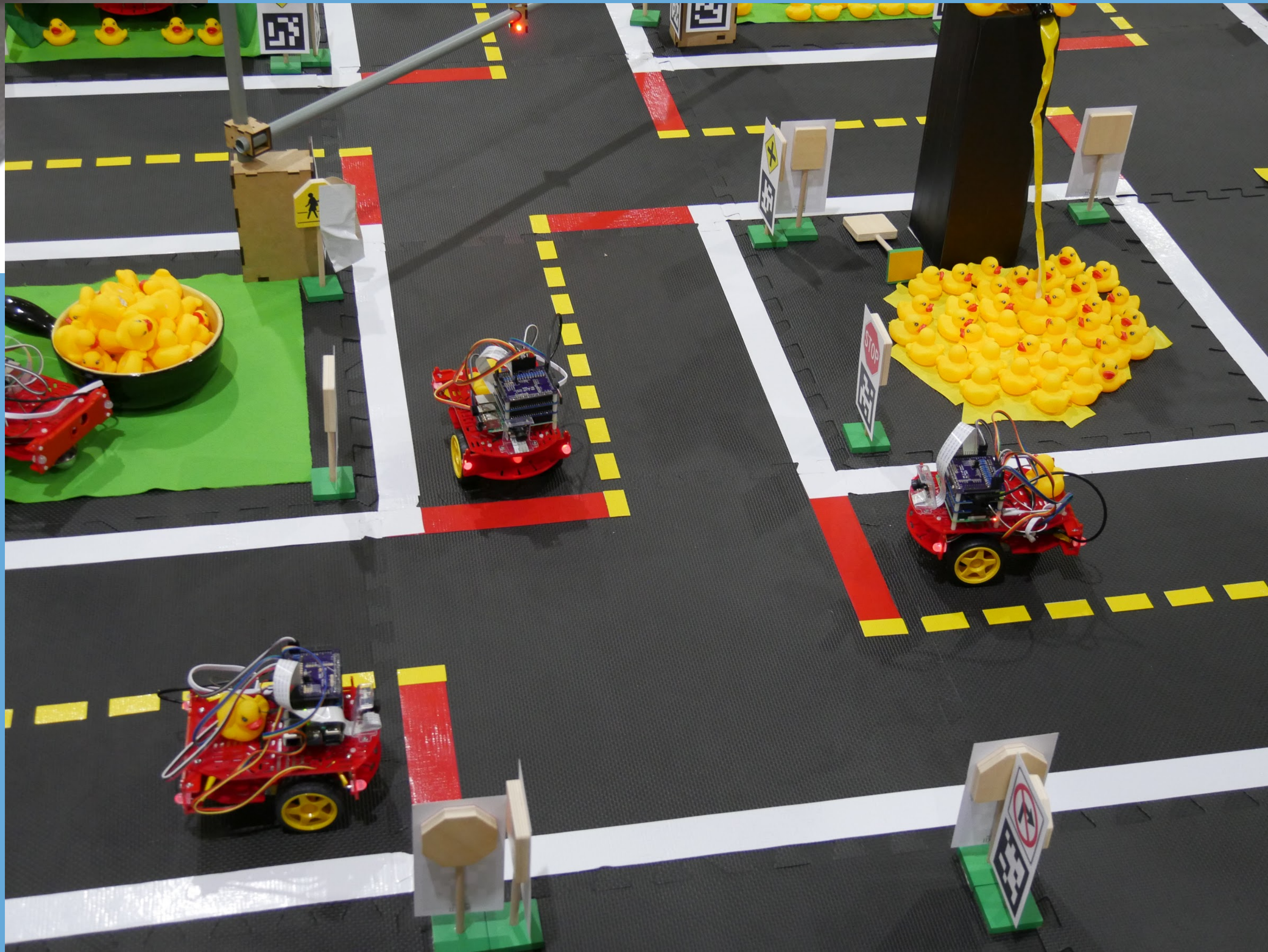
CS 3630!



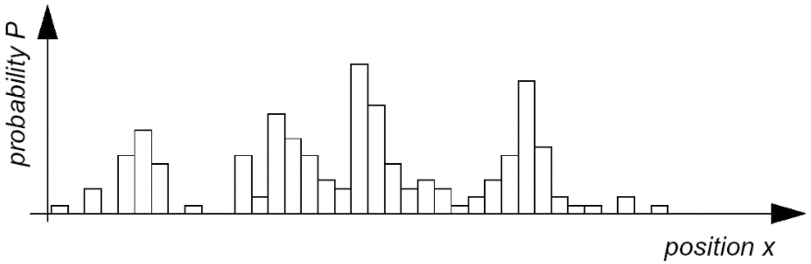
Lecture 10: Continuous Densities

Topics

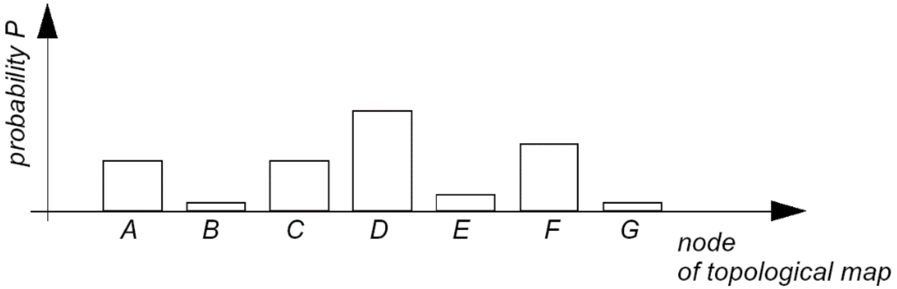
- **1. Continuous Densities**
- **2. Gaussian Densities**
- **3. Bayes Nets & Mixture Models**
- **4. Cont. Measurement Models**
- **5. Cont. Motion Models**
- **6. Simulating Cont. Bayes Nets**

Motivation: how do we represent our belief of where the robot is located?

