

**CS 3630**



***Lecture 21:***  
**Structure from Motion**  
**and Visual SLAM**



# Overview

- Structure from Motion
- The Correspondence Problem
- Optimization
- Visual SLAM
- 4D Reconstruction

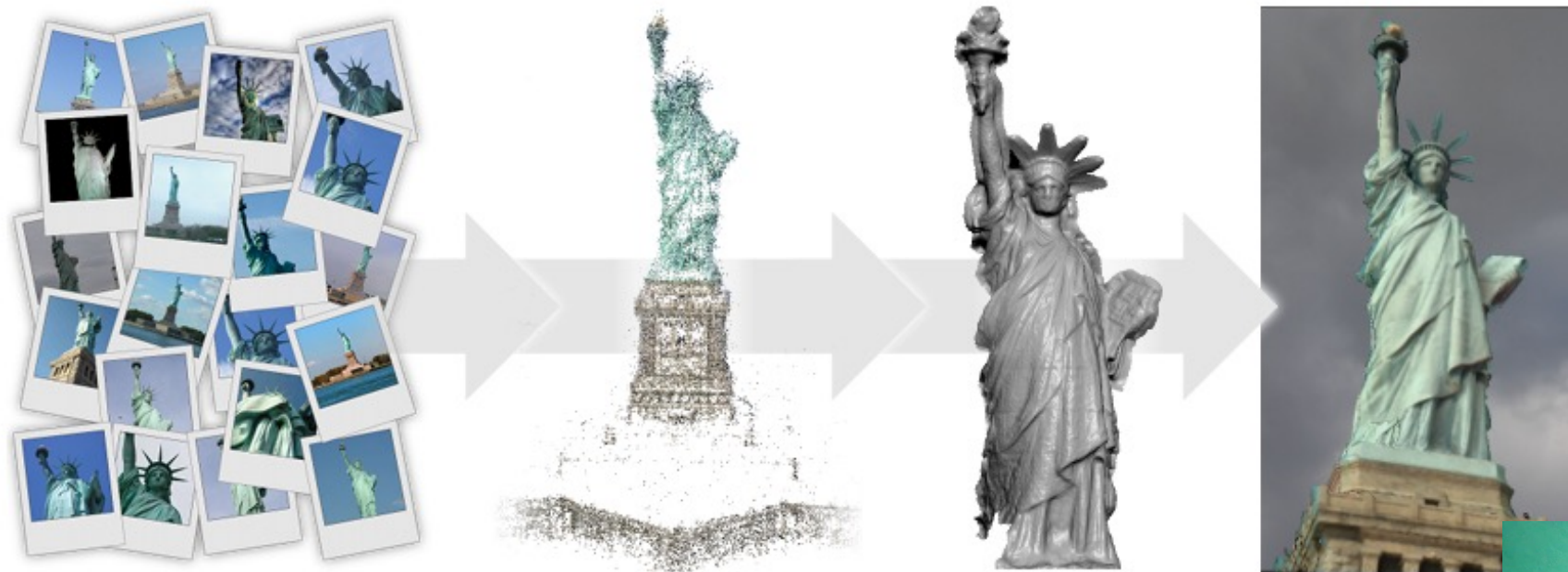


# Multi-view Stereo

Multi-View Stereo for Community Photo Collections

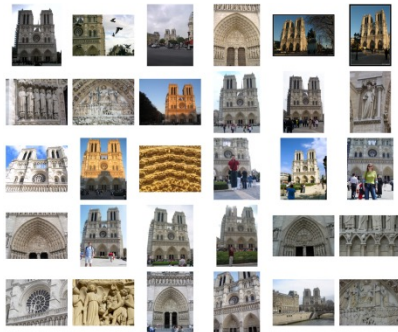
Michael Goesele, Noah Snavely, Brian Curless, Hugues Hoppe, and Steven M. Seitz

ICCV 2007

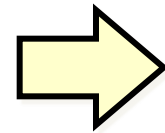


# Photo Tourism

Noah Snavely, Steven M. Seitz, Richard Szeliski, [Photo tourism: Exploring photo collections in 3D,](#) ACM Transactions on Graphics (SIGGRAPH Proceedings), 25(3), 2006, 835-846.



Input photographs



Scene reconstruction

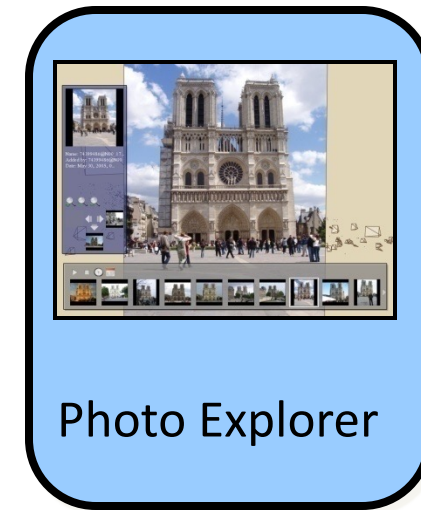
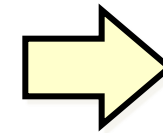
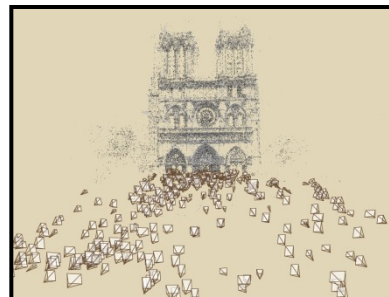


Photo Explorer

<http://phototour.cs.washington.edu/>



# 3D Models from Community Databases

- E.g.,

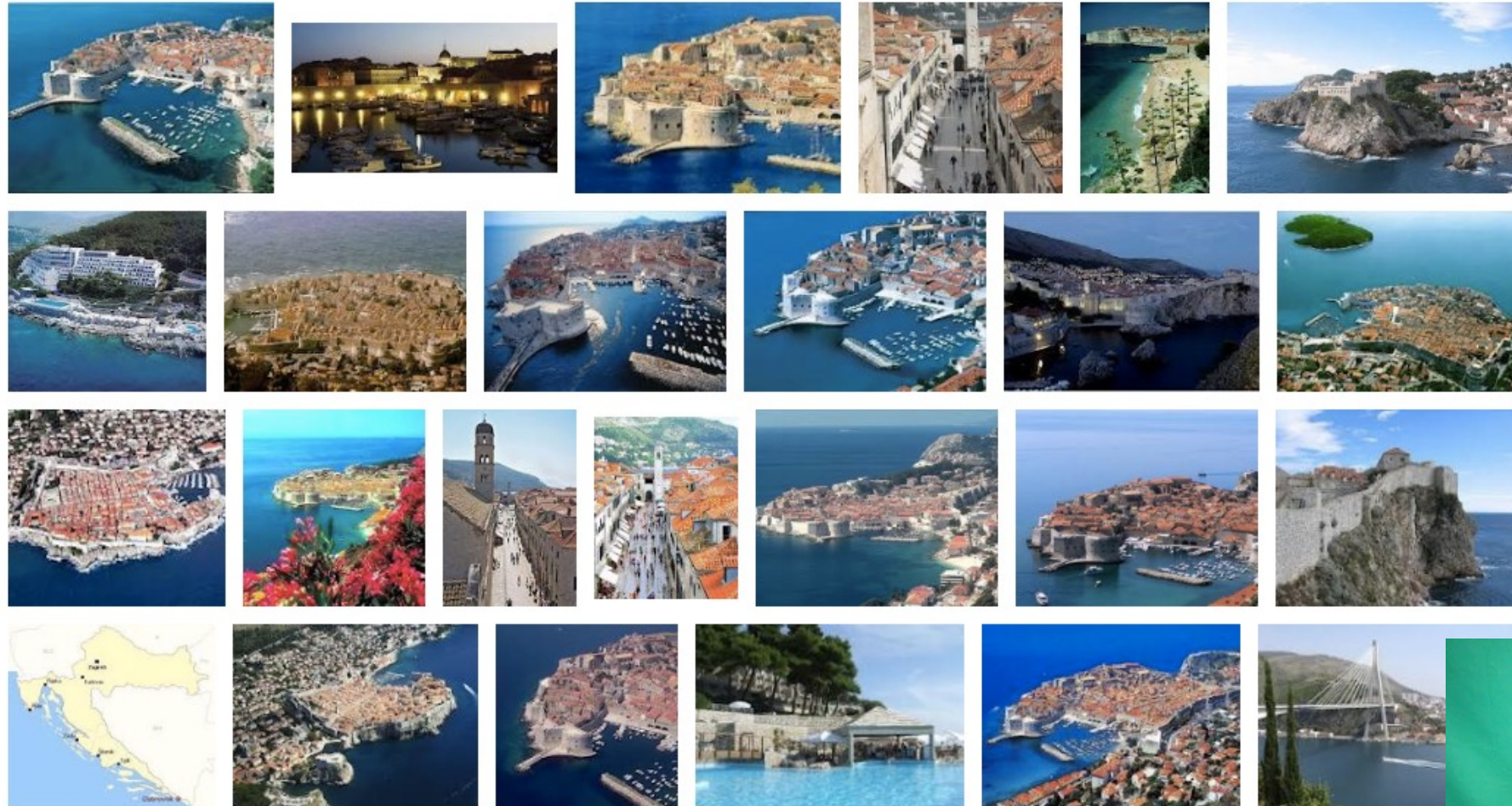


Figure by Aggarwal et al.



