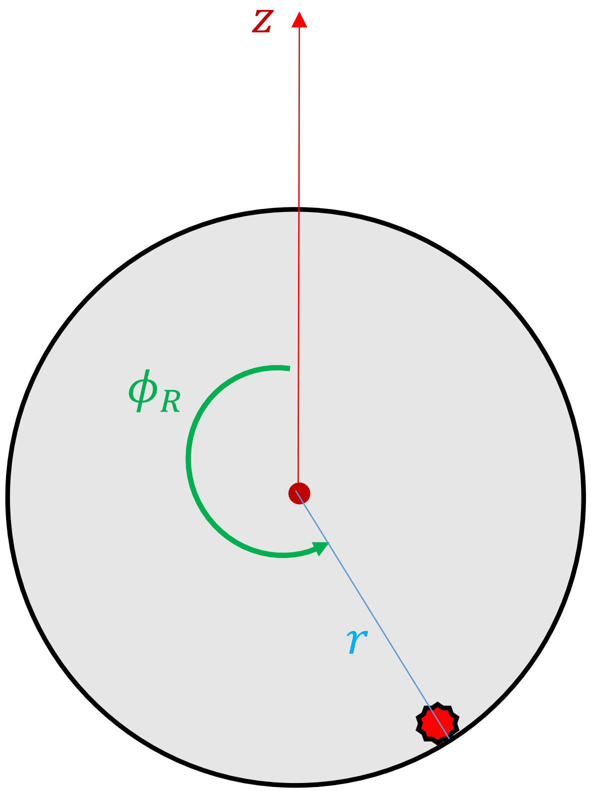
- The velocity of the DDR includes both the linear and angular velocities.
- We stack these into a single vector to describe the robot's instantaneous velocity w.r.t. the body frame of the world frame:

$$v^{body} = \begin{bmatrix} v_x \\ 0 \\ \dot{\theta} \end{bmatrix}, \qquad v^{world} = \begin{bmatrix} v_x \cos\theta \\ v_x \sin\theta \\ \dot{\theta} \end{bmatrix}$$

➢ Note that the z-axis of the body-attached frame ***<u>is the same as</u>*** the z-axis of the world frame, so that the angular velocity is given by $\dot{\theta}$ for both of these coordinate frames.

Wheel Actuation and DDR Velocity

- The two wheels of the DDR are independently actuated, and able to spin in both directions.
- Let ϕ_R and ϕ_L denote the instantaneous orientation of the right and left wheels (e.g., the angle from the world z-axis to some identifiable mark on the wheel.
- The angular speeds of the wheels are therefore given by $\dot{\phi}_R$ and $\dot{\phi}_L$.



As we saw with the omni-directional robot, the relationship between forward speed of the wheel and its angular speed is given by

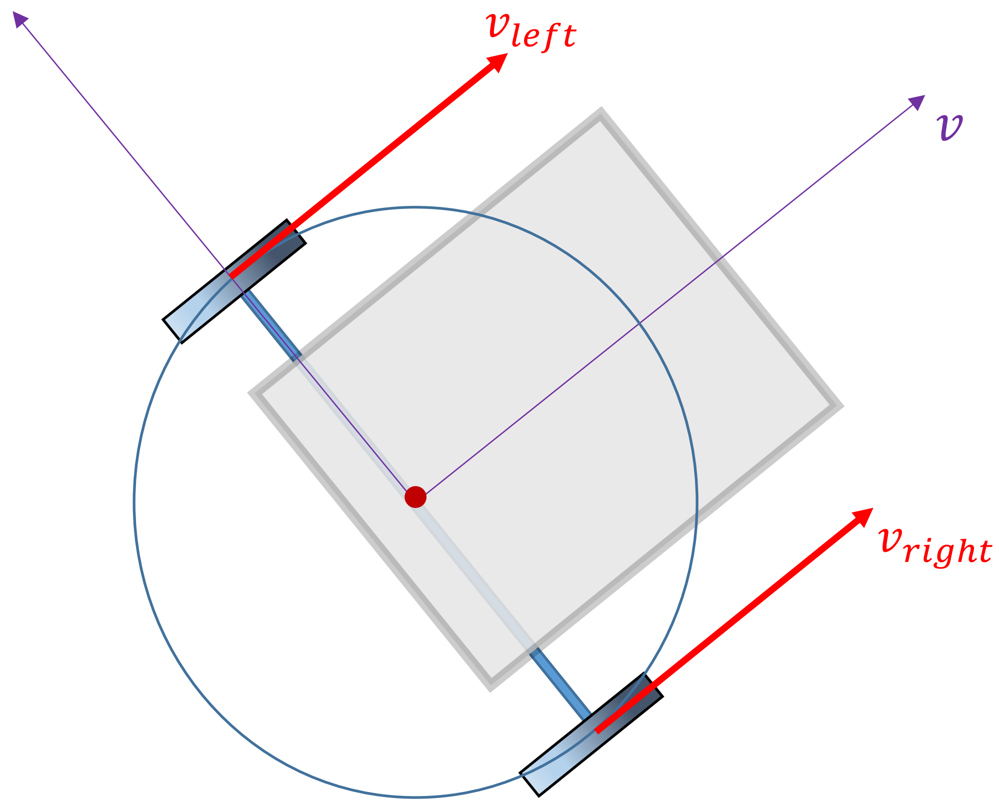
$$\frac{\mathrm{d}}{\mathrm{dt}}x = r\dot{\phi}$$

