Deep Learning

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CS x476 Computer Vision

Many slides from Stanford’s CS231N by Fei-Fei Li, Justin Johnson, Serena Yeung, as well as some slides on filtering from Devi Parikh and Kristen Grauman, who may in turn have borrowed some from others

Image Classification

Supervised Learning

CNN Review

Training CNNs

Loss Functions

Stochastic Gradient Descent

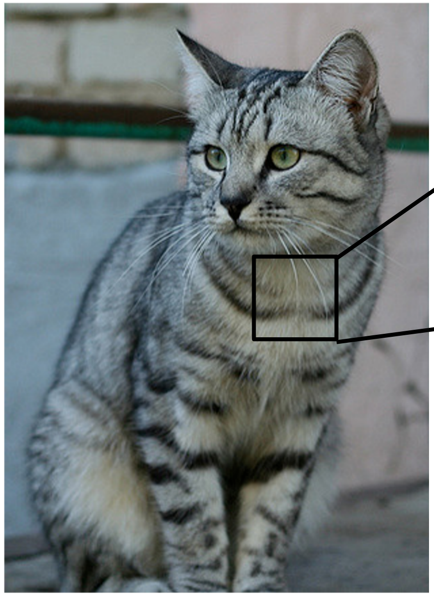
Computing Gradients

Image Classification: A core task in Computer Vision

(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

→ cat

The Problem: Semantic Gap




```
[[105 112 108 111 104  99 106  99  96 103 112 119 104  97  93  87]
 [ 91  98 102 106 104  79  98 103  99 105 123 136 110 105  94  85]
 [ 76  85  90 105 128 105  87  96  95  99 115 112 106 103  99  85]
 [ 99  81  81  93 120 131 127 100  95  98 102  99  96  93 101  94]