# 3D Models from Community Databases



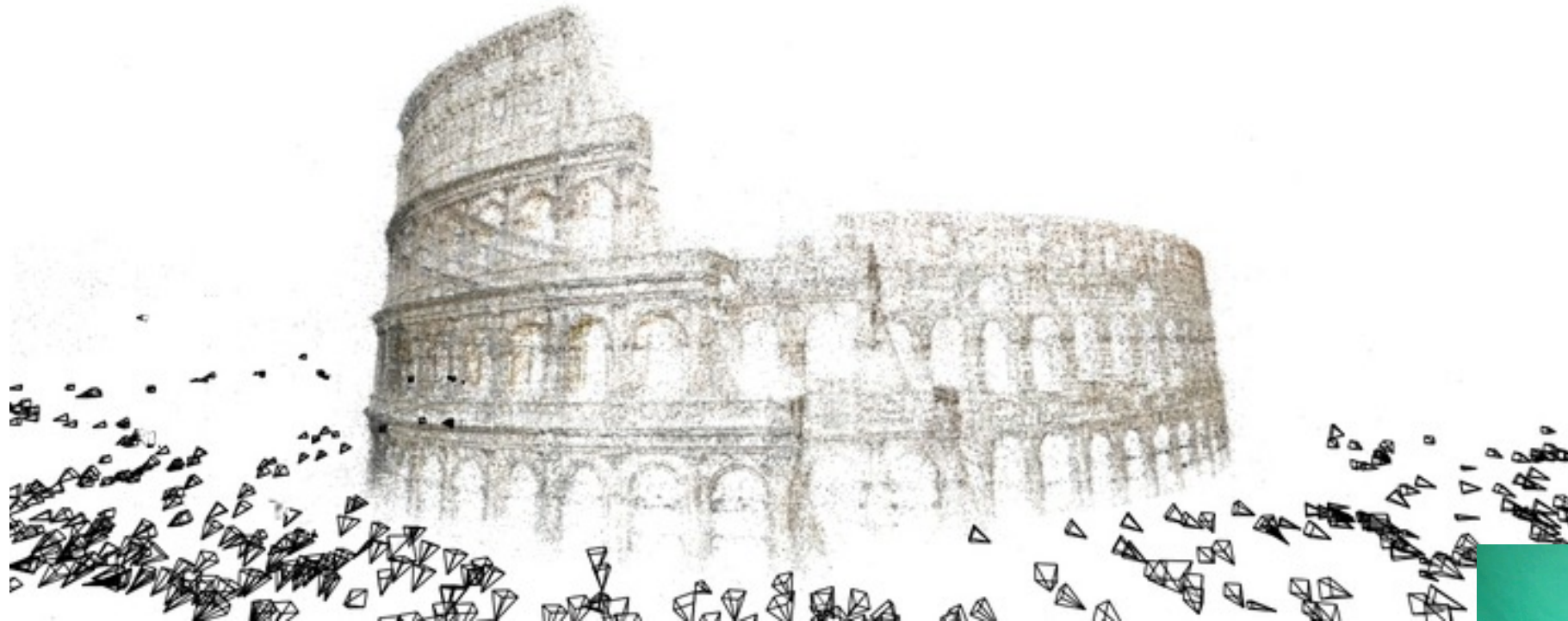
5K images, 3.5M points, >10M factors

Movie by Aggarwal et al.



# Building Rome in a Day

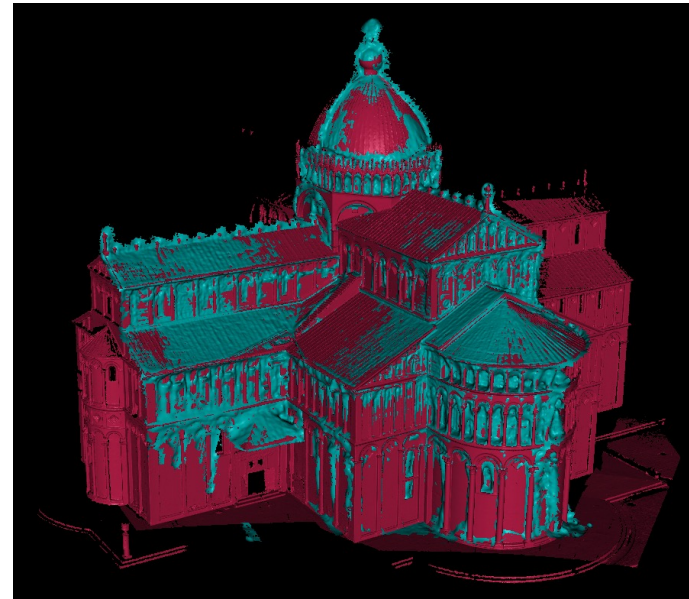
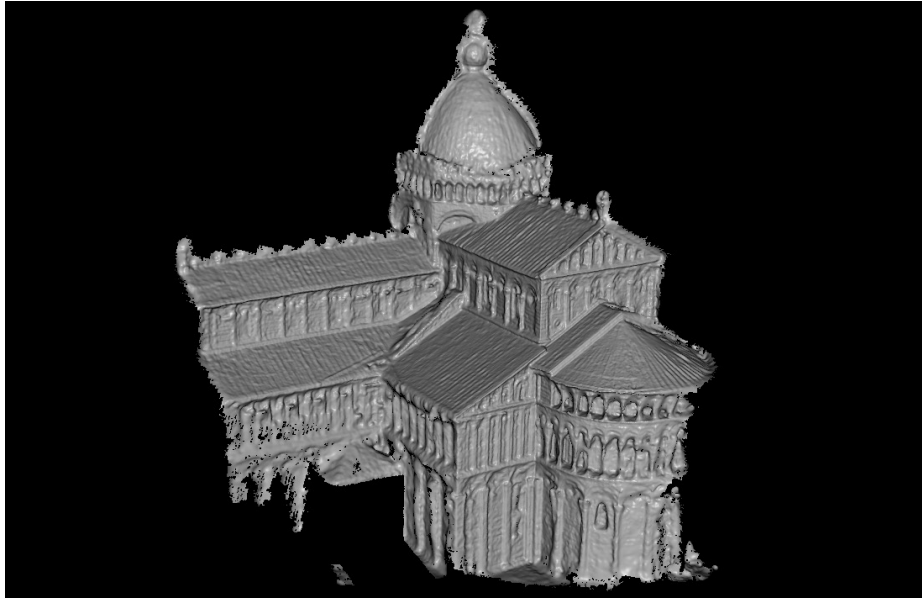
[Building Rome in a Day](#) Sameer Agarwal, Noah Snavely, Ian Simon, Steven M. Seitz and Richard Szeliski International Conference on Computer Vision, 2009, Kyoto, Japan.