and therefore, since the wheel rolls without slipping, and $v_y = 0$, we have

$$\dot{\phi} = \frac{v_x}{r}$$

Wheel Actuation and DDR Velocity

When the two wheels turn with the same angular speed, $\dot{\phi}_R = \dot{\phi}_L$, the robot moves with pure translation.



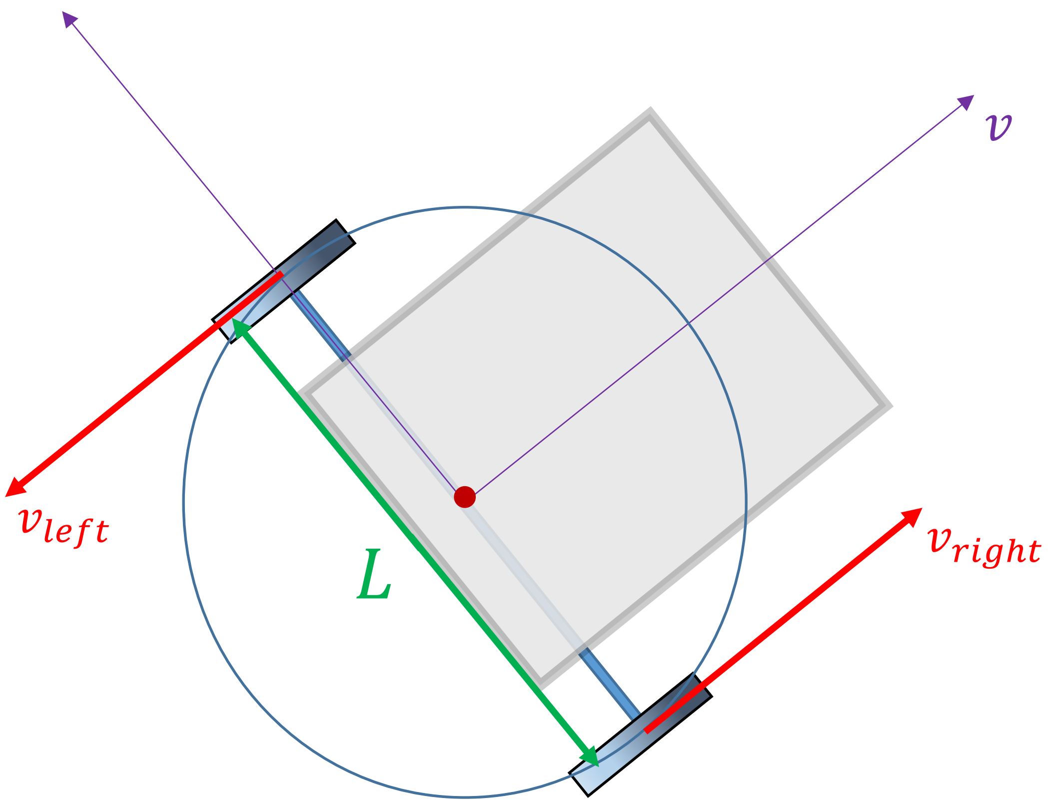
In this case,

- $v_{left} = v_{right}$
- Both v_{left} and v_{right} are parallel to the steering direction.
- The robot's angular velocity is zero (i.e., $\omega = 0$).
- The angular wheel speed is related to the robot's linear velocity by

