CS 3630!

Lecture 6: Inference in Factor Graphs

Topics

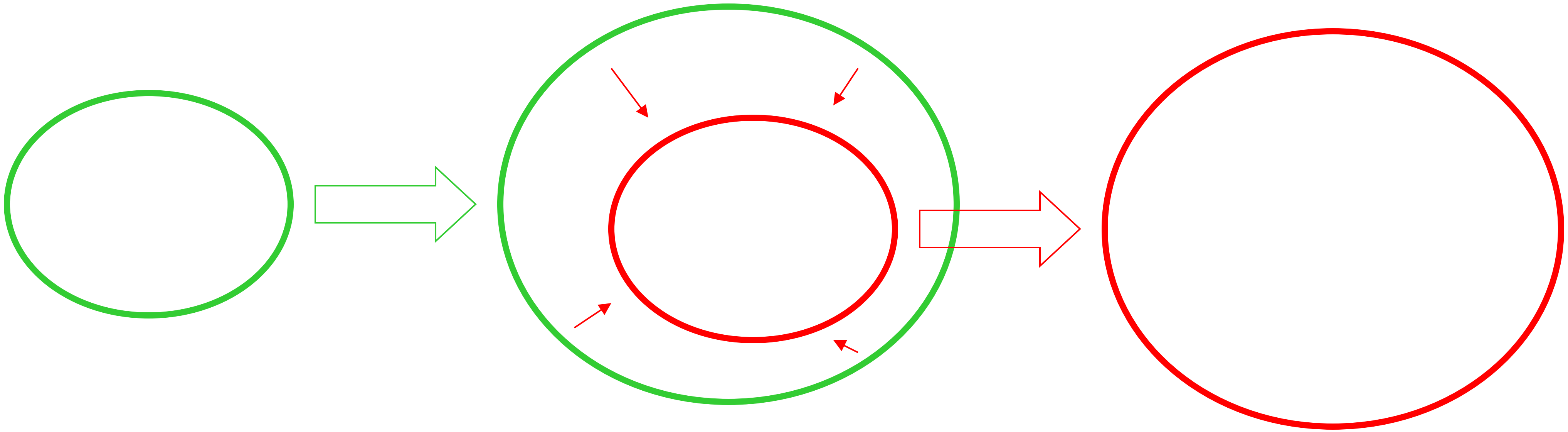
- **1. Bayes Filter**
- **2. HMMs**
- **3. Factor Graphs**
- **4. Converting Bayes Nets into Factor Graphs**
- **5. The Max-Product Algorithm for HMMs**

Motivation

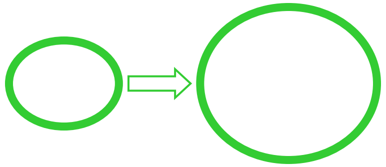
- Probability -> simulate robots !
- Our example: grid world
- Probabilistic statements about state: Bayesian inference