<http://grail.cs.washington.edu/rome/>



# Multi-view Stereo



Compared with Laser-Scanner





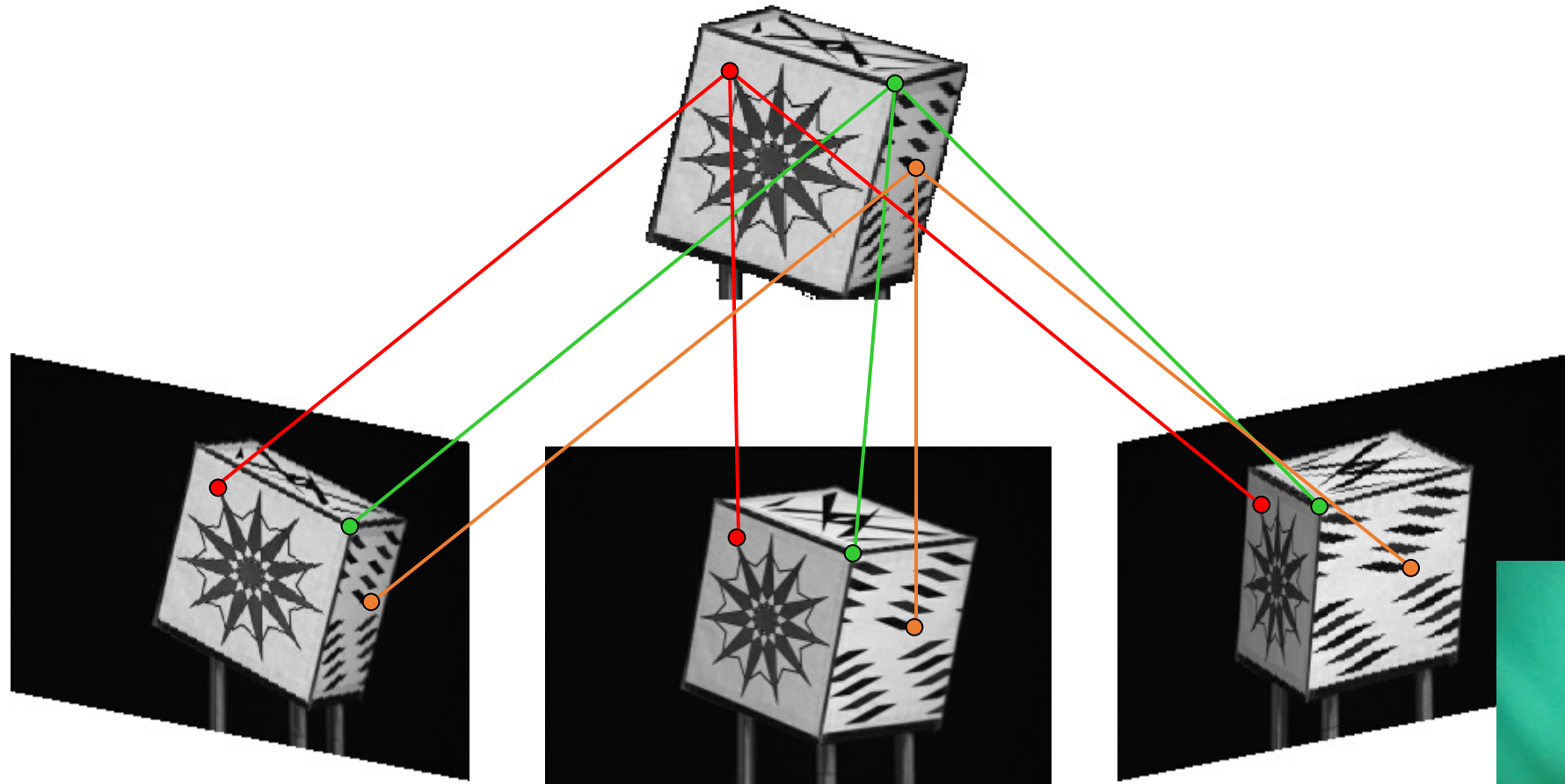
# 2 Problems !

Correspondence

Optimization

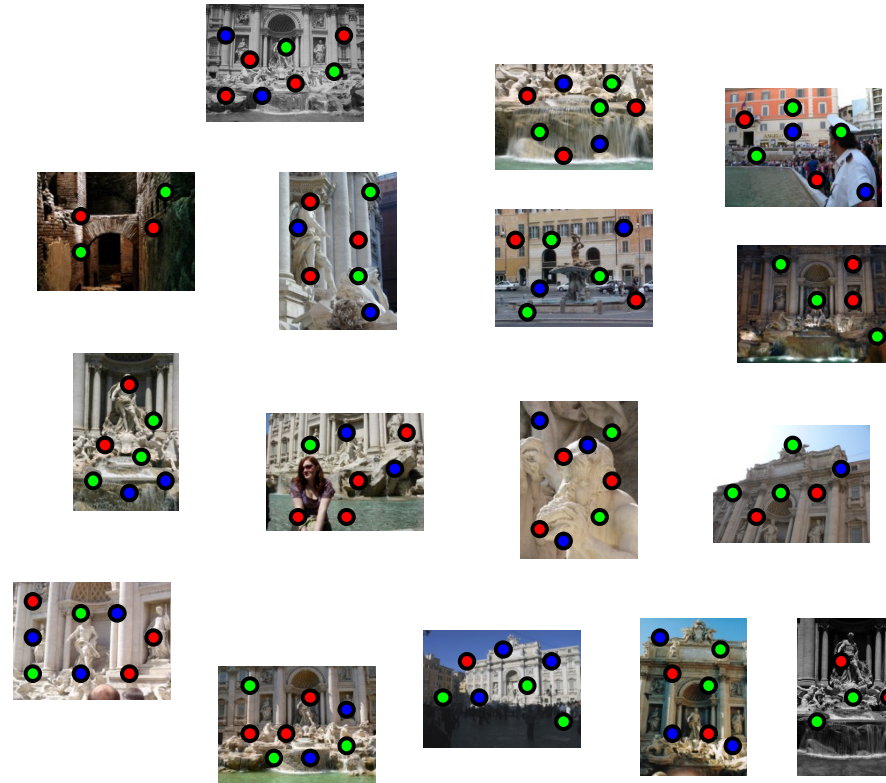


# A Correspondence Problem



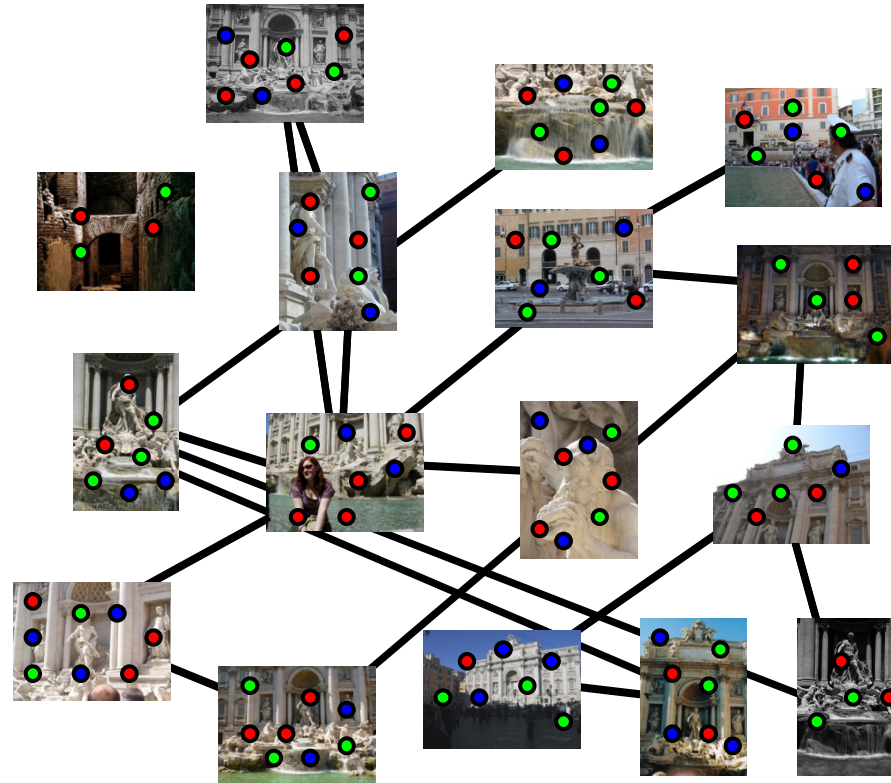
# Feature detection

- Detect features using SIFT [Lowe, IJCV 2004]