$$\dot{\phi}_R = \dot{\phi}_L = \frac{v_x}{r}$$

Total DDR Velocity

When the two wheels turn with the opposite angular velocity, $\dot{\phi}_R = -\dot{\phi}_L$, the robot moves with pure rotation.



In this case,

- $v_{left} = -r\dot{\phi}_L$ and $v_{right} = r\dot{\phi}_R$
- Both v_{left} and v_{right} are tangent to the circle centered at the origin of the body-attached frame w/radius $0.5L$.
- The robot's angular velocity satisfies *the eqn for circular motion:*

$$\frac{L}{2}\omega = v_{right} = r\dot{\phi}_R$$

and

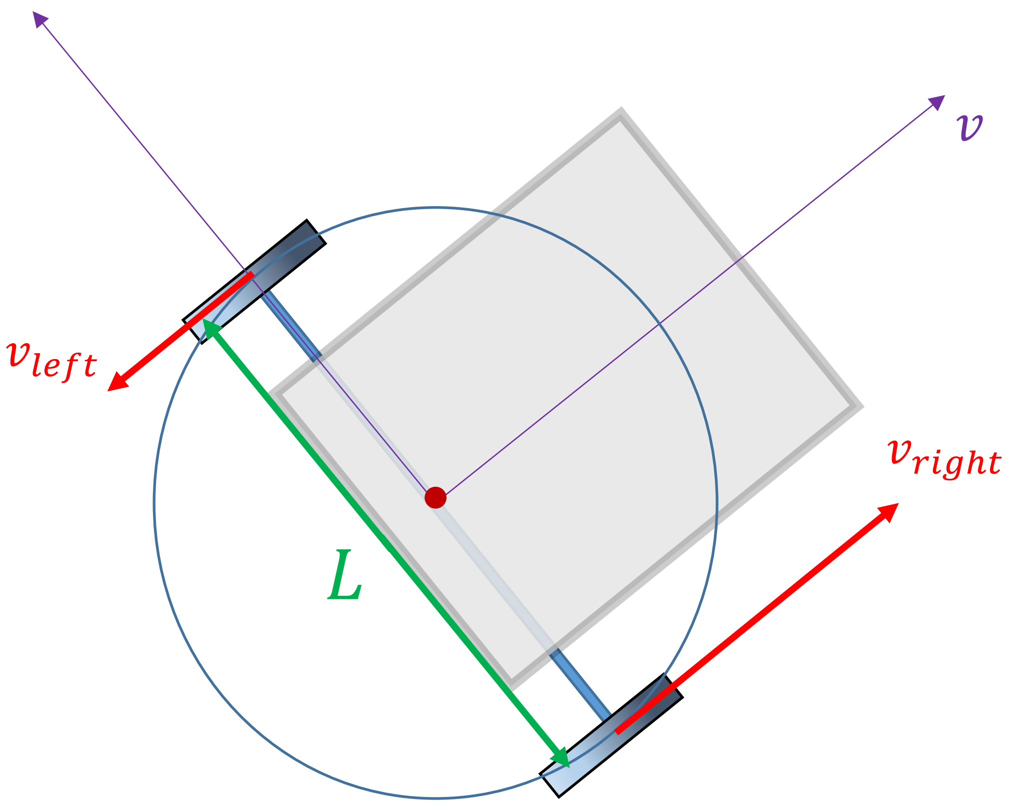
$$\frac{L}{2}\omega = -v_{left} = -r\dot{\phi}_L$$

- Rearranging terms, we obtain:

$$\dot{\phi}_R = \frac{L}{2}\frac{\omega}{r} \quad \text{and} \quad \dot{\phi}_L = -\frac{L}{2}\frac{\omega}{r}$$

Wheel Actuation and DDR Velocity

All other velocities are linear combinations of these two cases, and therefore we can apply superposition.