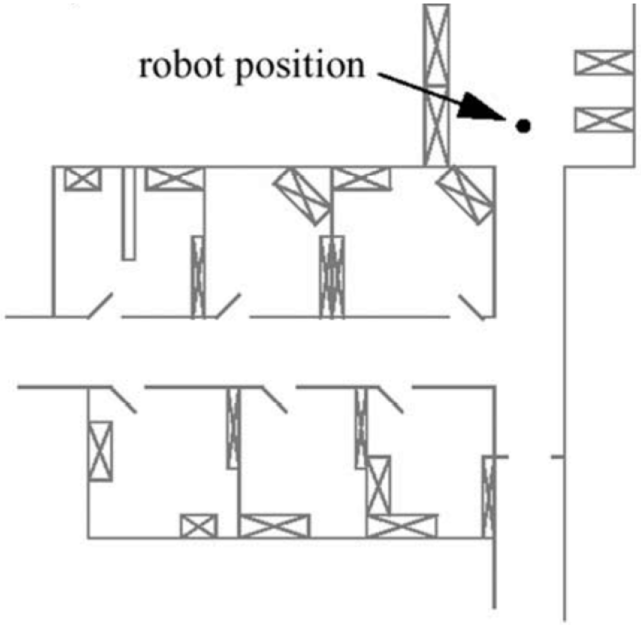
Discretized map with multiple hypotheses probability distribution



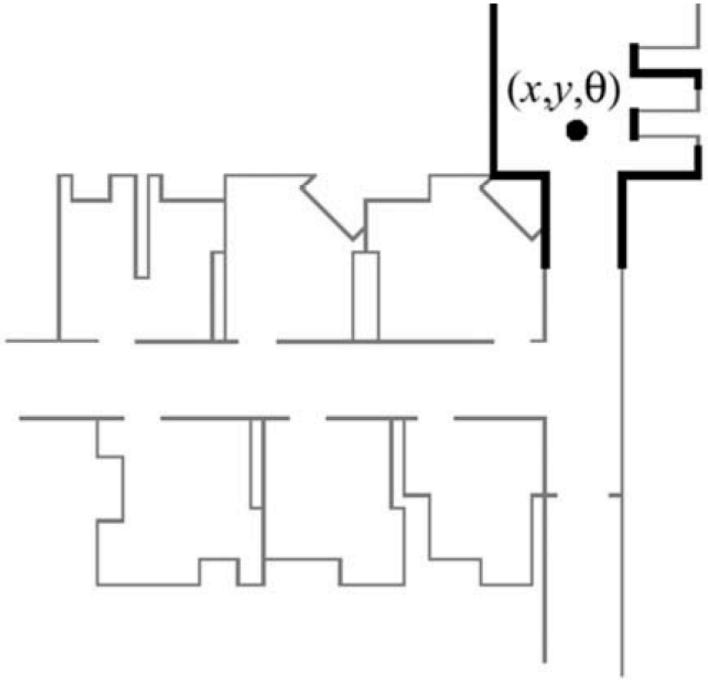
Discretized topological map with with multiple hypotheses probability distribution



Representations

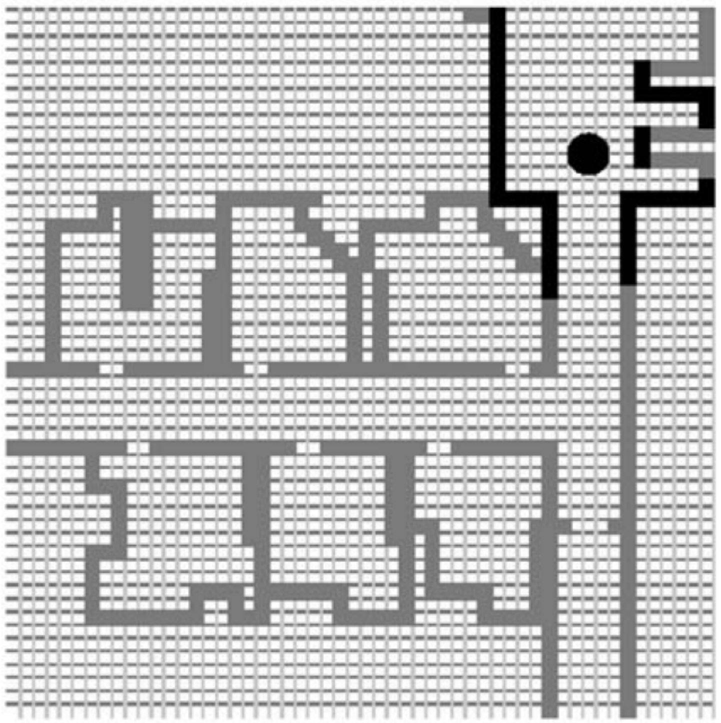


The real map with walls, doors and furniture.

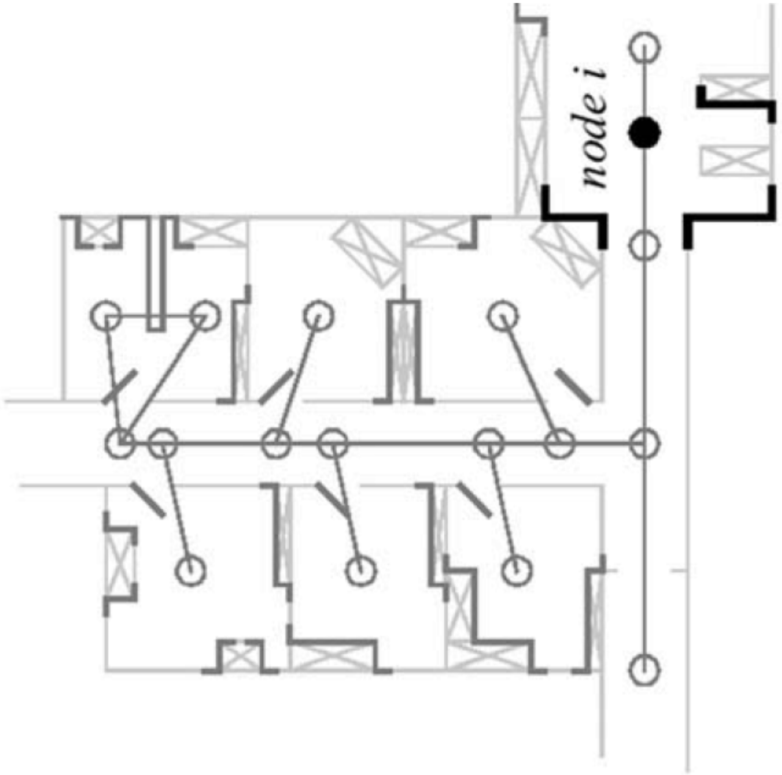


Line-based map (~100 lines)