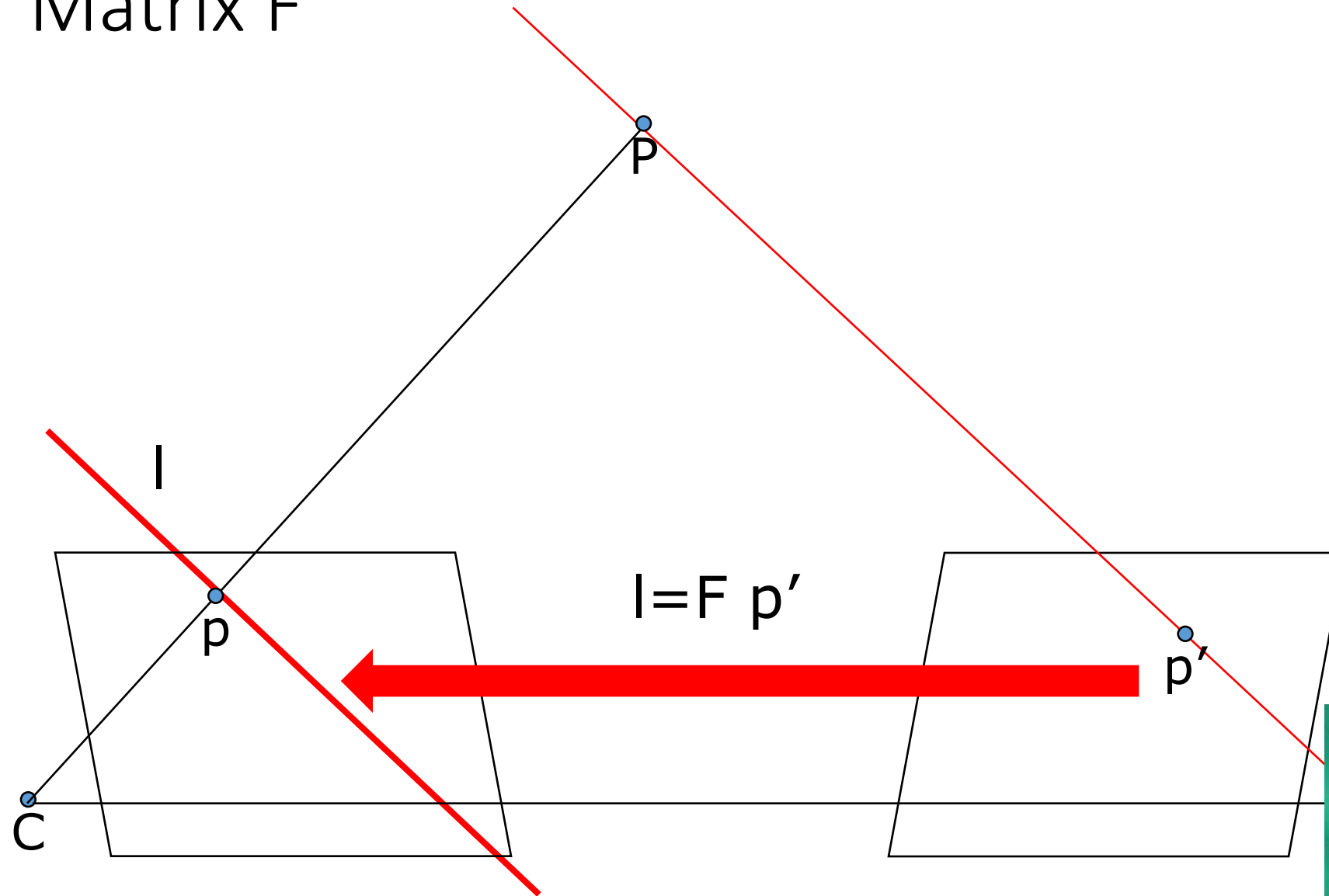


# Feature matching

Refine matching using RANSAC [Fischler & Bolles 1987] to estimate fundamental matrices between pairs



# Recap: Two views and Fundamental Matrix F



# 2 Problems !

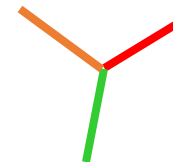
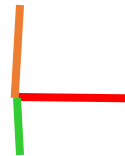
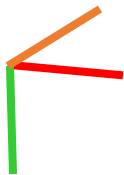
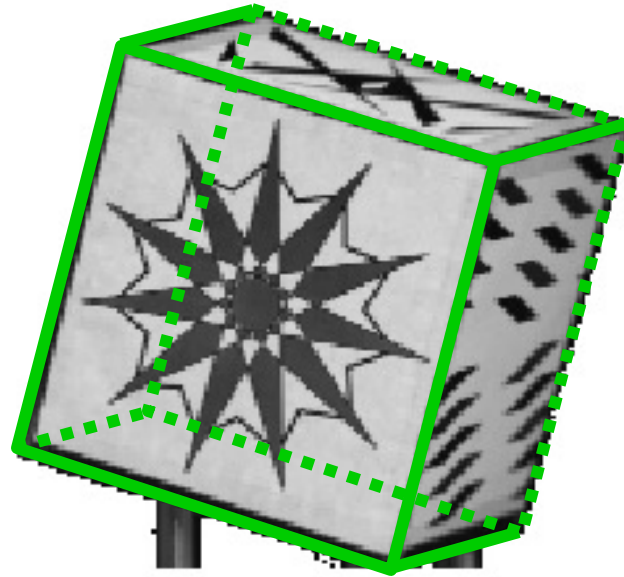
## Correspondence

Optimization



# An Optimization Problem

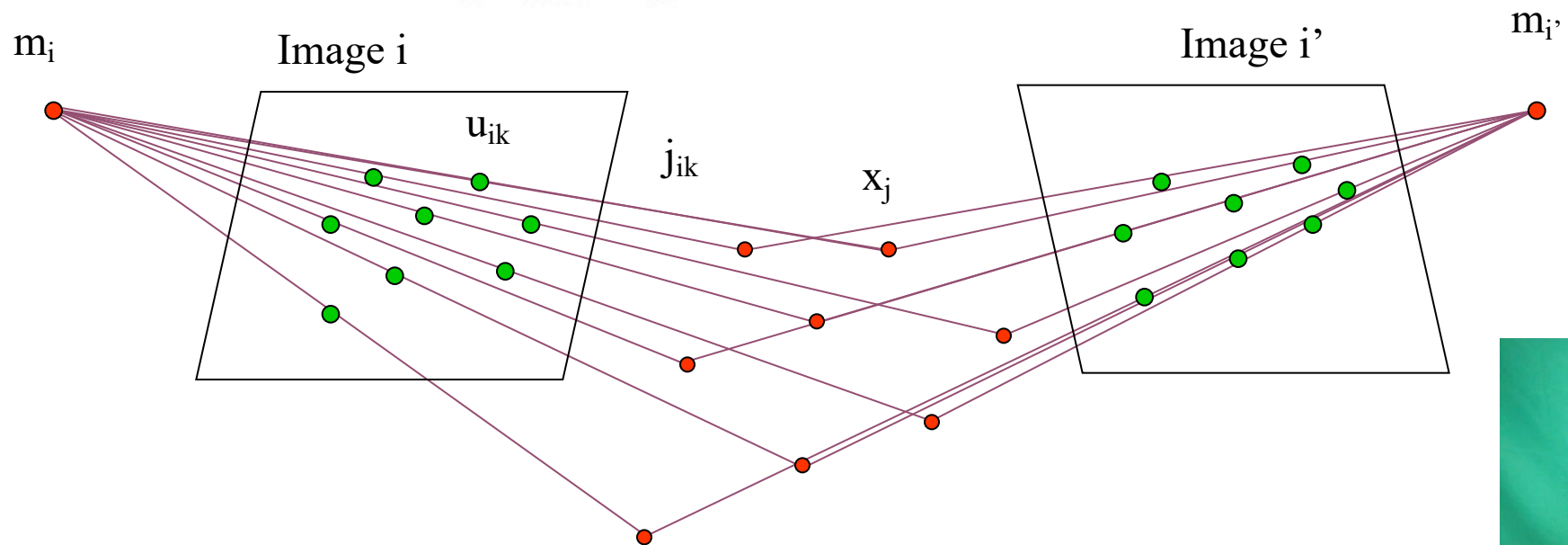
- Find the **most likely** structure and motion  $\Theta$



# Optimization

=Non-linear Least-Squares !

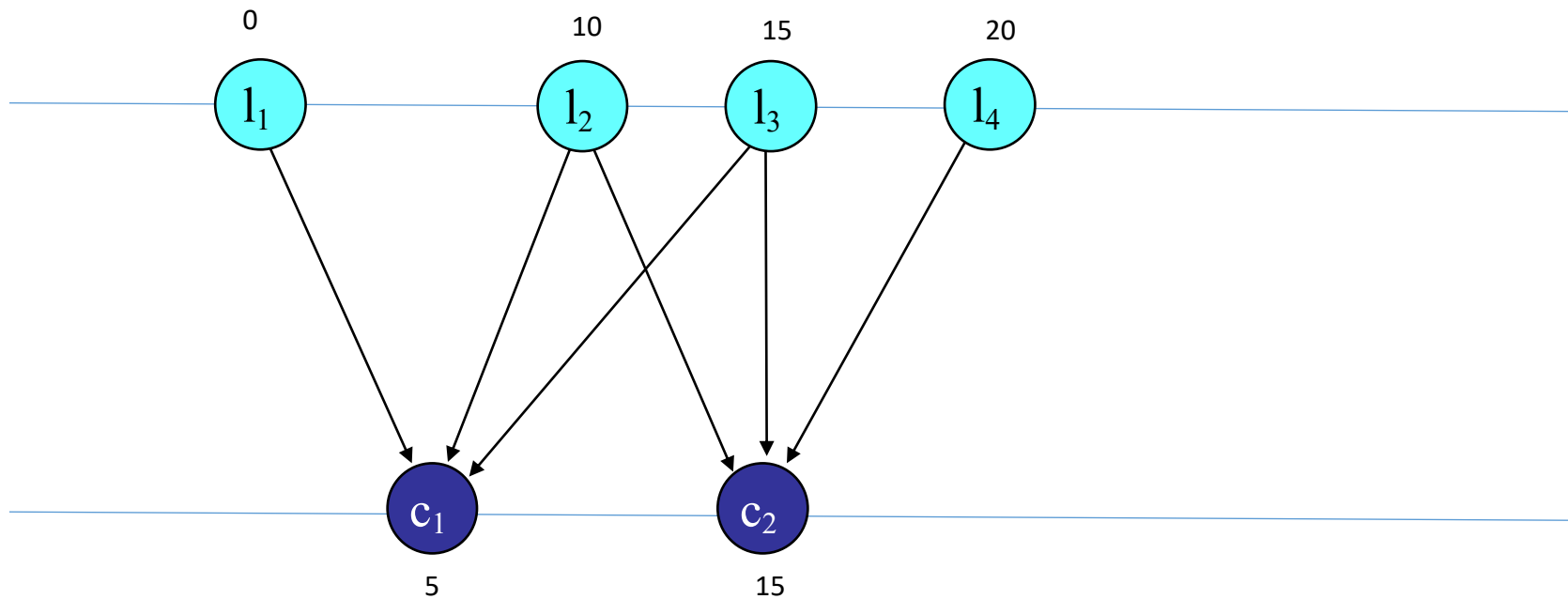
$$\sum_{i=1}^m \sum_{k=1}^{K_i} \|\mathbf{u}_{ik} - \mathbf{h}(\mathbf{m}_i, \mathbf{x}_{j_{ik}})\|^2$$