- For pure translation we have:

$$\dot{\phi}_R = \frac{v_x}{r} \quad \text{and} \quad \dot{\phi}_L = \frac{v_x}{r}$$

- For pure rotation we have:

$$\dot{\phi}_R = \frac{L}{2}\frac{\omega}{r} \quad \text{and} \quad \dot{\phi}_L = -\frac{L}{2}\frac{\omega}{r}$$

- Combining (adding) the two equations for $\dot{\phi}_R$ and $\dot{\phi}_L$ we obtain:

$$\dot{\phi}_R = \frac{L}{2}\frac{\omega}{r} + \frac{v_x}{r} \quad \text{and} \quad \dot{\phi}_L = -\frac{L}{2}\frac{\omega}{r} + \frac{v_x}{r}$$

➢ ***These two equations tell us how to choose $\boldsymbol{\dot{\phi}_R}$ and $\boldsymbol{\dot{\phi}_L}$ to achieve a desired velocity $\boldsymbol{v}, \boldsymbol{\omega}$.***

Wheel Actuation and DDR Velocity

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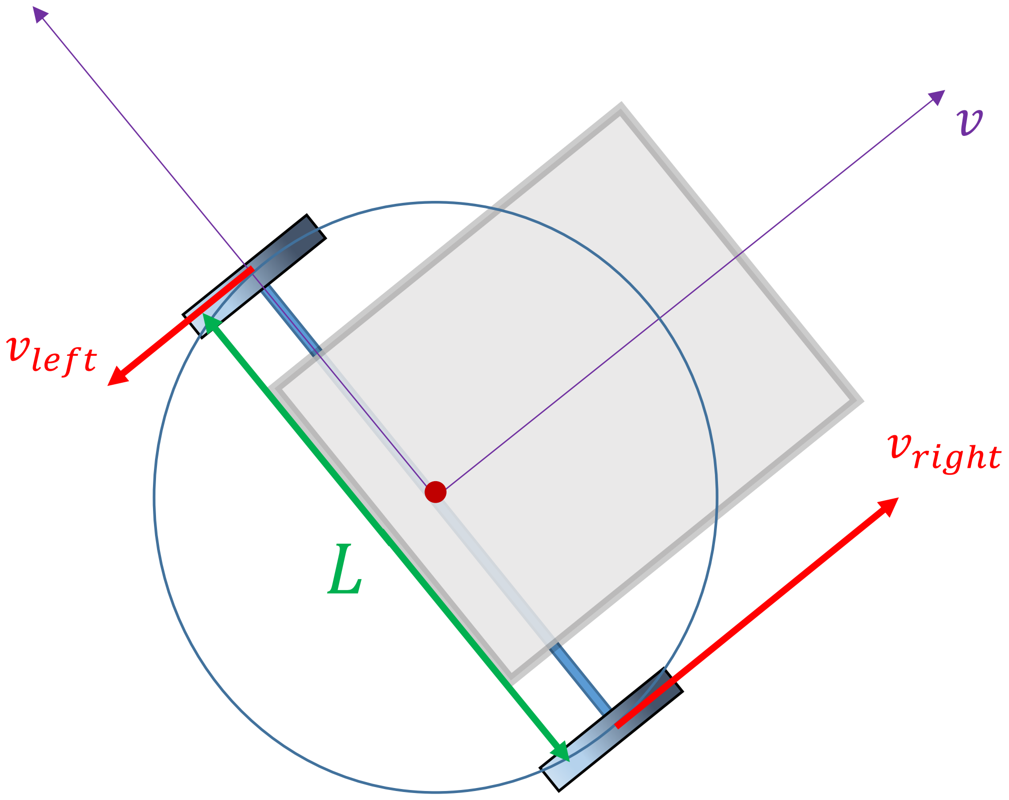