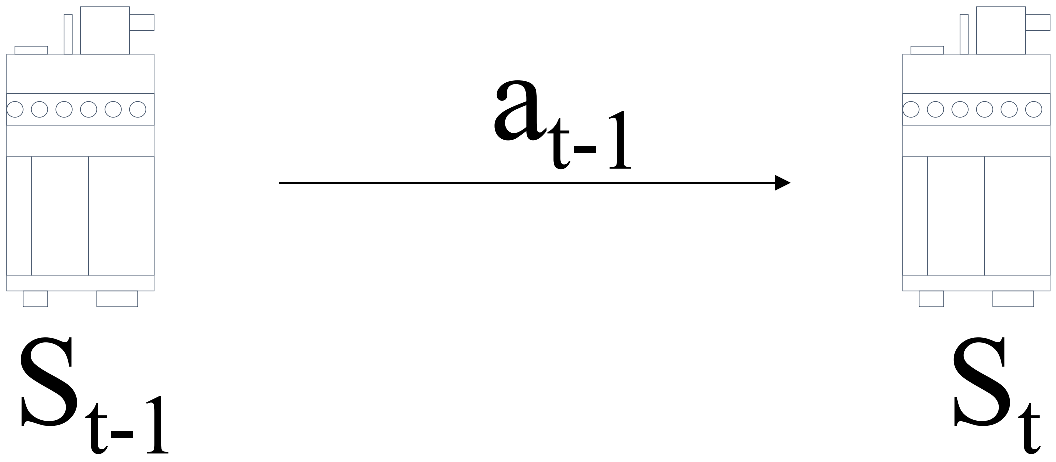
1. The Bayes filter

- Two phases: a. Prediction Phase
  b. Measurement Phase



a. Prediction Phase



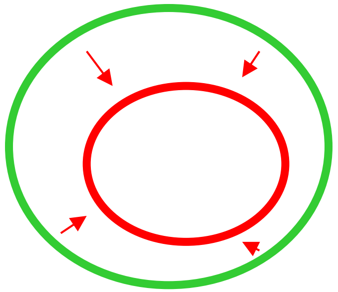


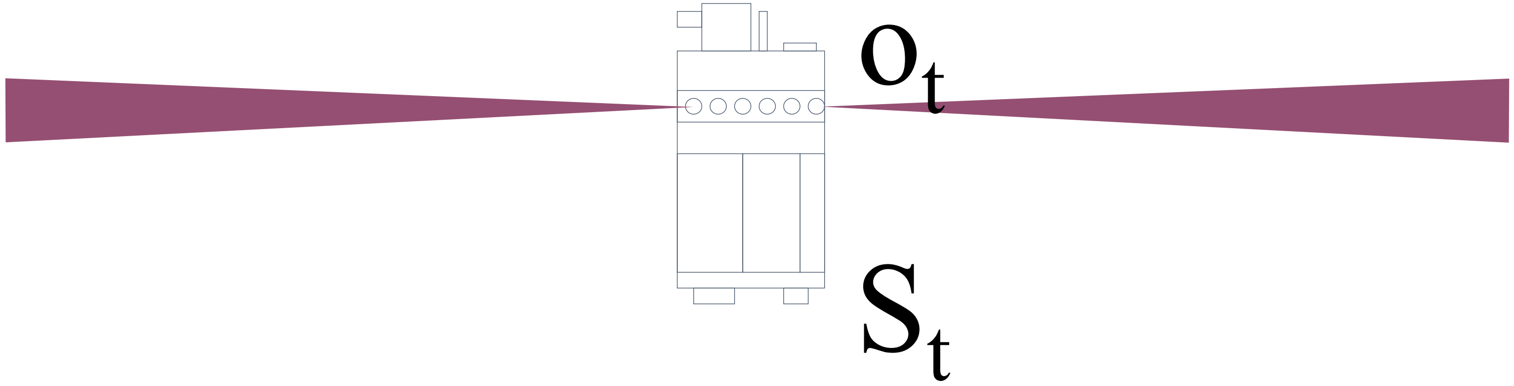
$$P(S_t) = \sum_{s_{t-1}} P(S_t | s_{t-1}, a_{t-1}) P(s_{t-1})$$