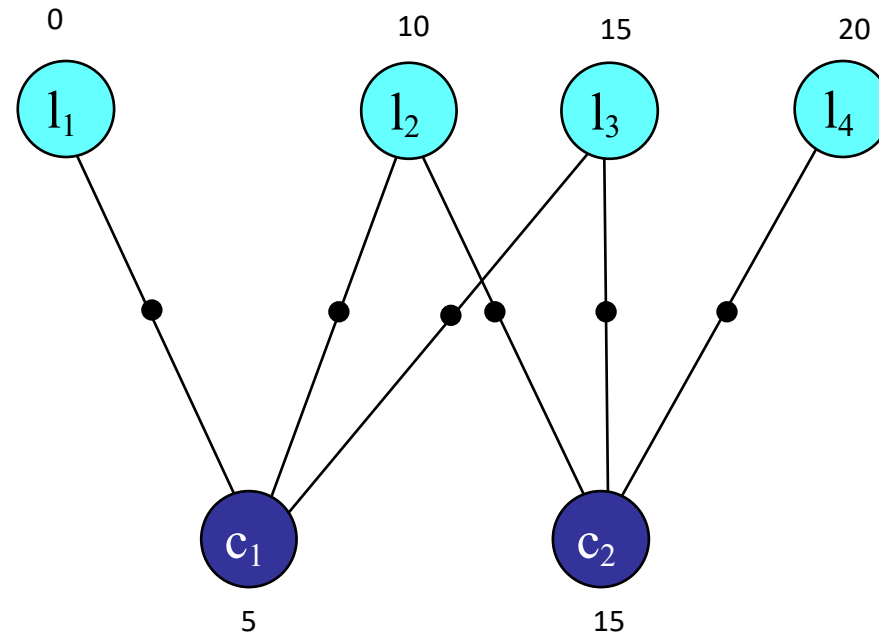
# Sparse nonlinear least squares

- Simple 1-Dimensional Example
- $p = 2$  cameras and 4 points:  $\{c_1, c_2, l_1, l_2, l_3, l_4\}$
- $f(u_{ik}; p) = \text{difference in x position} = l_{j(ik)} - c_i$

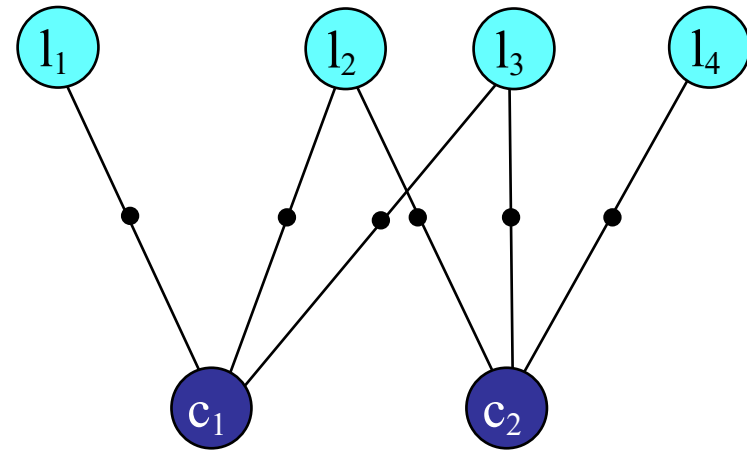


# Model with Factor Graphs

- Connectivity = sparsity!
- Factor is function of small set.



# Sparse Jacobian and Hessian



$$A = \begin{array}{c|cccccc} & c1 & c2 & l1 & l2 & l3 & l4 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{array}$$

b =

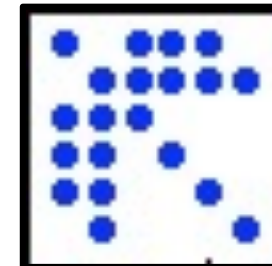
5  
-5  
5  
10  
-15  
-5  
0  
5



$$A' * A = \begin{array}{c|cccccc} & c1 & c2 & l1 & l2 & l3 & l4 \\ \hline 4 & 0 & -1 & -1 & -1 & 0 \\ 0 & 4 & -1 & -1 & -1 & -1 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ -1 & -1 & 0 & 2 & 0 & 0 \\ -1 & -1 & 0 & 0 & 2 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{array}$$

$(A' * A) \setminus A' * b =$

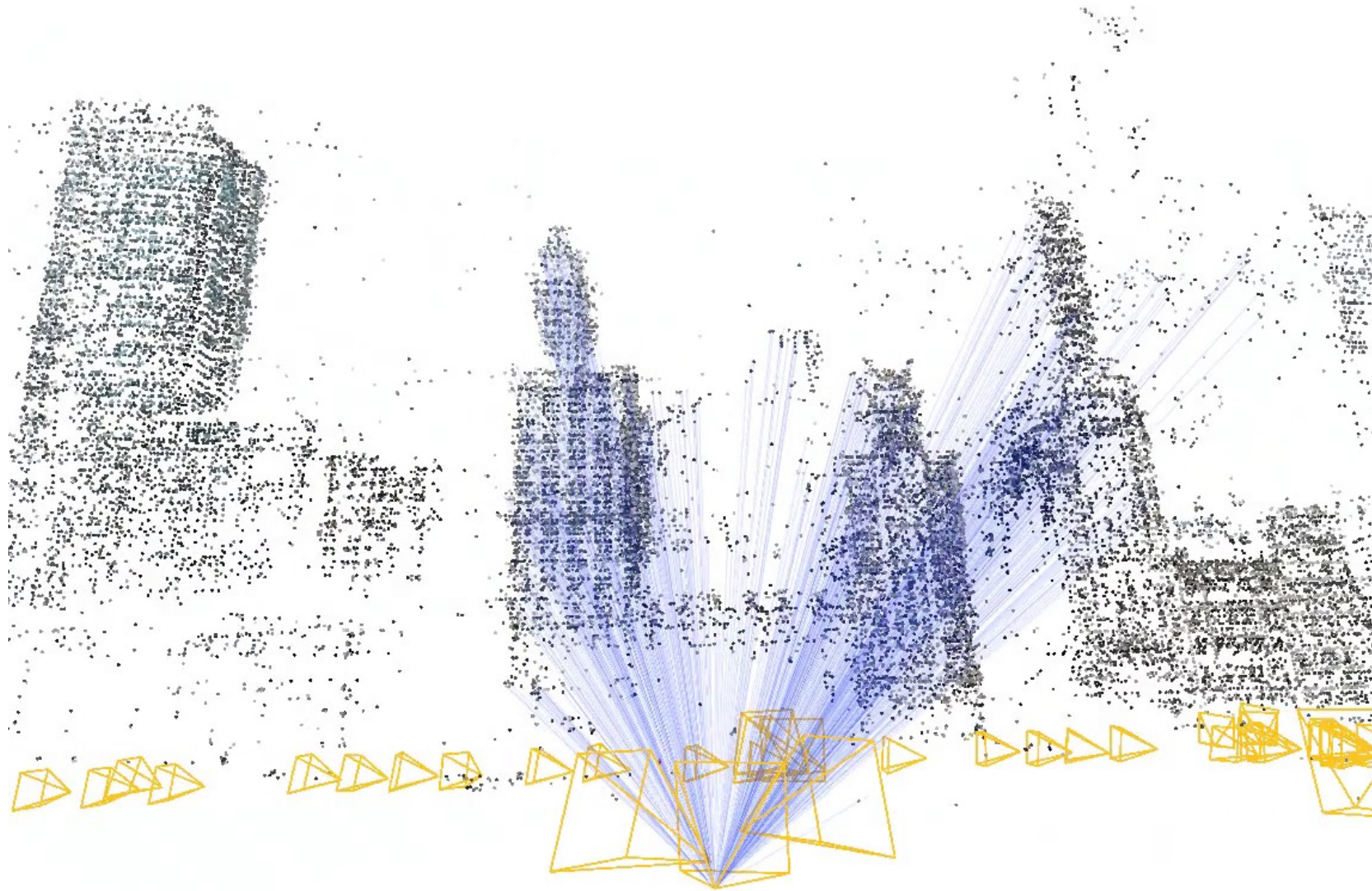
5.0000  
15.0000  
0.0000  
10.0000  
15.0000  
20.0000