- The equations

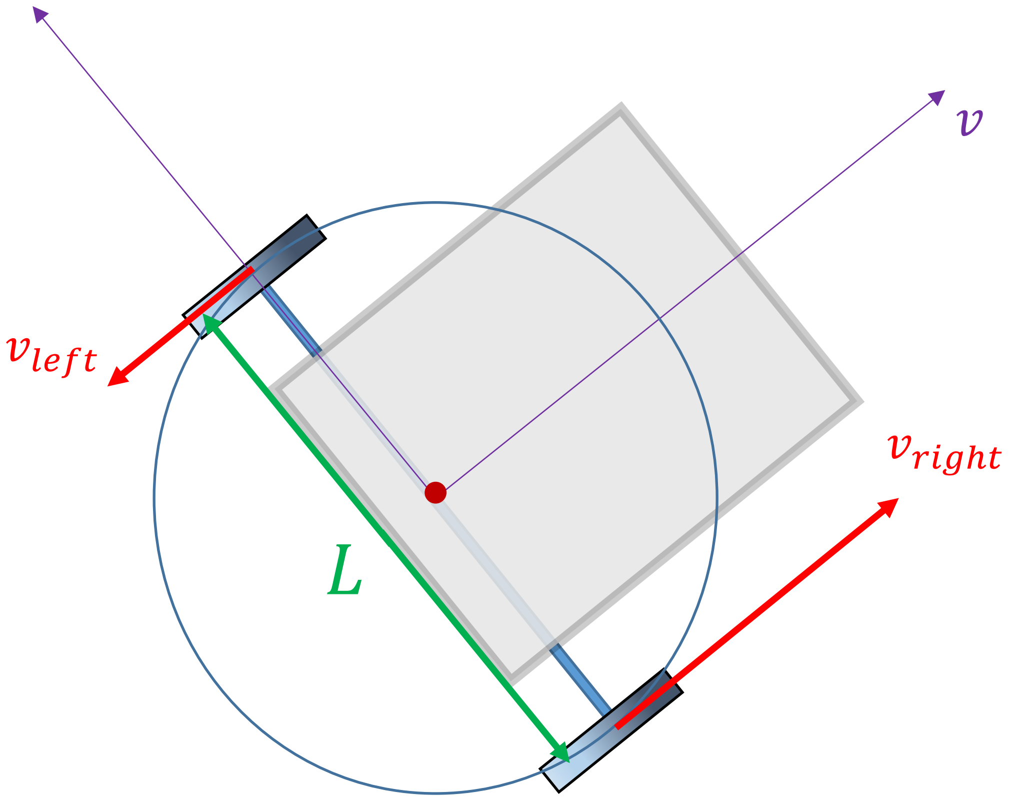
$$\dot{\phi}_R = \frac{L}{2}\frac{\omega}{r} + \frac{v_x}{r} \quad \text{and} \quad \dot{\phi}_L = -\frac{L}{2}\frac{\omega}{r} + \frac{v_x}{r}$$

  are sometimes called *inverse* equations, since they solve the inverse problem: "what wheel speed (*input*) to I need to achieve a desired robot behavior (*output*)?"
- We can easily compute the forward mapping, from $\boldsymbol{\dot{\phi}_R}$ and $\boldsymbol{\dot{\phi}_L}$ to $\boldsymbol{v}, \boldsymbol{\omega}$ using simple algebra:

$$\dot{\phi}_R + \dot{\phi}_L = 2\frac{v_x}{r} \Rightarrow v_x = \frac{r}{2}\left(\dot{\phi}_R + \dot{\phi}_L\right)$$

$$\dot{\phi}_R - \dot{\phi}_L = \frac{L\omega}{r} \Rightarrow \omega = \frac{r}{L}\left(\dot{\phi}_R - \dot{\phi}_L\right)$$

Wheel Actuation and DDR Velocity



We can express these equations relative to the body-attached frame or the world frame:

$$v^{body} = \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2}(\dot{\phi}_R + \dot{\phi}_L) \\ 0 \\ \frac{r}{L}(\dot{\phi}_R - \dot{\phi}_L) \end{bmatrix}$$

and

$$v^{world} = \begin{bmatrix} v_x \cos\theta \\ v_x \sin\theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r}{2}(\dot{\phi}_R + \dot{\phi}_L)\cos\theta \\ \frac{r}{2}(\dot{\phi}_R + \dot{\phi}_L)\sin\theta \\ \frac{r}{L}(\dot{\phi}_R - \dot{\phi}_L) \end{bmatrix}$$