CS 3630!

Lecture 13: Trajectory Optimization

Topics

- **A Motivating Example**
- **A Factor Graph Representation**
- **A 1-D Version is Linear!**
- **Trajectory Optimization and Bayes Law**
- **Trajectory Optimization as Least-Squares**

Motivation

- Navigation for drones, autonomous cars, mobile robots
- Need a way to do MAP estimation in continuous spaces
- Because this involves fusing information from multiple sensors, is an instance of "sensor fusion"

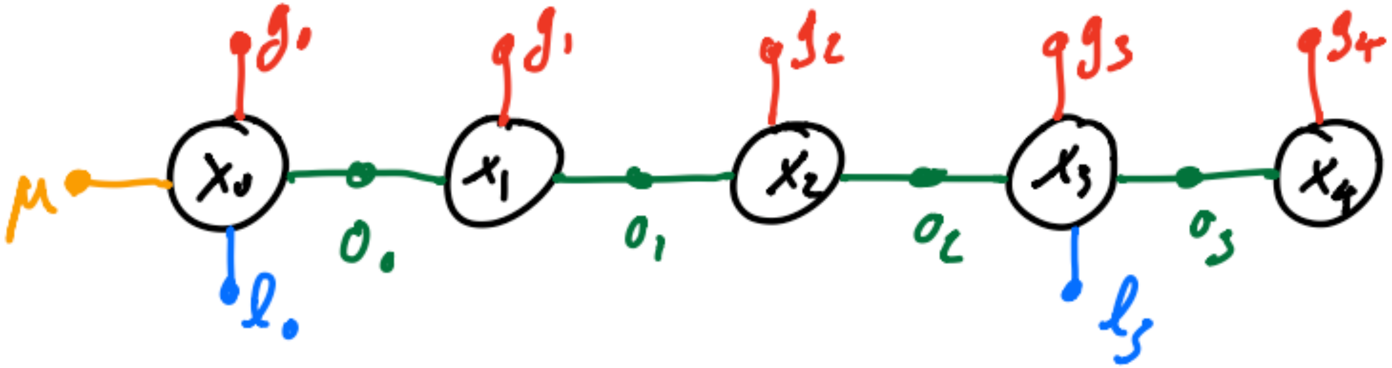
A Motivating Example

Autonomous vehicle, driving on the highway, and

- regular GPS measurements
- odometry measurements from sensors on the wheels
- from time to time, we observe a landmark
- Optionally: we know that GPS is biased

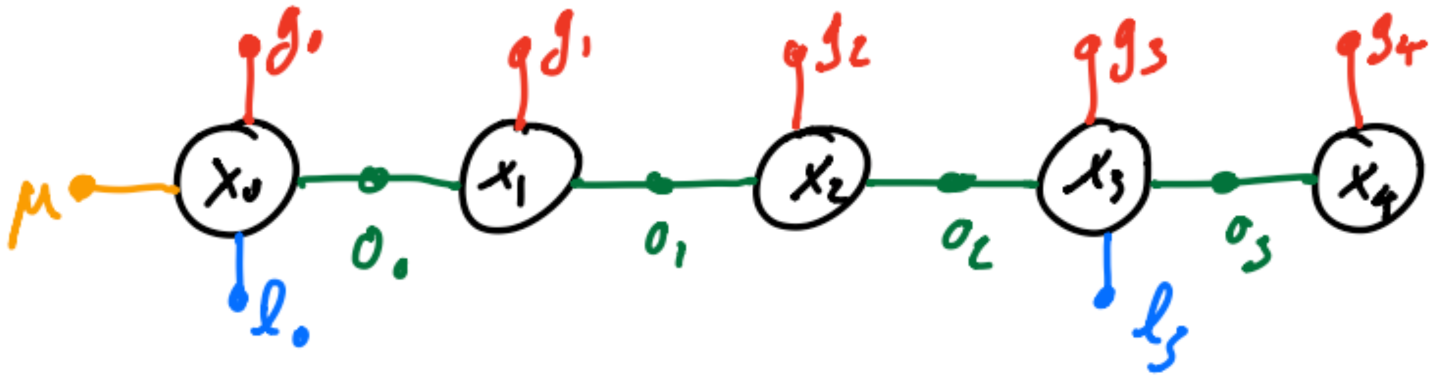
Factor Graph Representation

- 5 variables
- Factors for all measurement types
- Extra factor for prior
- Some factors are:
  - Binary (odometry)
  - Unary (all others)



- Exercise: what will happen if we model GPS bias?

1-D Version is Linear!