Representations



Grid-based map (3000 cells, each 50cm x 50cm)



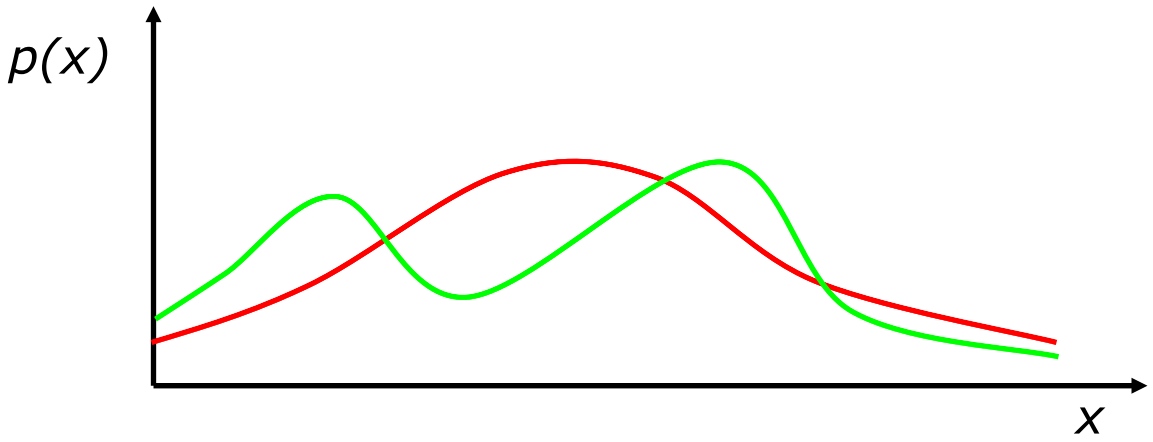
Topological map (50 features, 18 nodes)

1. Continuous Probability Densities

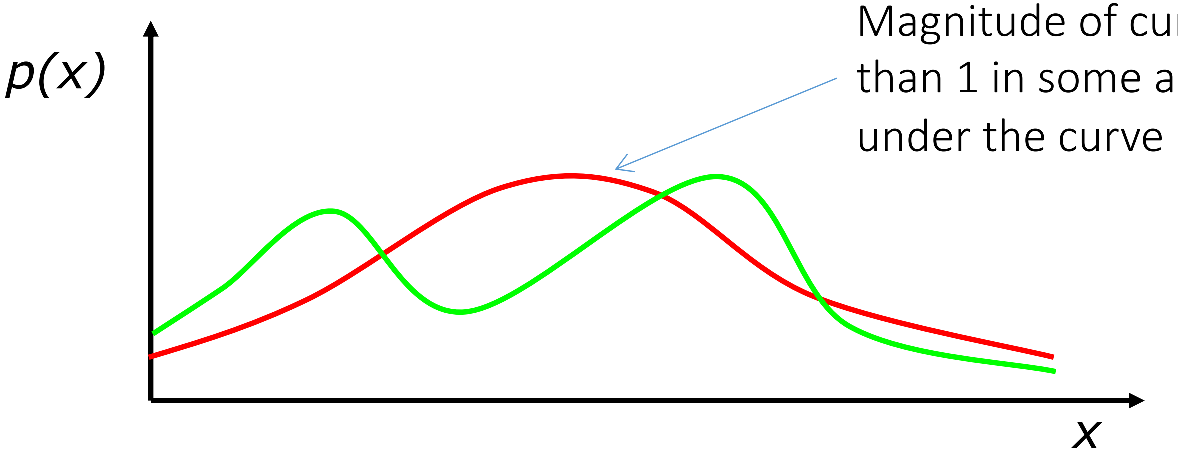
- X takes on values in the continuum.
- $p(X = x)$, or $p(x)$, is a probability density function.

$$P(x \in (a,b)) = \int_a^b p(x)dx$$

- E.g.



Probability Density Function



Magnitude of curve could be greater than 1 in some areas. The total area under the curve must add up to 1.

Since continuous probability functions are defined for an infinite number of points over a continuous interval, the probability at a single point is always 0.

2. Gaussian Densities

A Gaussian probability density is given by

$$\mathcal{N}(\theta; \mu, \Sigma) = \frac{1}{\sqrt{|2\pi\Sigma|}} \exp\left\{ -\frac{1}{2} \|\theta - \mu\|_\Sigma^2 \right\},$$

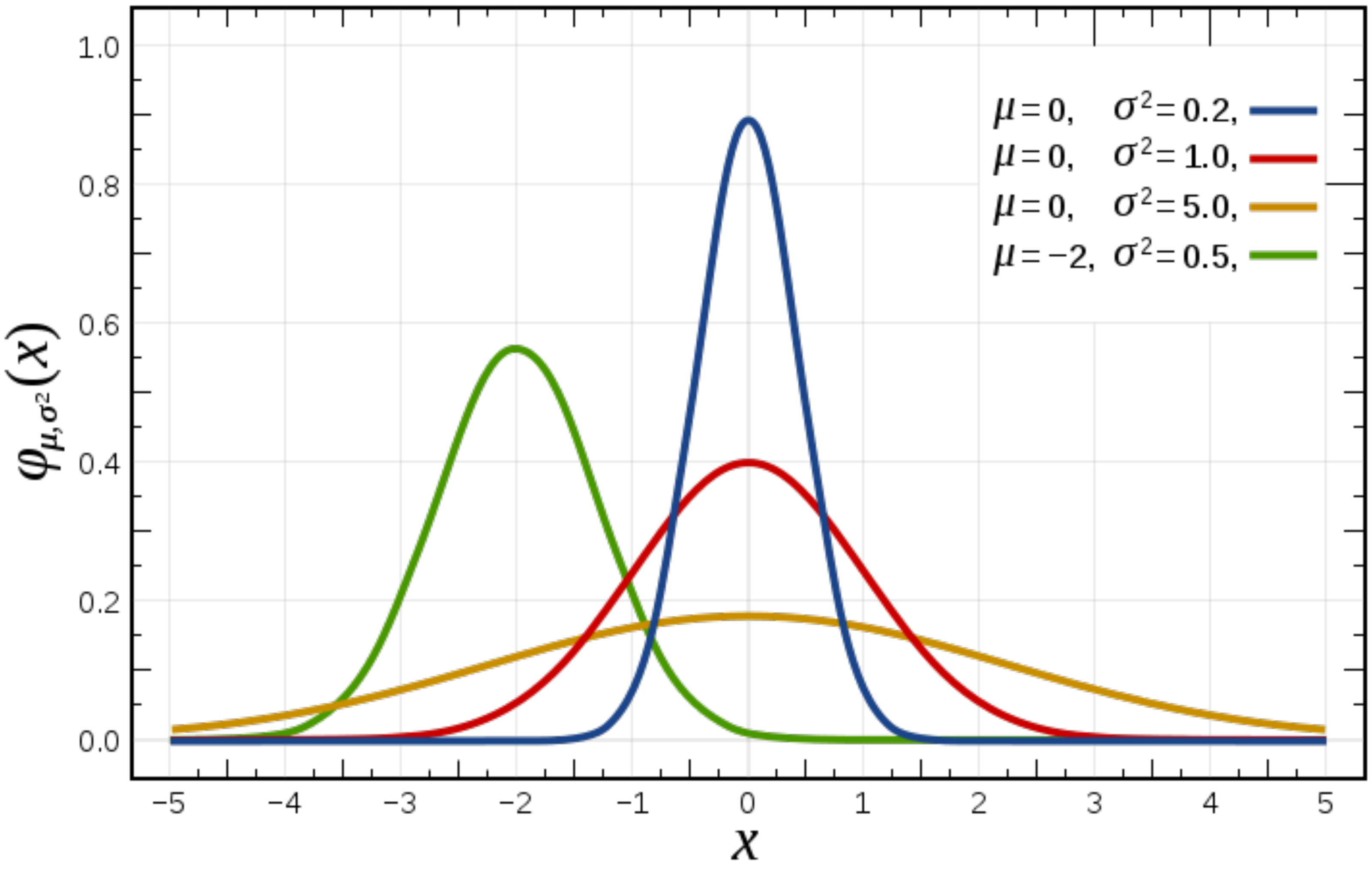
where $\mu \in \mathbb{R}^n$ is the mean, Σ is an $n \times n$ covariance matrix, and