State transition model

Previous state prior

b. Measurement Phase

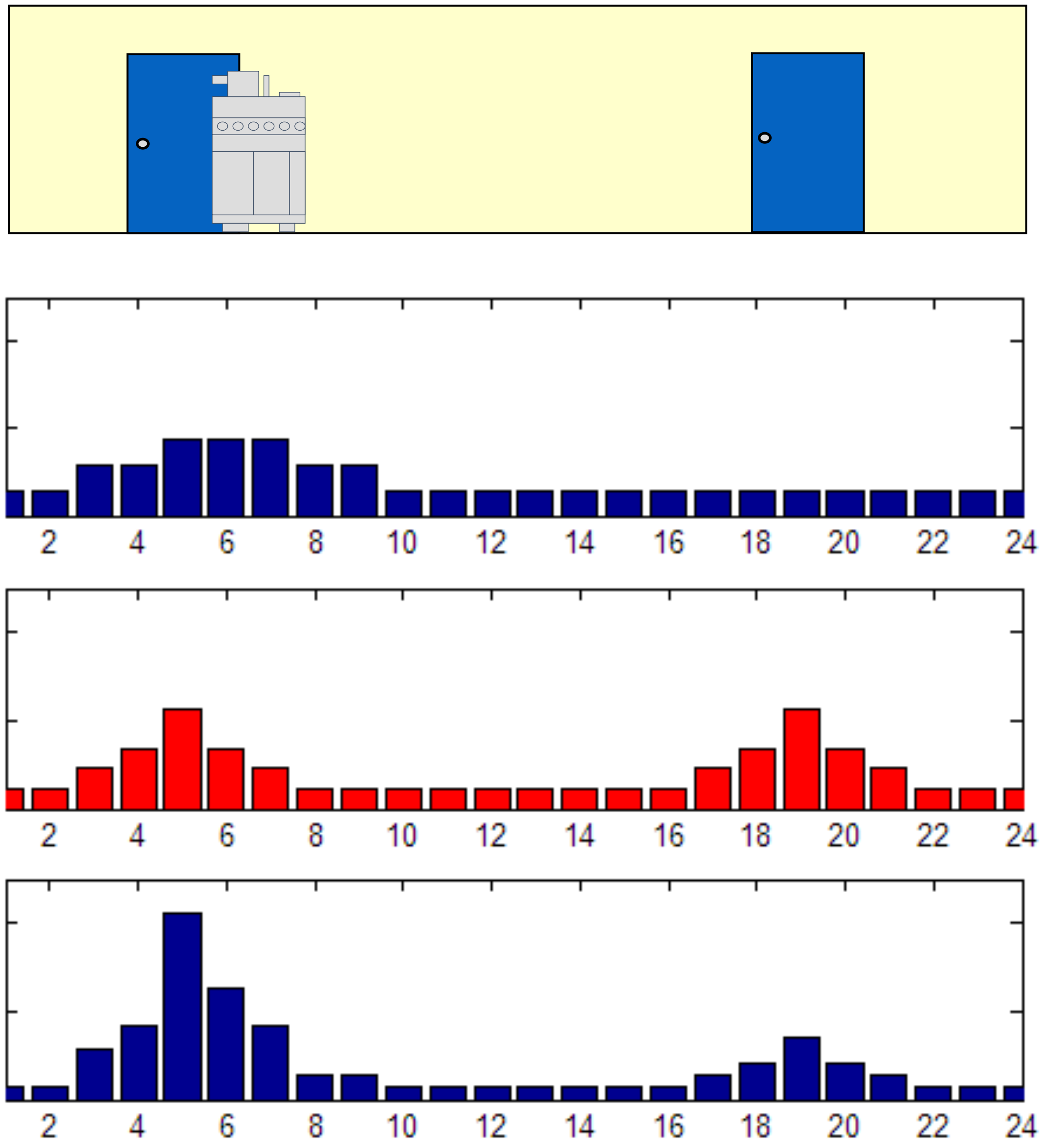




$$P(S_t|O_t = o_t) \propto P(o_t|S_t)P(S_t)$$
$$\propto L(S_t|o_t)P(S_t)$$