# Structure from Motion (Chicago, movie by Yong Dian Jian)

180 cameras, 88723 points  
458642 projections  
active camera: 4

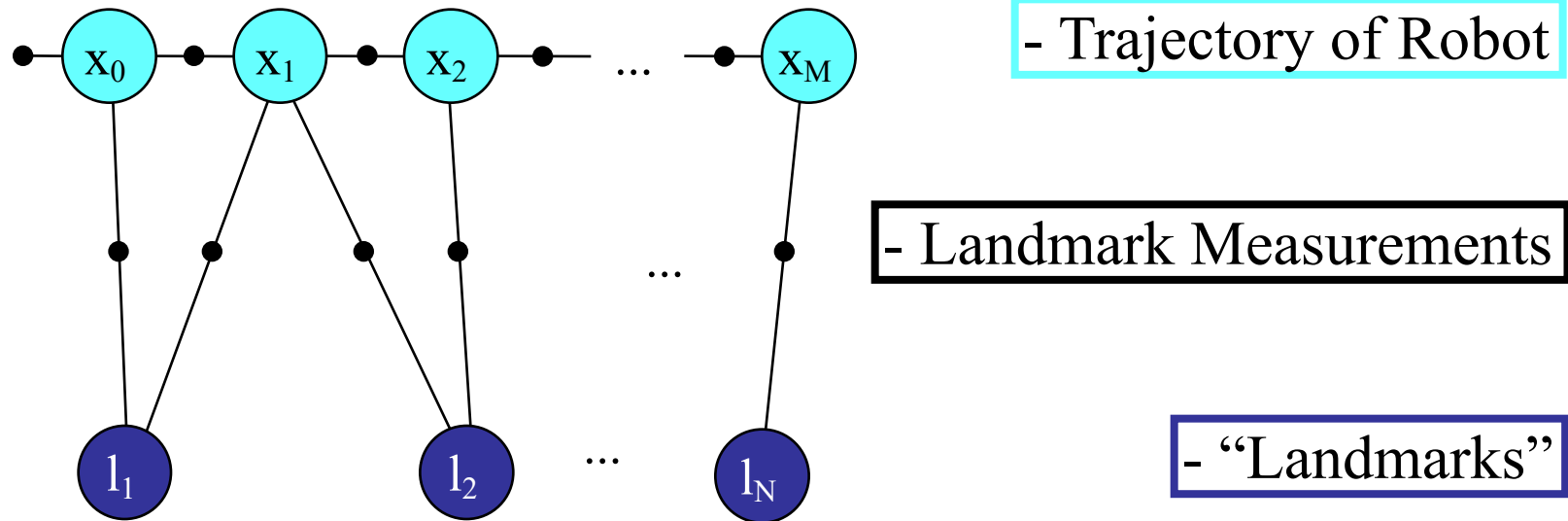
Original graph



# Visual SLAM: SfM for Robots



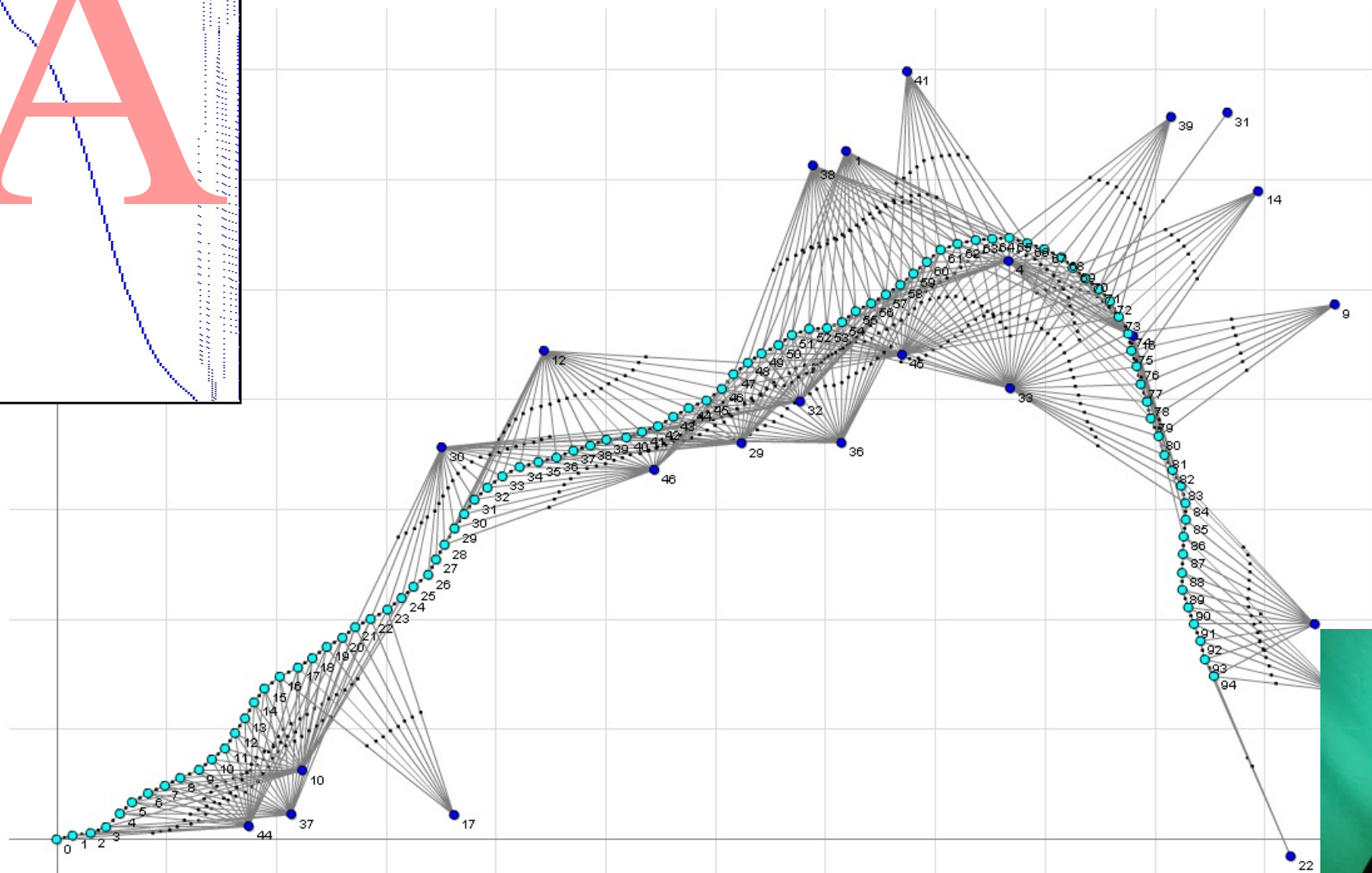
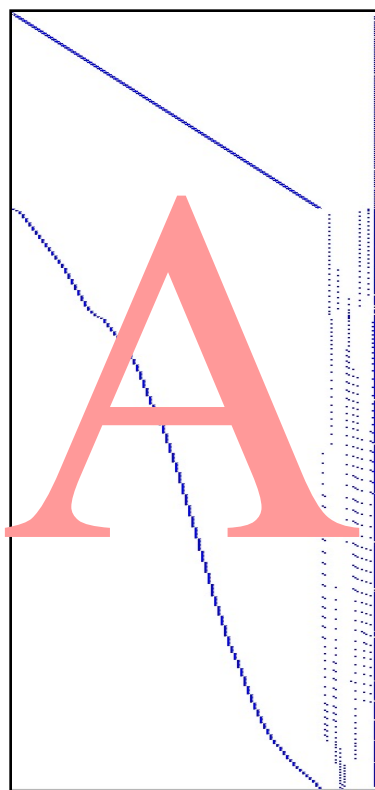
# Visual SLAM Factor Graph