$$\|\theta - \mu\|_\Sigma^2 \triangleq (\theta - \mu)^\top \Sigma^{-1} (\theta - \mu)$$

denotes the squared Mahalanobis distance.

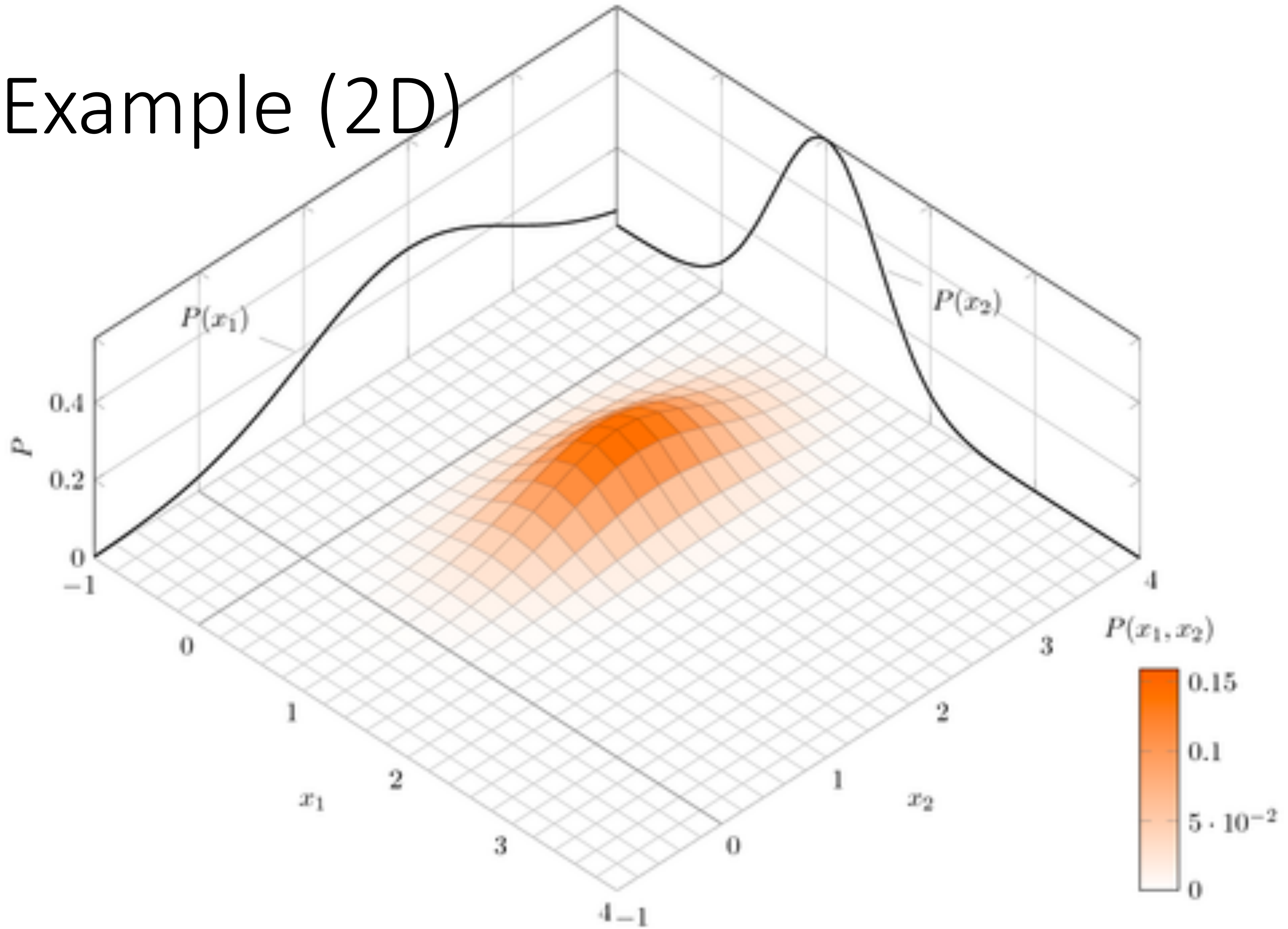
- Easy: negative log is quadratic
- Also known as the "bell curve"
- One of a few densities for which sampling is *easy*

1D examples



- From Wikipedia!