Sensor Model   State prior

Markov Localization: a 1D Example

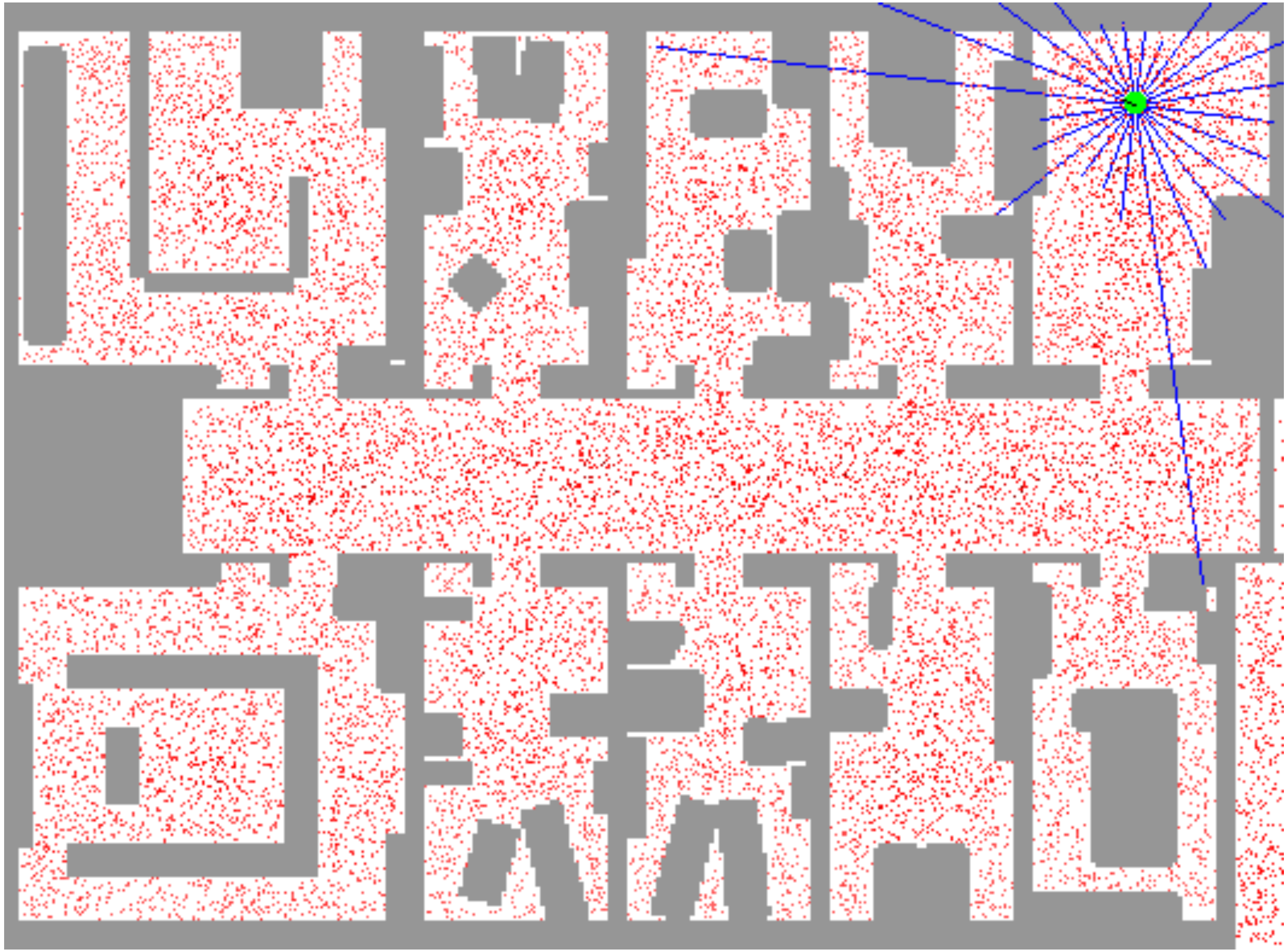


Prediction
P(S)

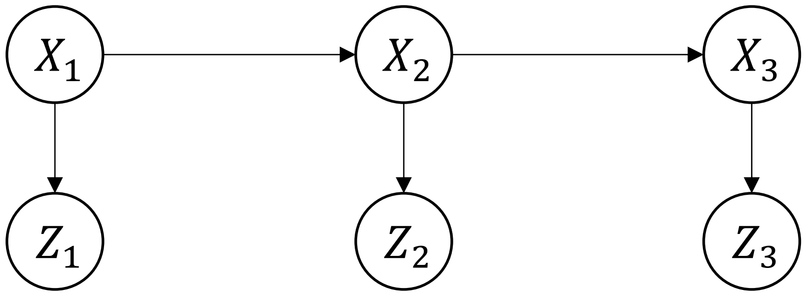
Likelihood
L(S;o)

Posterior
P(S|o)