 [106  91  61  64  69  91  88  85 101 107 109  98  75  84  96  95]
 [114 108  85  55  55  69  64  54  64  87 112 129  98  74  84  91]
 [133 137 147 103  65  81  80  65  52  54  74  84 102  93  85  82]
 [128 137 144 140 109  95  86  70  62  65  63  63  60  73  86 101]
 [125 133 148 137 119 121 117  94  65  79  80  65  54  64  72  98]
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 [ 63  65  75  88  89  71  62  81 120 138 135 105  81  98 110 118]
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 [118  97  82  86 117 123 116  66  41  51  95  93  89  95 102 107]
 [164 146 112  80  82 120 124 104  76  48  45  66  88 101 102 109]
 [157 170 157 120  93  86 114 132 112  97  69  55  70  82  99  94]
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 [128 112  96 117 150 144 120 115 104 107 102  93  87  81  72  79]
 [123 107  96  86  83 112 153 149 122 109 104  75  80 107 112  99]
 [122 121 102  80  82  86  94 117 145 148 153 102  58  78  92 107]
 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```
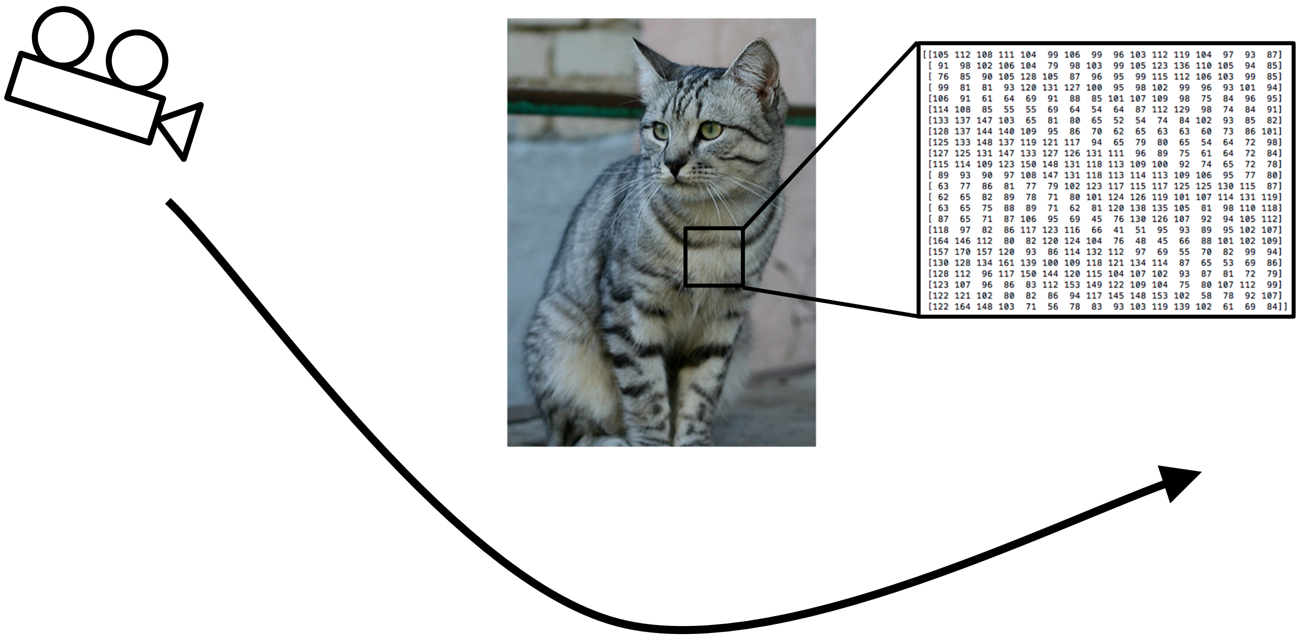

What the computer sees

An image is just a big grid of numbers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

Challenges: Viewpoint variation



All pixels change when
the camera moves!

Challenges: Illumination

This image is CC0 1.0 public domain

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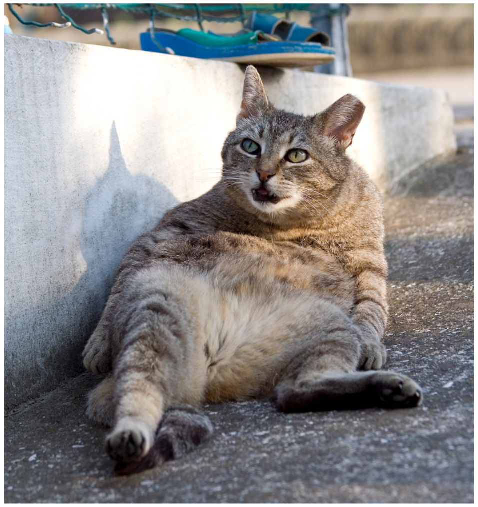
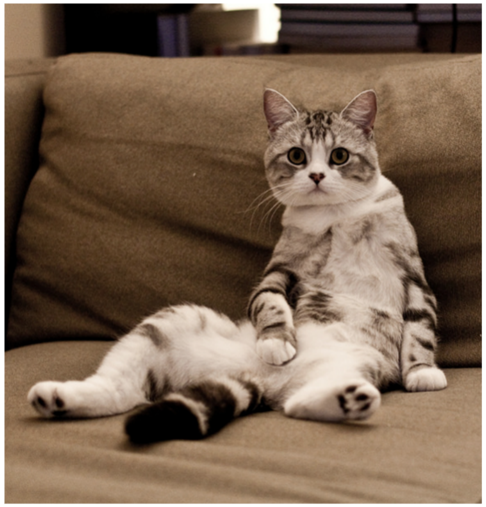
This image is CC0 1.0 public domain

Challenges: Deformation





Challenges: Occlusion

Challenges: Background Clutter

This image is CC0 1.0 public domain

This image is CC0 1.0 public domain

Challenges: Intraclass variation

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

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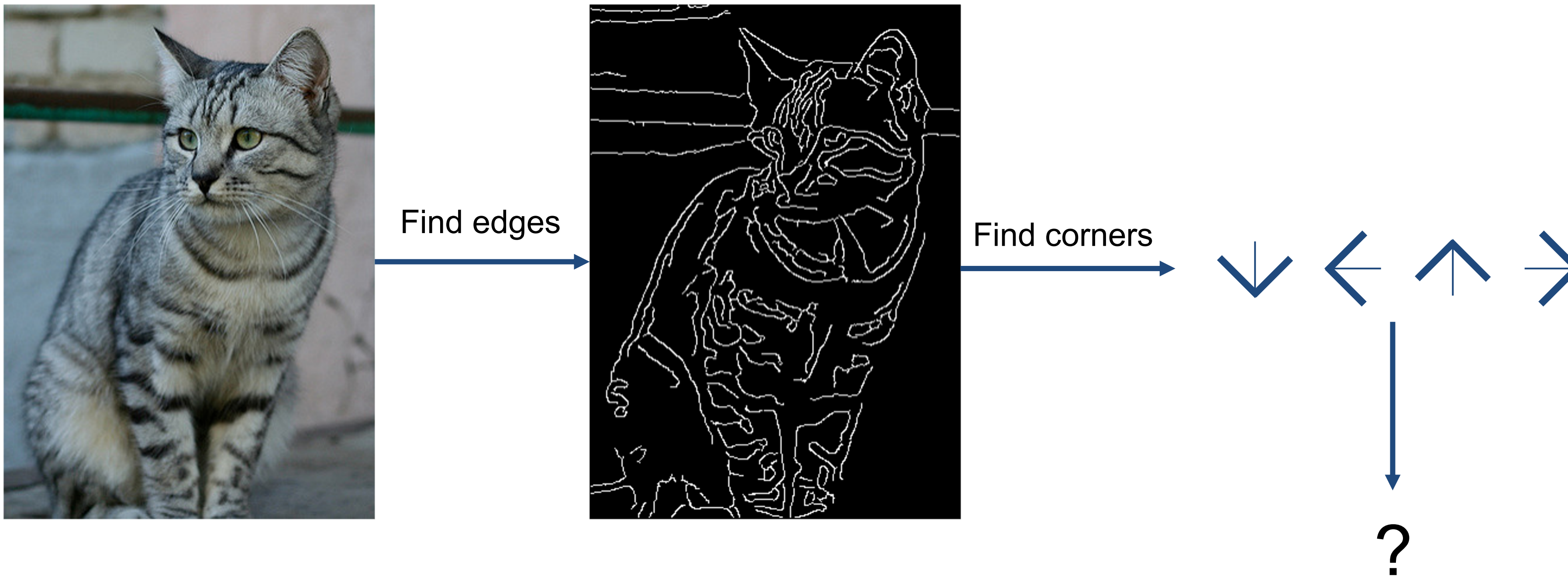
An image classifier