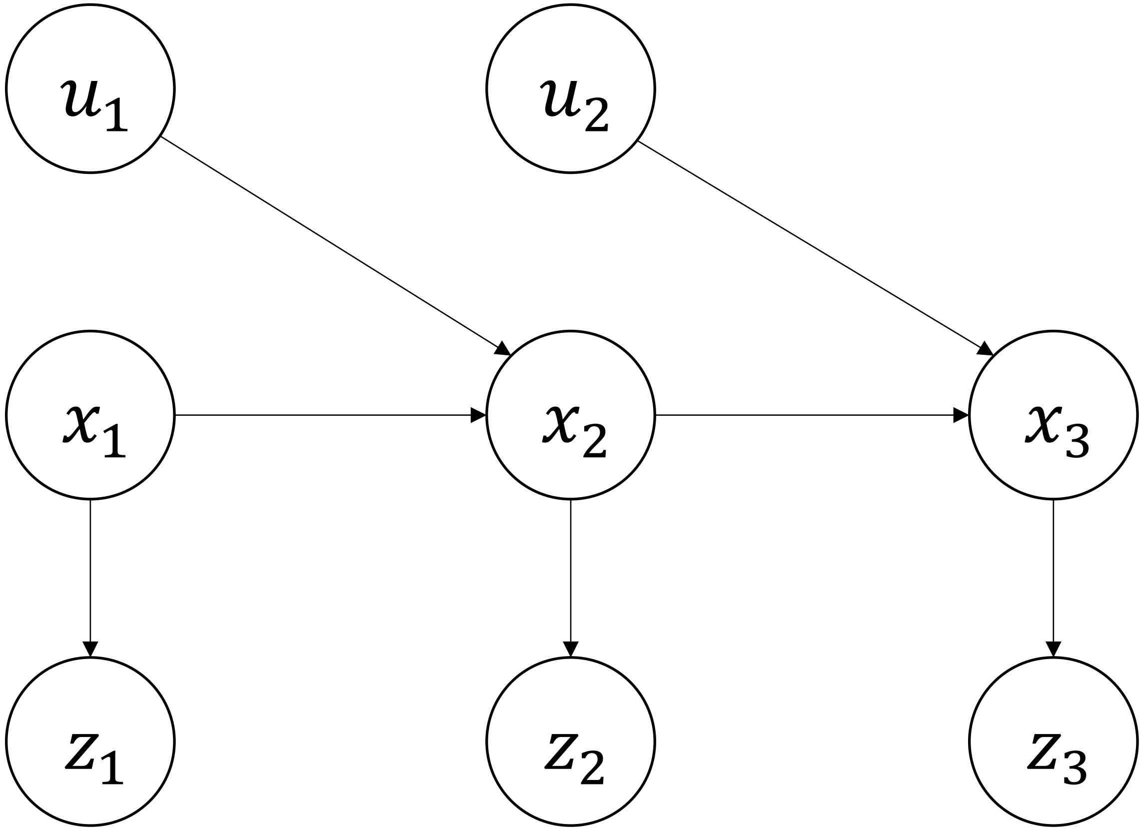
Multivariate Example (2D)



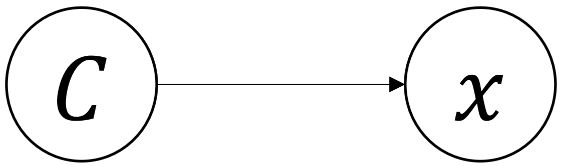
- http://pgfplots.net/tikz/examples/bivariate-normal-distribution/

2. Continuous Bayes Nets

- As before, but now states S, observations O, and action A can all be continuous.
- Terminology: x, z, u
- Hence: measurement models and state transition models are continuous.