Later: Monte Carlo Localization



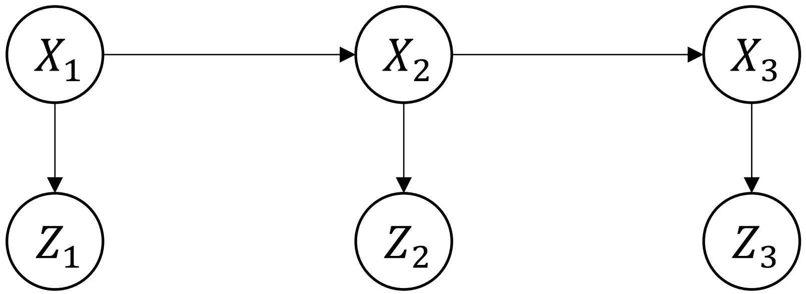
2. Hidden Markov Models



- Hidden states and given measurements.
- Joint distribution:

$$P(\mathcal{X}, \mathcal{Z}) = P(X_1)P(Z_1|X_1)P(X_2|X_1)P(Z_2|X_2)P(X_3|X_2)P(Z_3|X_3)$$

Bayes' rule for inference



- Consider states and measurements as sets:

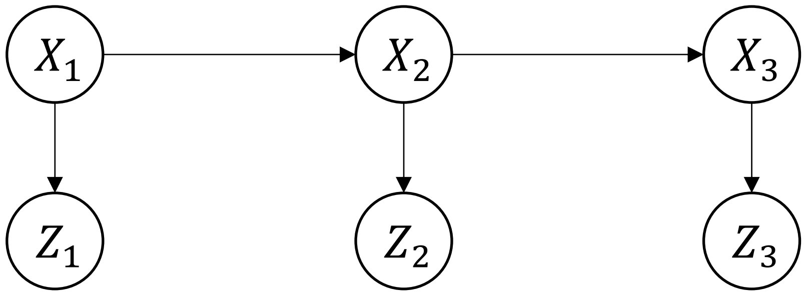
$$P(\mathcal{X}, \mathcal{Z}) = P(\mathcal{Z}|\mathcal{X})P(\mathcal{X})$$

- Bayes' rule:

$$\begin{aligned} P(\mathcal{X}|\mathcal{Z}) &\propto P(\mathcal{Z} = \mathfrak{z}|\mathcal{X})P(\mathcal{X}) \\ &= L(\mathcal{X}; \mathcal{Z} = \mathfrak{z})P(\mathcal{X}) \end{aligned}$$

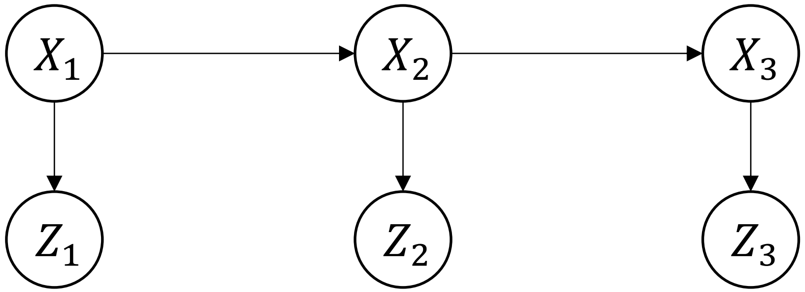
$$\begin{aligned} L(\mathcal{X}; \mathcal{Z} = \mathfrak{z}) &= P(z_1|X_1)P(z_2|X_2)P(z_3|X_3) \\ &= L(X_1; Z_1)L(X_2; Z_2)L(X_3; Z_3) \end{aligned}$$