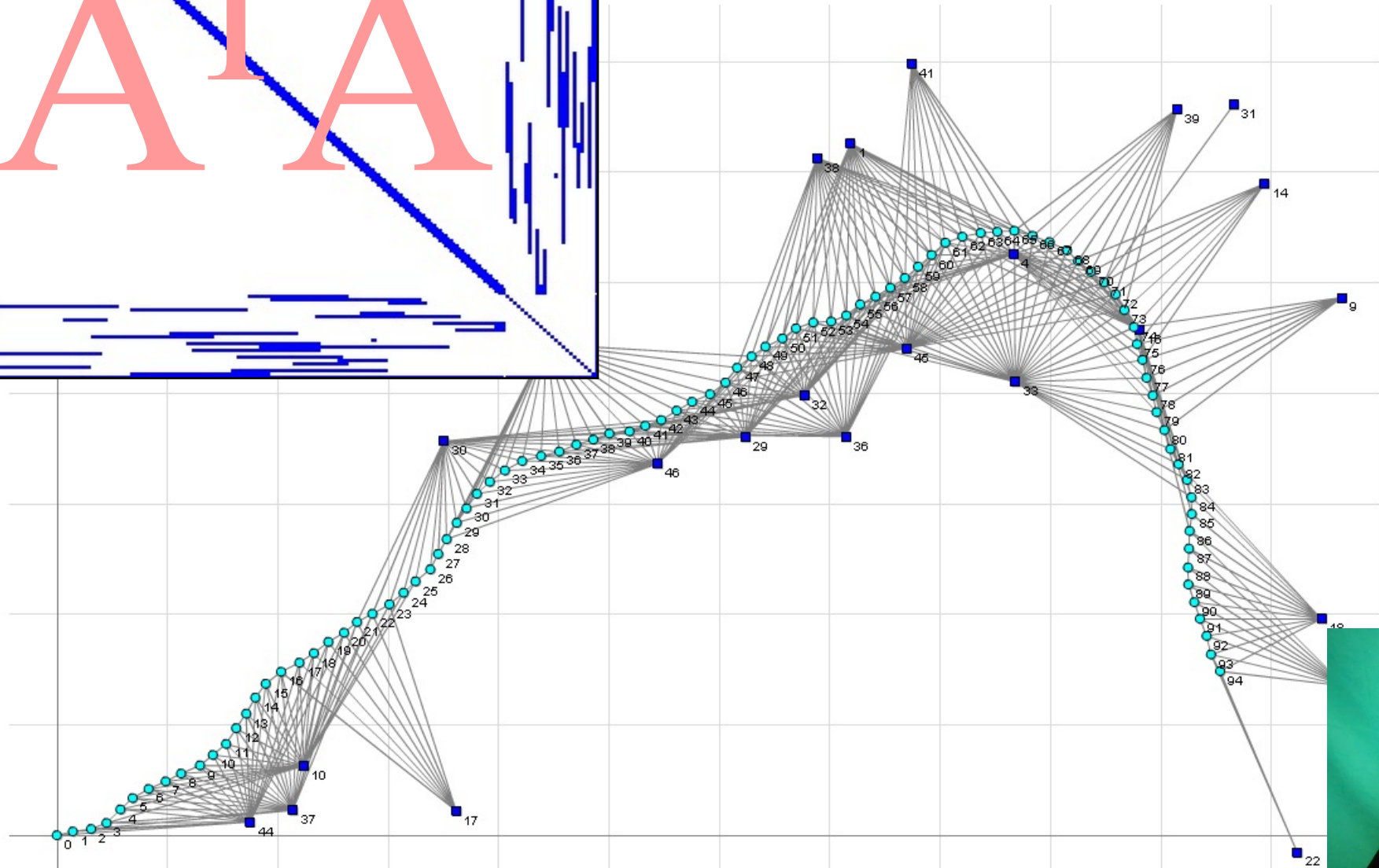
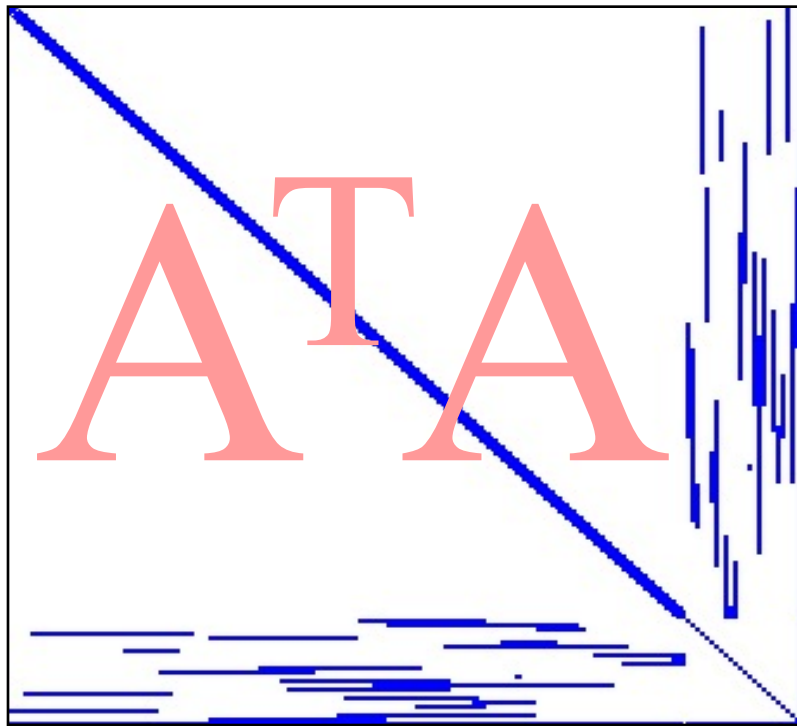
$$P(X, M) = k^* P(x_0) \prod_{i=1}^M P(x_i | x_{i-1}, u_i) \times \prod_{k=1}^K P(z_k | x_{i_k}, l_{j_k})$$