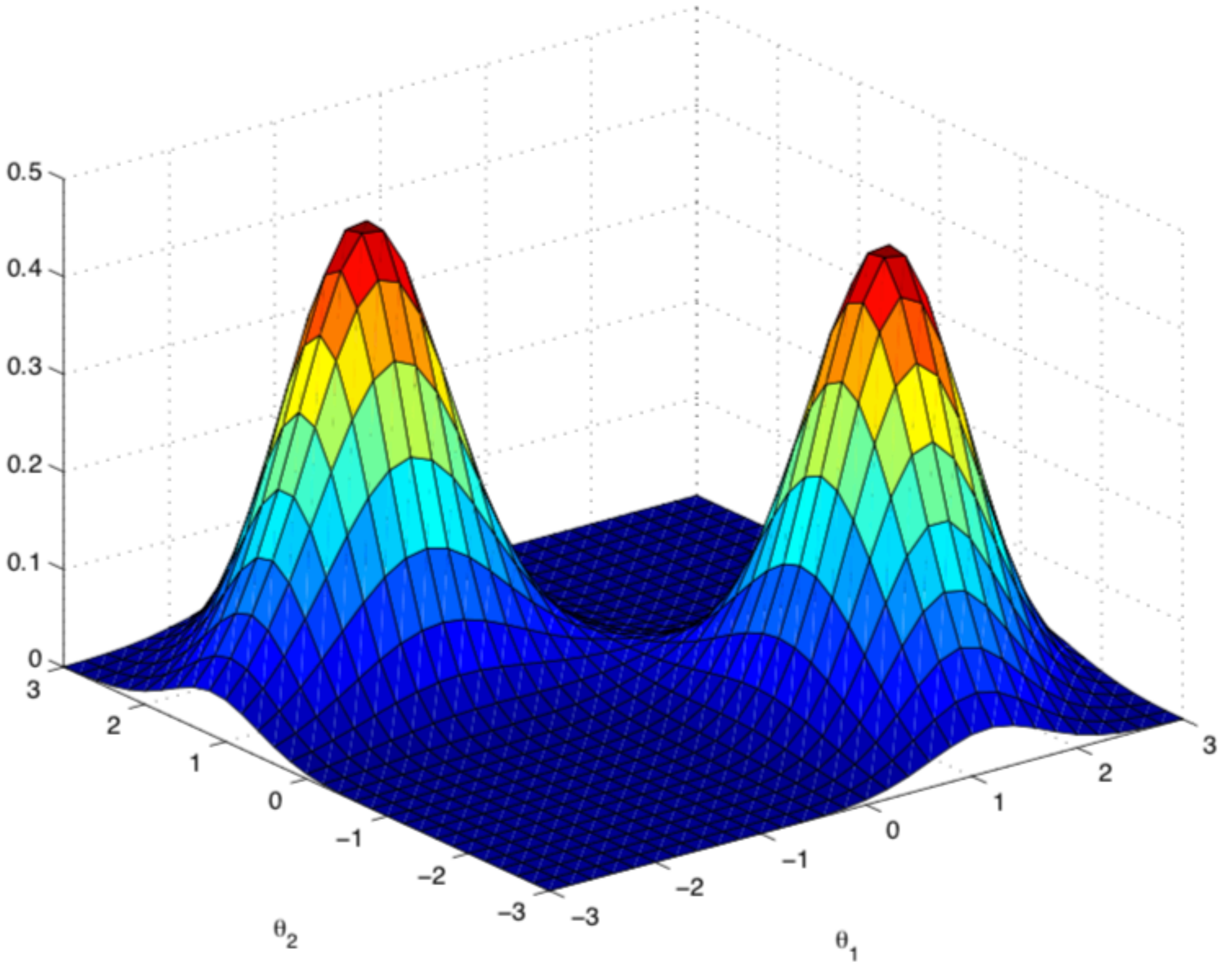


Important aside: Mixture Models



- We can mix discrete and continuous
- Most important example: mixture of continuous densities
- Example: Gaussian mixture model
- Sampling: sample **component**, then sample from Gaussian:

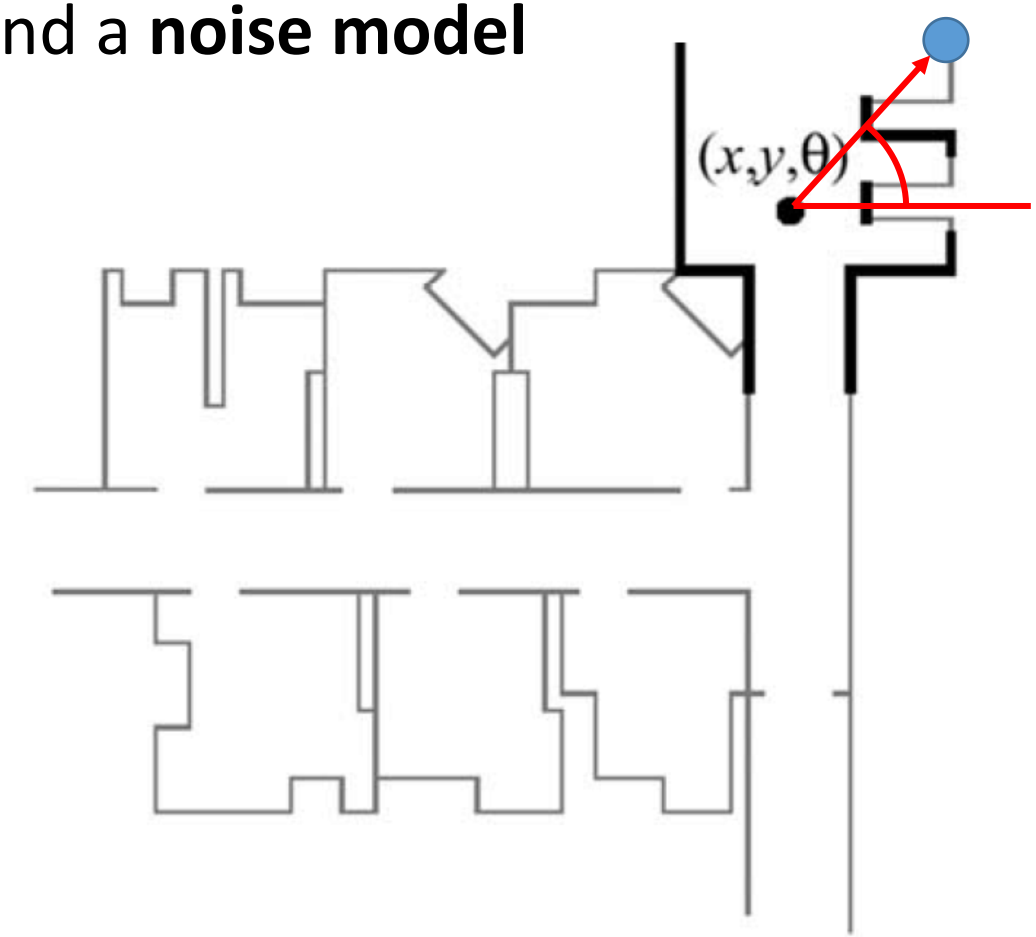
$$p(x,C)=p(x|C)P(C)$$



4. Continuous Measurement Models

- We need a **measurement function** and a **noise model**
- Example: bearing to a landmark l:

$$h(x,l) = atan2(l_y - x_y, l_x - x_x)$$



Adding a noise model

- Generative model of measuremen $z = h(x, l) + \eta,$
- Assuming Gaussian noise:

$$p(z|x,l) = \mathcal{N}(z; h(x,l), R) = \frac{1}{\sqrt{|2\pi R|}} \exp\left\{-\frac{1}{2}\|h(x,l) - z\|_R^2\right\}$$