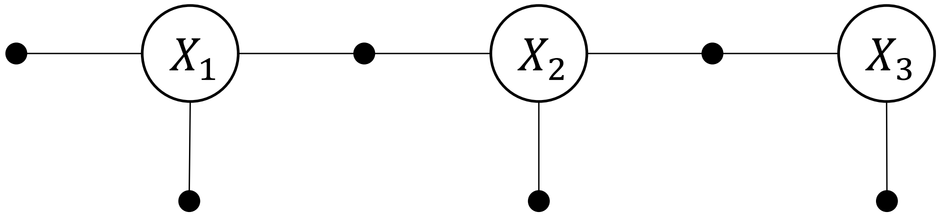
Three efficient inference methods



1. Branch & bound
2. Dynamic programming
3. Inference using factor graphs

3. Factor graphs





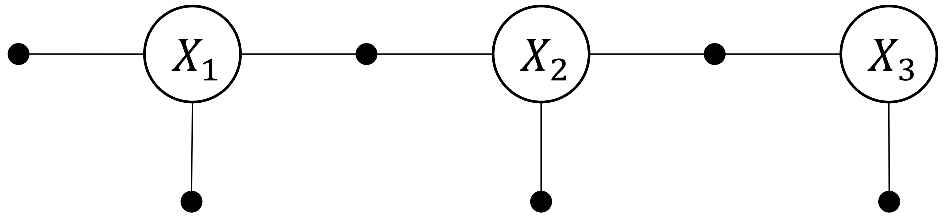
- Measurements are given: get rid of them!

$$P(\mathcal{X}|\mathcal{Z}) \propto P(X_1)L(X_1; z_1)P(X_2|X_1)L(X_2; z_2)P(X_3|X_2)L(X_3; z_3)$$

- This becomes:

$$\phi(\mathcal{X}) = \phi_1(X_1)\phi_2(X_1)\phi_3(X_1, X_2)\phi_4(X_2)\phi_5(X_2, X_3)\phi_6(X_3)$$

General definition of Factor graphs



- Bipartite graph of variables and factors

$$\phi(\mathcal{X}) = \prod_i \phi_i(\mathcal{X}_i)$$

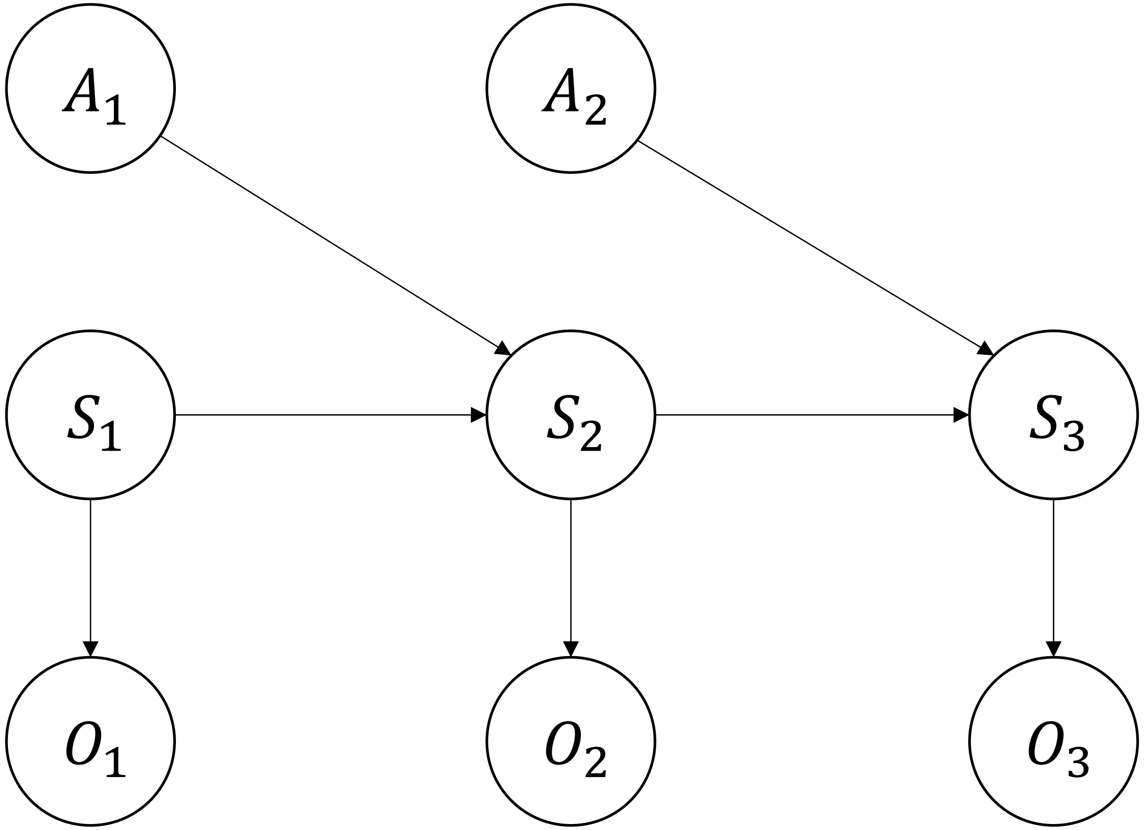
- Each $\mathcal{X}_i$ is the subset of variables connected to factor ϕ_i