# Visual SLAM Factor Graph

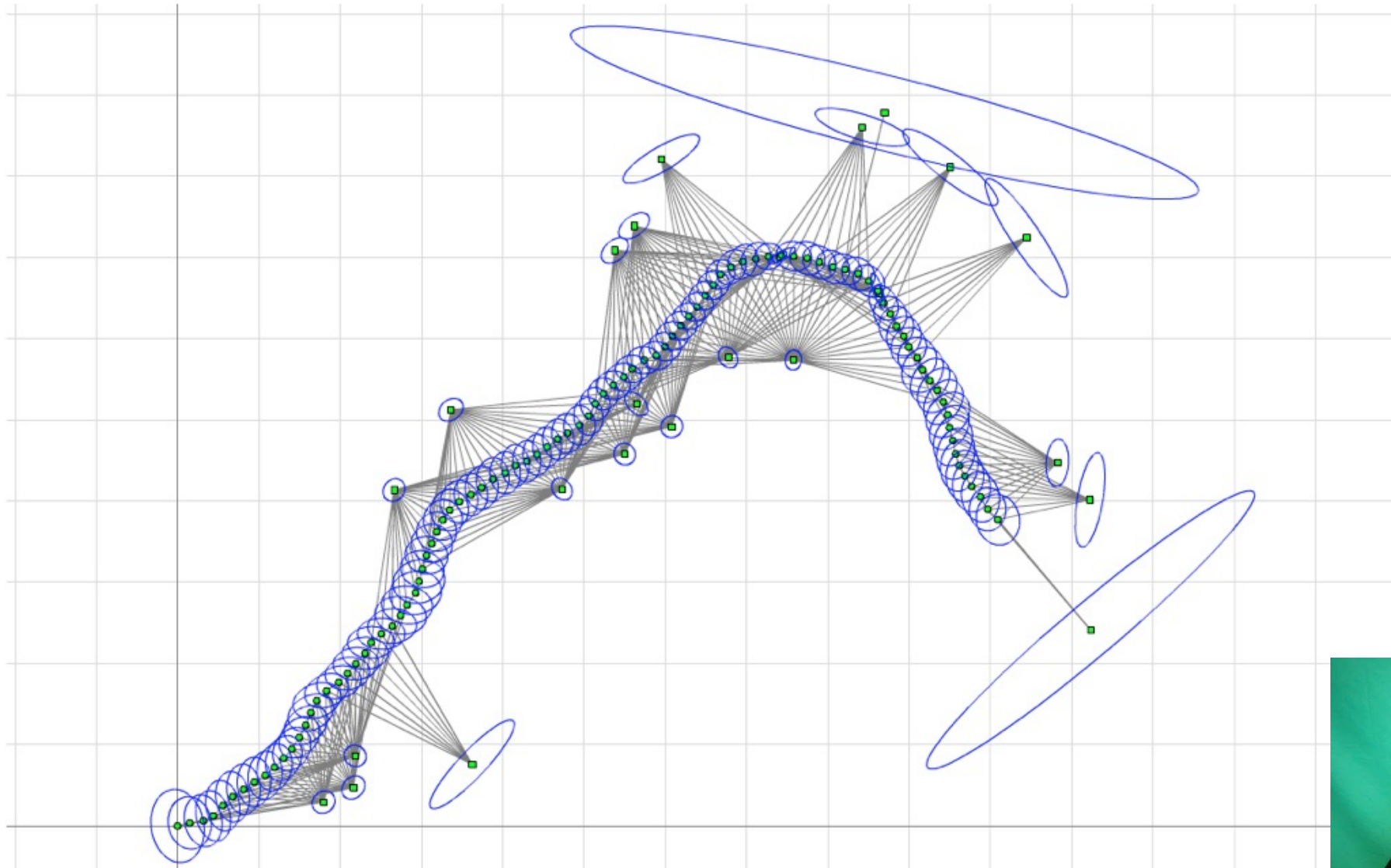


# Hessian



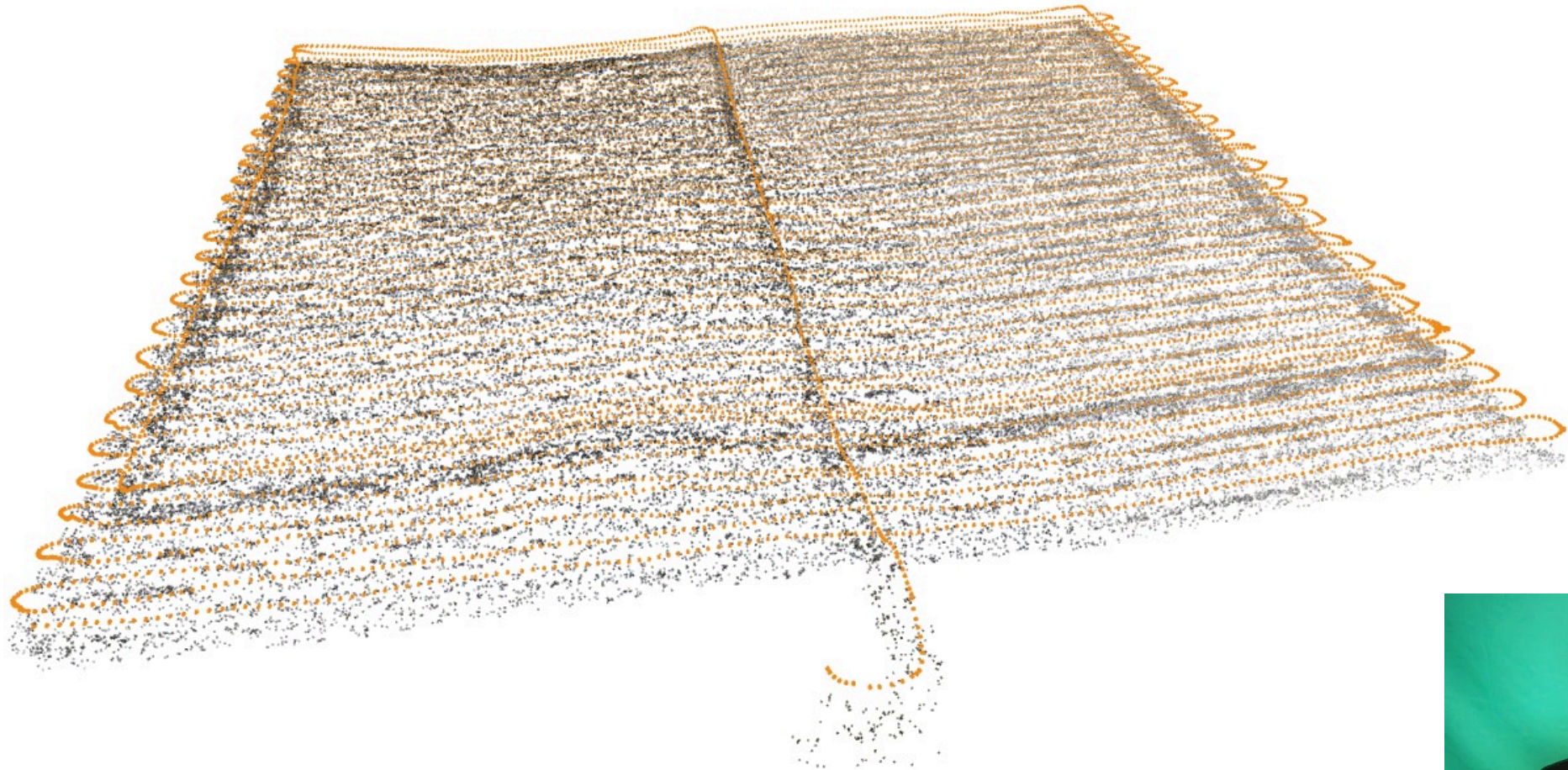


# End result: Solution + Uncertainty



# Example: Underwater SLAM

9831 camera poses, 185261 landmarks, and 350988 factors

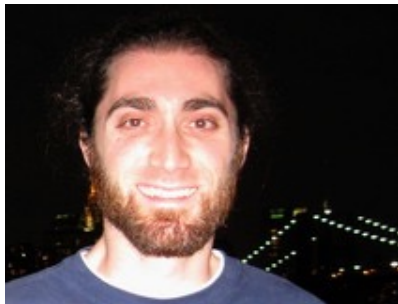
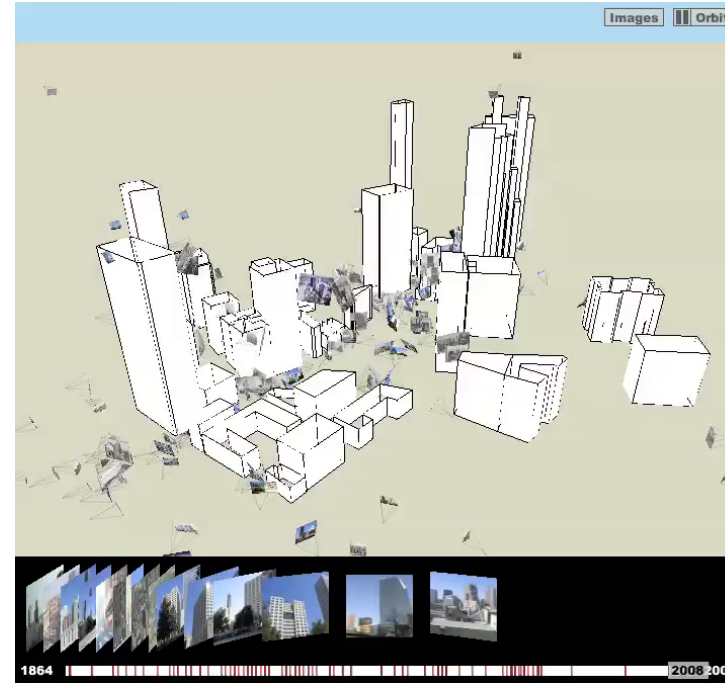
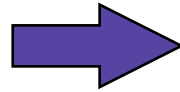


# Spatiotemporal Reconstruction

4D Cities: 3D + Time



Historical Image Collection



Grant Schindler

Supported by NSF CAREER, Microsoft  
Recent revival: NSF NRI award on 4D  
crops for precision agriculture...



# 4D Reconstruction of Lower Manhattan



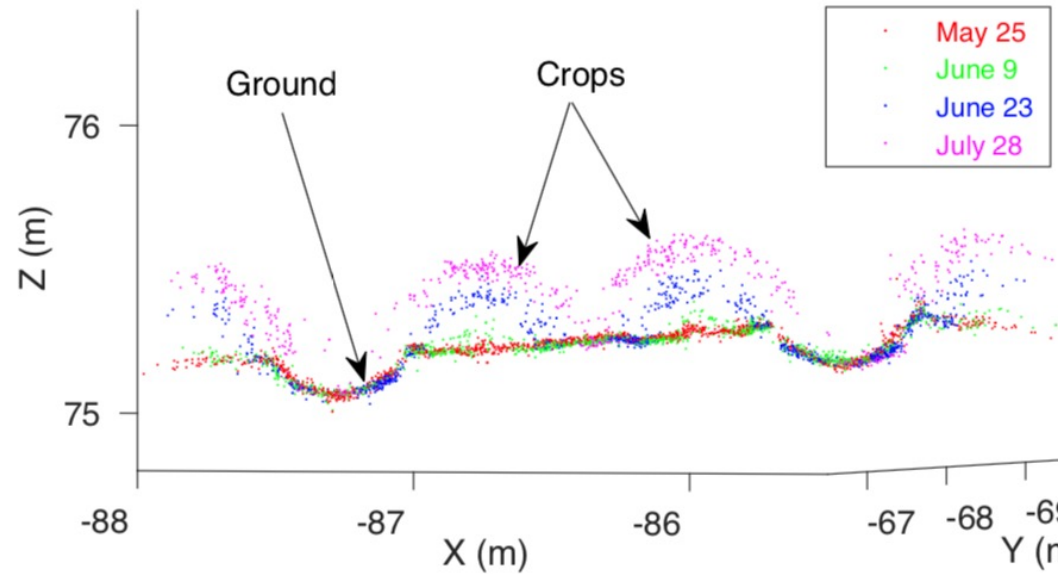
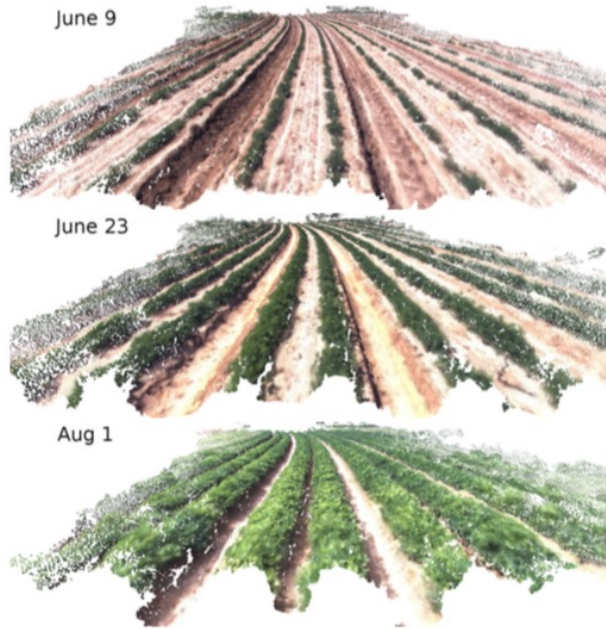
[Probabilistic Temporal Inference on Reconstructed 3D Scenes](#), G. Schindler and F. Dellaert, IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR), 2010.



# 4D Structure over Time



# 4D crop monitoring (Jing Dong)



# Results: video (by Jing Dong)

May 25, 2016



May 25, 2016



4D reconstruction results (by PMVS)  
and its cross section