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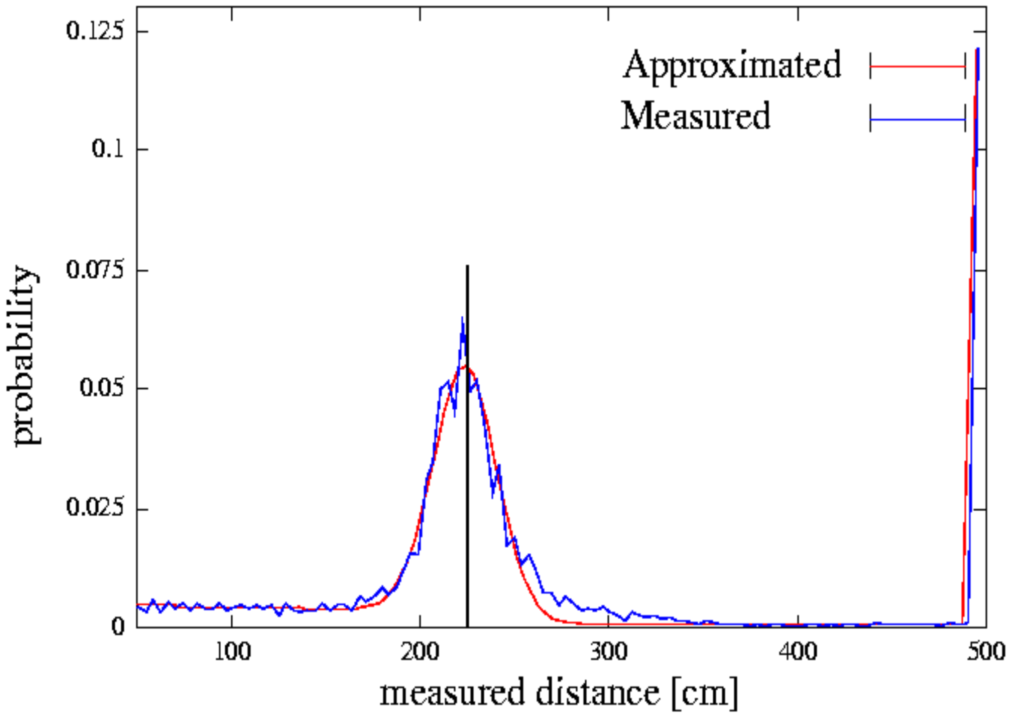
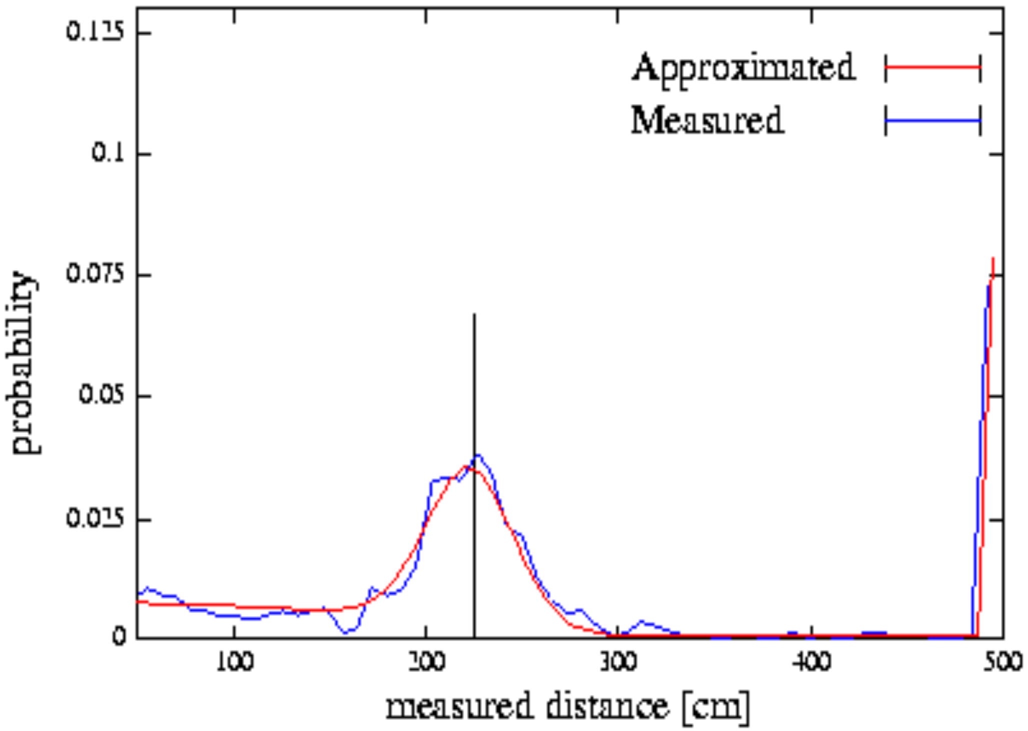
- Putting it together:

$$p(z|x,l) = \frac{1}{\sqrt{|2\pi R|}} \exp\left\{-\frac{1}{2}\|atan2(l_y - x_y, l_x - x_x) - z\|_R^2\right\}$$