Subsets here are:

$\mathcal{X}_1 = \{X_1\}$
$\mathcal{X}_2 = \{X_1\}$
$\mathcal{X}_3 = \{X_1, X_2\}$
$\mathcal{X}_4 = \{X_2\}$
$\mathcal{X}_5 = \{X_2, X_3\}$
$\mathcal{X}_6 = \{X_3\}$

4. Converting Bayes nets to factor graphs

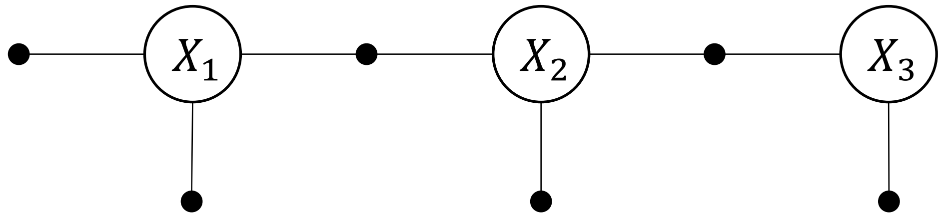
Every node in Bayes net becomes a factor, incl. evidence nodes.



Try converting when:

1. No known quantities
2. Observations and actions known
3. States known ??

5. The Max-product Algorithm for HMMs



Done in class:

- Eliminate X_1
- Eliminate X_2
- Eliminate X_3
- *Back-substitute*

Eliminate X_1

- Form product factor

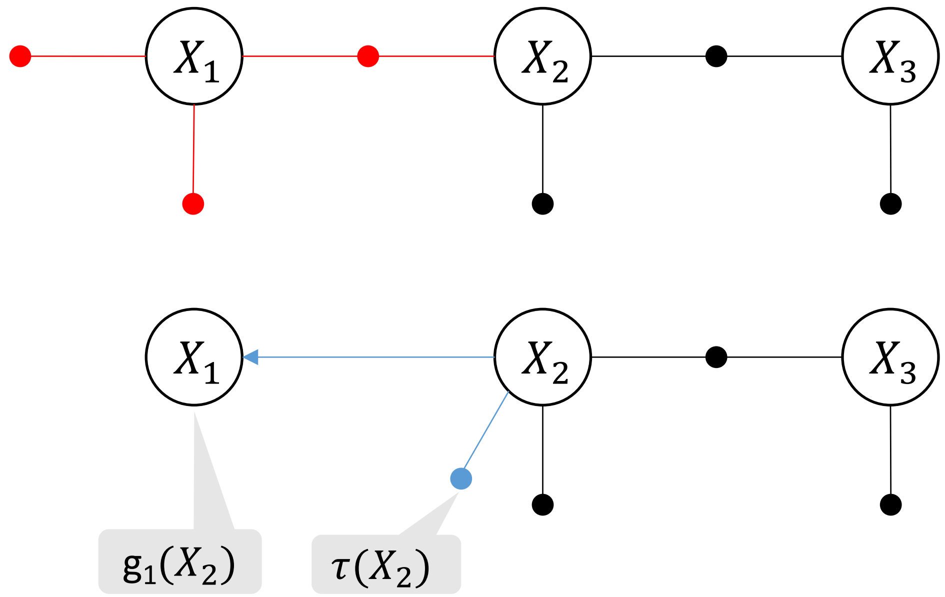
$$\psi(X_1, X_2) = \phi_1(X_1)\phi_2(X_1)\phi_3(X_1, X_2).$$

