WHOLE BODY MOTION PLANNING AND CONTROL

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Below I summarize a number of papers I have read about whole body motion planning and control. This is an incomplete list but a good start to see the evolution in the field over the past 20 years.

- (Vukobratović and Stepanenko, 1972) is the seminal reference for the Zero Moment Point (ZMP), which I could not find online, but an entertaining retrospective is given by Vukobratović and Borovac (2004). The ZMP is simply the point of application of the ground reaction forces, which for any dynamically stable gait lies *within* the support polygon of the feet, and identical to the Center of Pressure (CoP). When the CoP lies *outside* the support polygon the net moment is no longer zero (hence no longer a ZMP) and the robot tips over.
- Takanishi et al. (1990) use the ZMP to generate a stable Center of Mass (CoM) trajectory, and then track it with inverse kinematics. Cited by Stephens and Atkeson (2010).
- Wooten and Hodgins (2000) show impressive simulations using a combination of four controllers. The intuition is based on the ”funnels” from Burridge et al. (1999), where each controller feeds into the basin of attraction (funnel) of another controller. Aside: first author and then Georgia Tech student Wayne Wooten later helped create RenderMan at Pixar.
- Kajita et al. (2003a) combines the linear inverted pendulum (LIP) model with ZMP, developing a ”preview” controller that controls the CoM in order to track a pre-specified ZMP trajectory. The LIP simplification tracks almost perfectly even for a complex humanoid, at least in simulation.
- I really like the paper on ”Resolved Momentum Control” by Kajita et al. (2003b), as it focuses on the desired *momentum* of the system. For example, for a walking humanoid it is desirable that the angular momentum is equal to zero, and the linear momentum is forward and constant. given footholds, it is relatively easy to calculate the base and joint trajectories that realize this, minimizing joint velocities (through a pseudo-inverse). In a followup journal paper (Neo et al., 2007) the dynamics are addressed as well, and the ZMP is shown to be a function of the CoM and the time derivative of the momentum (a force!), although the paper does not yet describe dynamic/torque *control*.
- Sentis and Khatib (2005) approach whole body control (WBC) in humanoids by null-space projection to order tasks by priority: respect joint limits, maintain balance, then other tasks that are recursively handled in decreasing priority. The key equation is:

$$\Gamma = \Gamma_{constraints} + N^T_{constraints}(\Gamma_{task(1)} + N^T_{task(1)}(\Gamma_{task(2)} + \dots)) \tag{1}$$

- Mistry et al. (2010) introduce a simple WBC approach based on *elimination*, or as they call it, ”Orthogonal decomposition”, i.e., QR factorization. By eliminating the constraint forces and only then the torques, the latter can be computed as a function of the desired acceleration trajectory, automatically satisfying the contact constraints. This is quite similar to null-space projection.
- Stephens and Atkeson (2010) also eliminate the constraint forces as in (Mistry et al., 2010), but rather than desired accelerations use a momentum-style approach as in (Kajita et al., 2003b) to calculate the needed ground forces. After reconciling those with the contact constraints the torques are solved for.
- Righetti et al. (2013) have realized that the QR decomposition/variable elimination from (Mistry et al., 2010) is equivalent (and nicer) than the null-space projection in (Sentis and Khatib, 2005), although they cannot yet solve the *recursive* task ordering. They show that any linear or quadratic cost in the constraint forces and torques can be minimized. This is not surprising from a factor graph point of view. They also seem to not be aware of (Stephens and Atkeson, 2010) which deals exactly with inequality constraints.

- (Orin et al., 2013) is the landmark paper on ”centroidal dynamics”, following up on the intuitions in (Kajita et al., 2003b) with a solid theory. They define the 6×1 centroidal momentum vector

  $$\mathcal{P}_G(q, \dot{q}) = A_G(q)\dot{q} \tag{2}$$

  which comprises of the angular and linear momentum with respect to the center of mass. It is purely a function of the configuration $[q, \dot{q}]$ (including the 6 floating base coordinates), and can only be changed by external forces and moments. Curiously, while they present a balance controller at the end, the actual *dynamics* part of the theory is lacking a bit.

- (Dai et al., 2014) is a beautiful paper where a lot of the preceding work comes together: they start by stating the centroidal dynamics in the spatial frame:

  $$\dot{\mathcal{P}}_S = \mathcal{F}^{\mathcal{G}}_{\mathcal{S}} + \sum [Ad_{T^{C}_{S}}]^T \mathcal{F}_{\mathcal{C}} \tag{3}$$

  $$\mathcal{P}_S(q, \dot{q})s = A_S(q)\dot{q}. \tag{4}$$

  They then develop an optimization-based motion planner using this overall dynamics model, but using a *full* kinematic model of the robot. They can optionally optimize over the contact states using the ideas from (Posa et al., 2014). Optimization takes a few hours, but the results are impressive, and have the Atlas robot do all types of tricks, including challenging tasks with many flight phases.

- Winkler et al. (2017) optimizes for the CoP (center of pressure) over time, which us treated as an input u to a simplified LIP (linear inverted pendulum) model of a quadruped robot. It cleverly constrains the CoP to be a ZMP by making sure it is within the convex hull of the feet. Also has an excellent literature review.

- However, the key paper from that group is (Winkler et al., 2018), in which they propose “a single [Trajectory Optimization] formulation for legged locomotion that automatically determines the gait-sequence, step timings, footholds, swing-leg motions, 6D body motion and required contact forces over non-flat and inclined terrain. No prior footstep planning is necessary.” The kinematics setup is quite similar to (Dai et al., 2014) but assuming massless legs which means the CoM is fixed in the body, but the handling of contact is vastly different: timings of alternating swing/stance phases are optimized over as continuous parameters, independently for each leg. The result is quite elegant and yields impressive results, including locomotion over inclined planes, with various body types.

- A good antidote against all the hardcore TO work is the paper by Hubicki et al. (2018), where they discuss the ATRIAS robot. It’s control architecture comprises of three behaviors implemented on top of a compliant walking mechanism. A key concept is the injection of force in the second half of stance that counteracts damping by the compliance, which accomplishes a *physical* velocity feedback loop.

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