1. GPS: $h_{GPS}(x_k) = x_k$
2. Odometry from time t_k to time t_{k+1}: $h_{ODO}(x_k, x_{k+1}) = x_{k+1} - x_k$
3. Landmark observations: $h_{LM}(x_k; l_k) = l_k - x_k$, where $l_k \in R$ is the location of the landmark at time t_k. In other words, we just measure the *signed* distance to the landmark[1].
4. In case GPS is biased, we modify the GPS measurement model: $h_{GPS}(x_k) = x_k + b$

Trajectory Optimization and Bayes Law

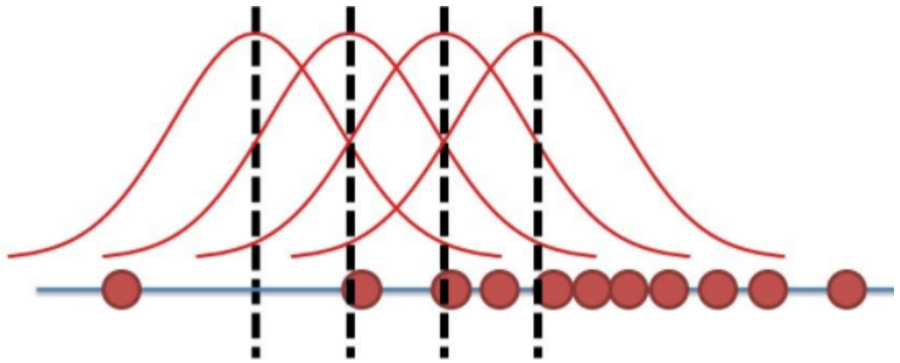
- Want to maximize the posterior probability density

$$X^{K*} = \arg\max p(X^K | G^K, O^{K-1}, Z^K)$$

- Using Bayes law = prior x likelihood

$$\begin{aligned} p(X^K | G^K, O^{K-1}, Z^K) &\propto p(X^K) l(X^K; G^K) l(X^K; O^{K-1}) l(X^K; Z^K) \\ &= p(x_0) \prod_k l(x_k; g_k) \prod_k l(x_k, x_{k+1}; o_k) \prod_k l(x_k; z_k, l_j) \end{aligned}$$

Likelihood for continuous measurements



- Example: GPS measurement
- Assume corrupted by Gaussian noise
- Density is Gaussian:

$$p(g_k|x_k) = \mathcal{N}(z; h(x_k), R) = \frac{1}{\sqrt{|2\pi R|}} \exp\left\{-\frac{1}{2}\|h_{GPS}(x_k) - g_k\|_R^2\right\}$$

- Likelihood: proportional, but *given* g, and a function of x:

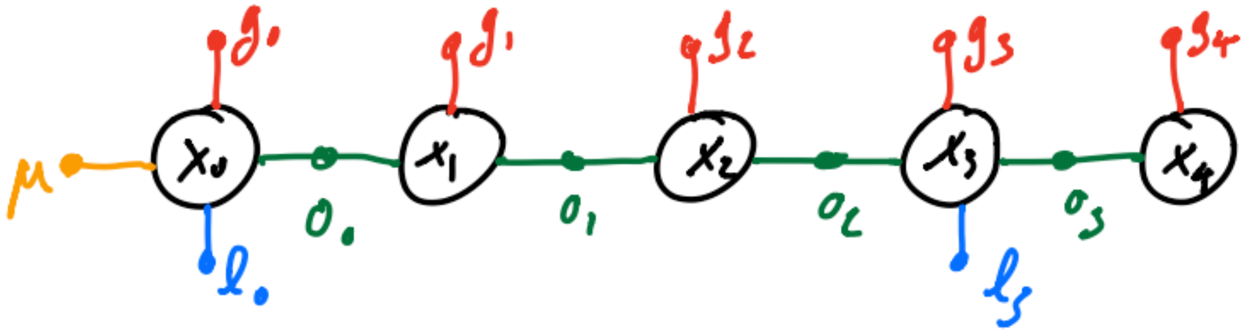
$$l(x_k; g_k) = \exp\left\{-\frac{1}{2}\|h_{GPS}(x_k) - g_k\|_R^2\right\} = \exp\left\{-\frac{1}{2}\|x_k - g_k\|_R^2\right\}$$

*Nice image is from https://www.youtube.com/watch?v=XepXtl9YKwc but unfortunately, they talk about "likelihood of measurement" rather than "likelihood of the parameters given the measurement", which is common mistake.

Trajectory Optimization as Least-Squares

Amazing trick: switch to (negative) log space:

$$-\log l(x_k; g_k) = -\log \exp\left\{-\frac{1}{2}\|x_k - g_k\|_R^2\right\} = \frac{1}{2}\|x_k - g_k\|_R^2$$



$$X^{K*} = \arg\min \frac{1}{2}\|x_0 - \mu\|_P^2 + \sum_k \frac{1}{2}\|x_k - g_k\|_R^2 + \sum_k \frac{1}{2}\|x_{k+1} - x_k - o_k\|_Q^2 + \sum_j \frac{1}{2}\|x_k - l_k - z_k\|_P^2.$$

Summary

- Autonomous driving provides a simple motivating example.
- We can represent the problem graphically using a factor graph.
- In 1-D, this problem is linear, although we will not be so lucky in 2D.
- We then turn the MAP estimate of the trajectory into a trajectory optimization problem.
- Finally, by converting to (negative) log-space, we obtain an easy linear least-squares problem.