- Create lookup table:

$$g_1(X_2) = \arg\max_{x_1} \psi(x_1, X_2)$$

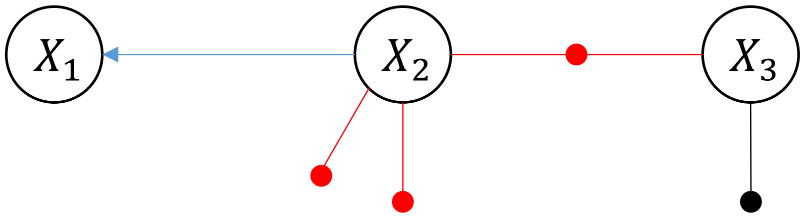
- Remember value:

$$\tau(X_2) = \max_{x_1} \psi(x_1, X_2)$$

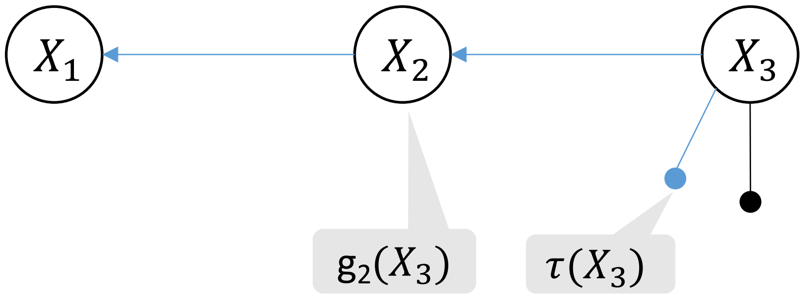


Eliminate X_2

- Form product factor



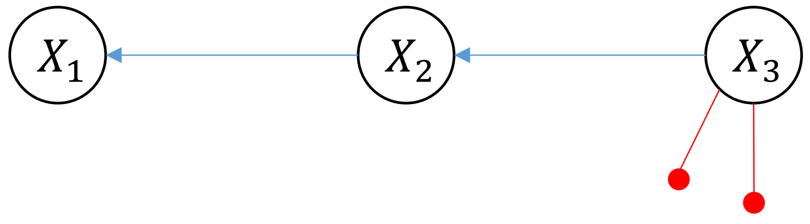
- Create lookup table:



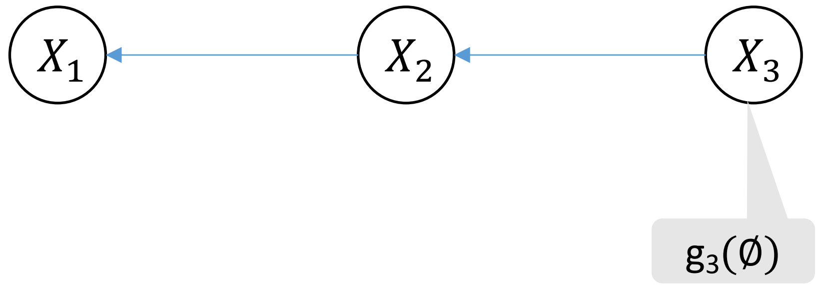
- Remember value:

Eliminate X_3

- Form product factor



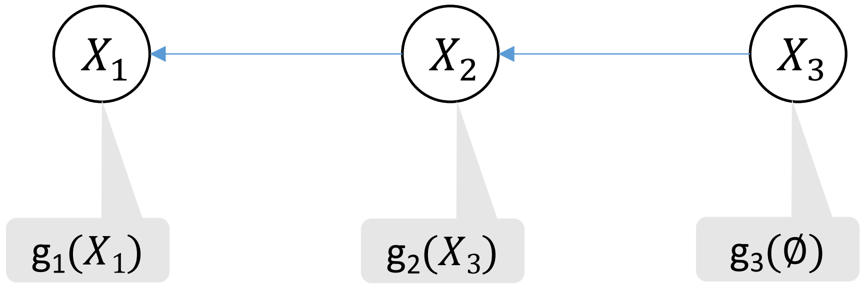
- Create lookup table:



- Remember value:

Back-substitute

- Get maximum probable explanation (MPE) in *reverse* elimination order.



Summary

- 1. Bayes Filter is great for localization
- 2. HMMs model entire trajectories
- 3. Factor Graphs make inference tractable
- 4. You can convert any Bayes Nets into a Factor Graph
- 5. The Max-Product Algorithm does MPE in linear time