Other sensor models



Laser sensor



Sonar sensor

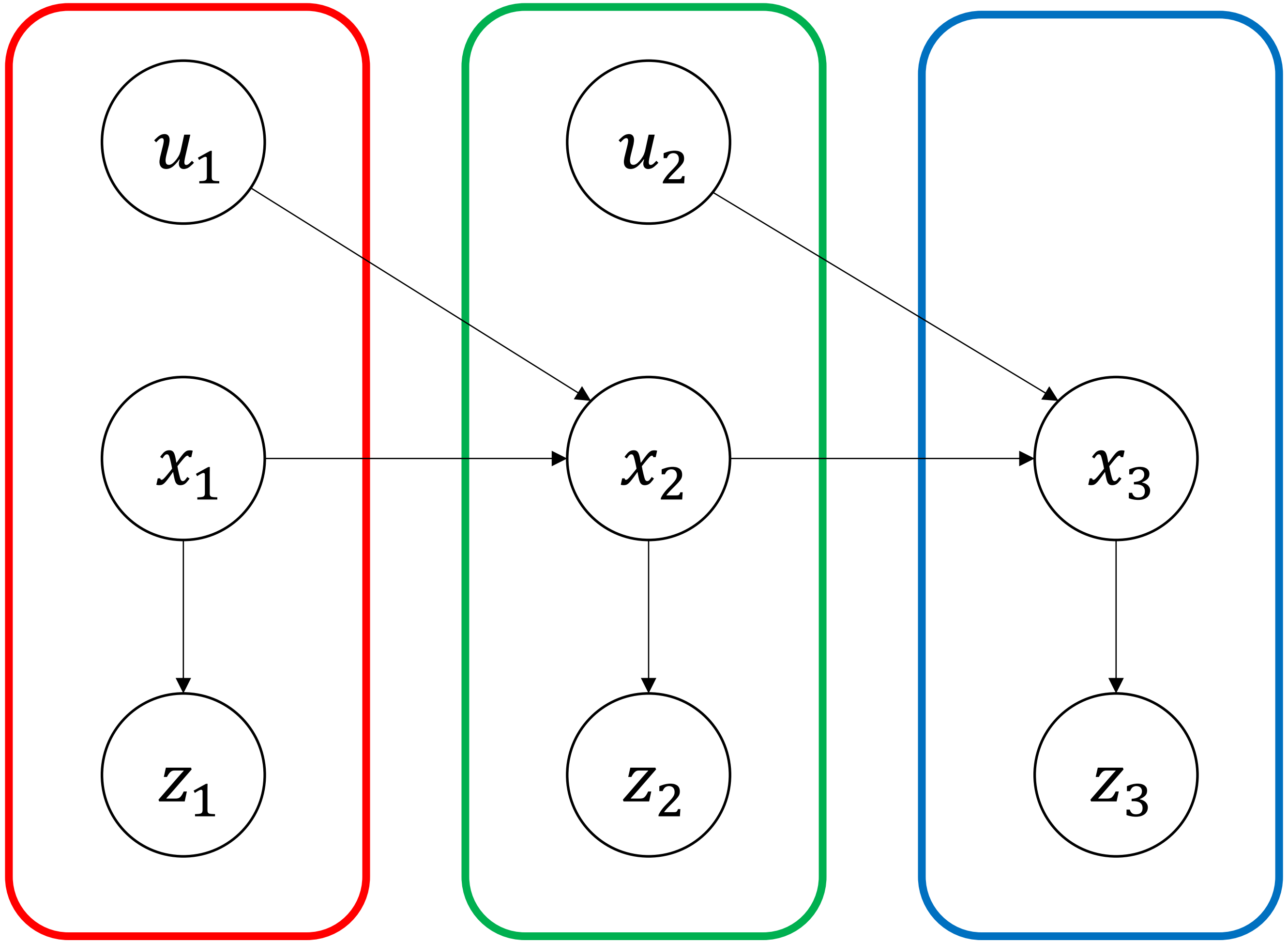
5. Continuous Motion Models

- Similar for state transition, but we now have a motion model
- Motion model $g(x,u)$ takes state x and control u
- Multivariate noise model with covariance Q:

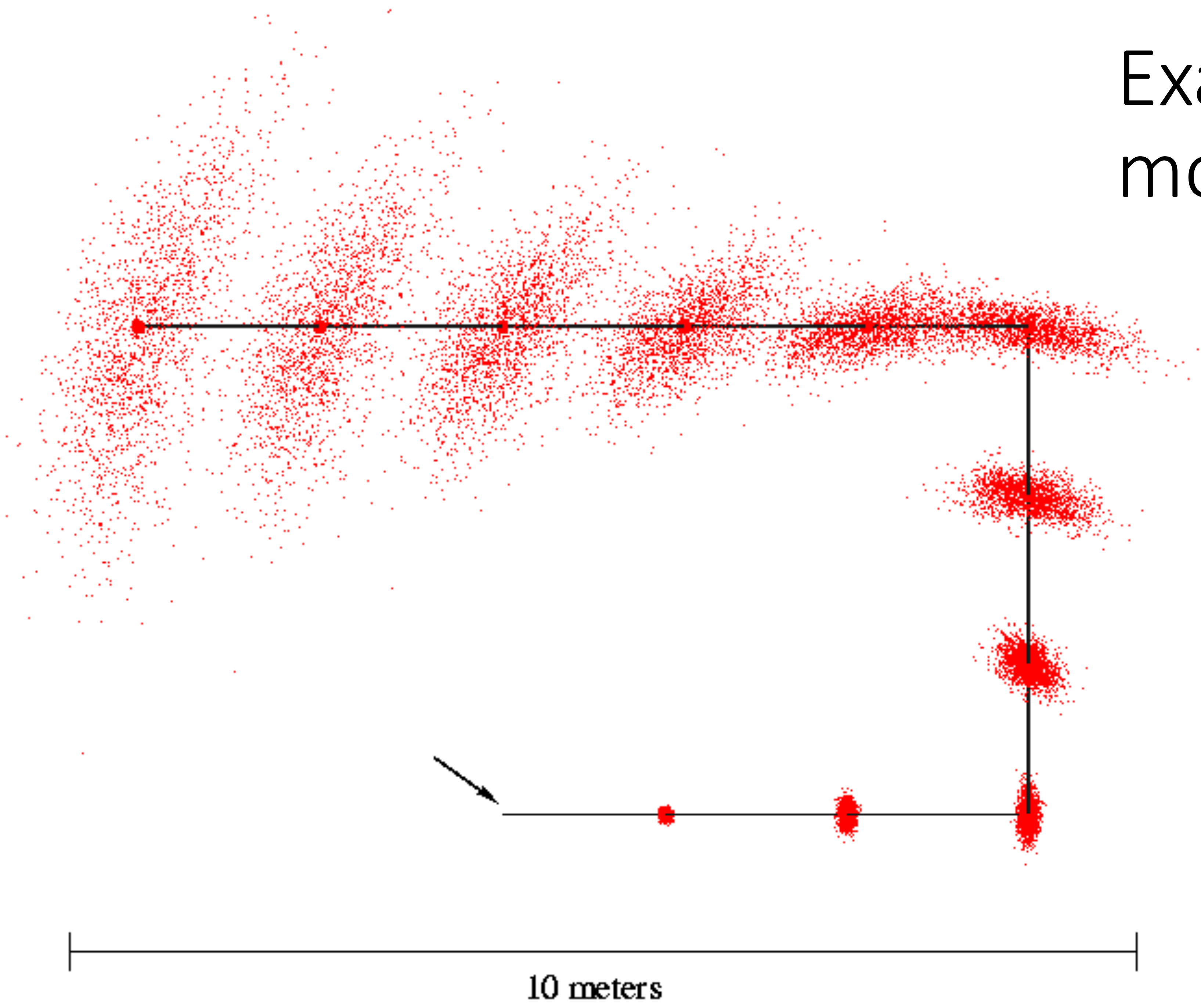
$$p(x_{t+1}|x_t, u_t) = \frac{1}{\sqrt{|2\pi Q|}} \exp\left\{-\frac{1}{2}\|g(x_t, u_t) - x_{t+1}\|_Q^2\right\}$$

6. Simulating from a Continuous Bayes Net

1. Slice 1:
   a) Sample from $p(x_1)$
   b) Sense $p(z_1|x_1)$
   c) Sample from $p(u_1)$
2. Slice 2:
   a) Act $p(x_2|x_1, u_1)$
   b) Sense $p(z_2|x_2)$
   c) Sample from $p(u_2)$
3. Slice 3:
   a) …



Example: motion model only



- The infamous "banana density"
- Happens because we also sample heading \theta
- Clearly non-Gaussian!

Summary

- Continuous Densities
- Gaussian Densities
- Bayes Nets & Mixture Models
- Cont. Measurement Models
- Cont. Motion Models
- Simulating Cont. Bayes Nets