```python
def classify_image(image):
  # Some magic here?
  return class_label
```

Unlike e.g. sorting a list of numbers,

no obvious way to hard-code the algorithm for recognizing a cat, or other classes.

Attempts have been made



John Canny, "A Computational Approach to Edge Detection", IEEE TPAMI 1986

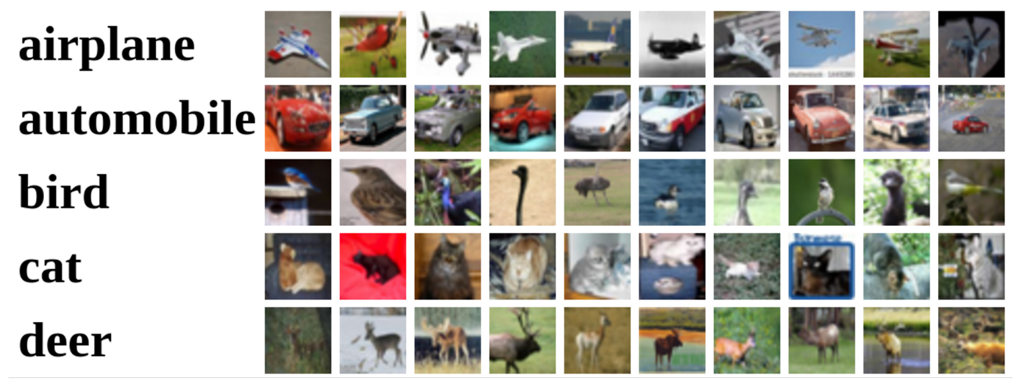
ML: A Data-Driven Approach

1. Collect a dataset of images and labels
2. Use Machine Learning to train a classifier
3. Evaluate the classifier on new images

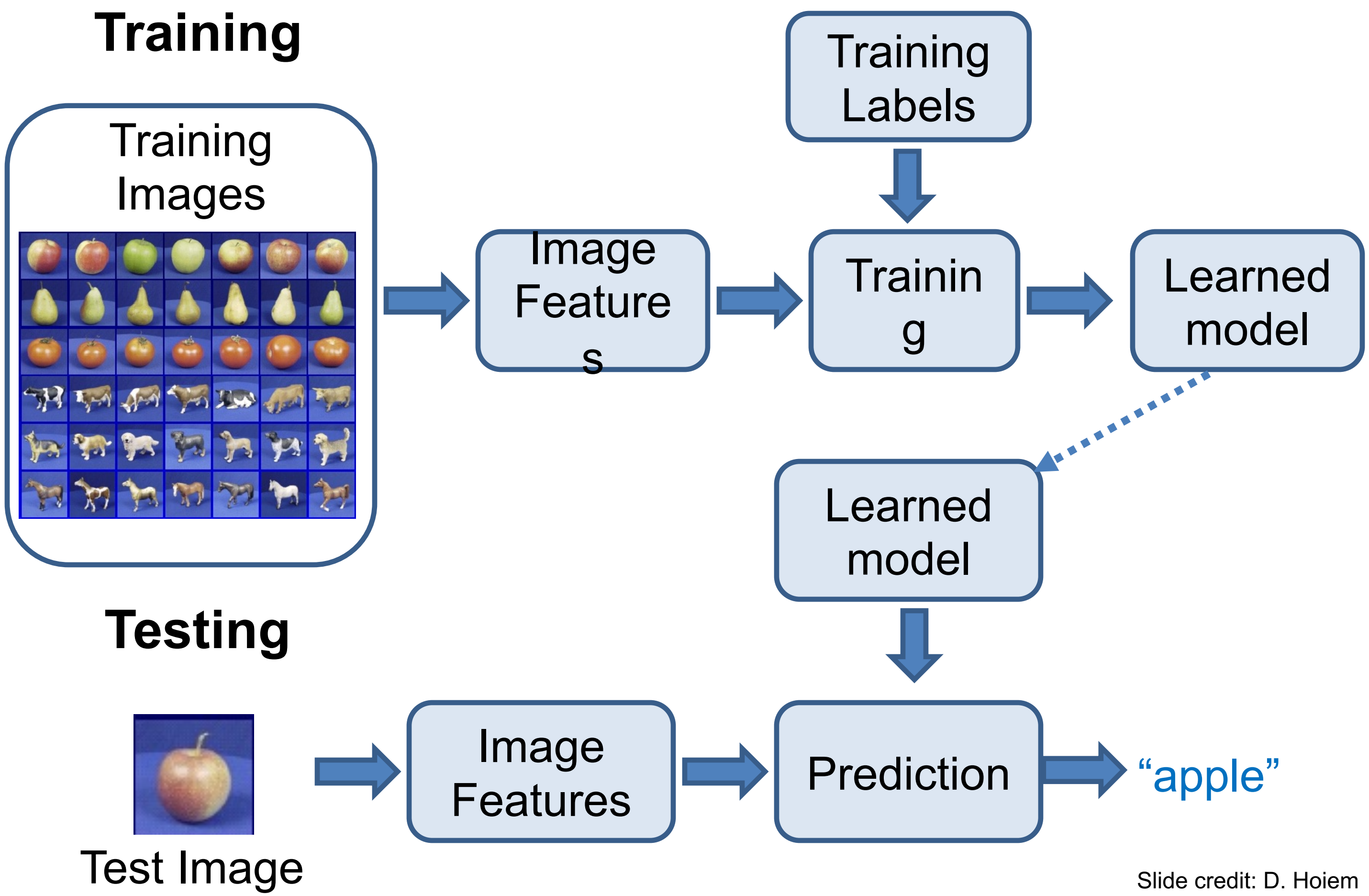
```
def train(images, labels):
  # Machine learning!
  return model
```

```
def predict(model, test_images):
  # Use model to predict labels
  return test_labels
```

Example training set



Steps

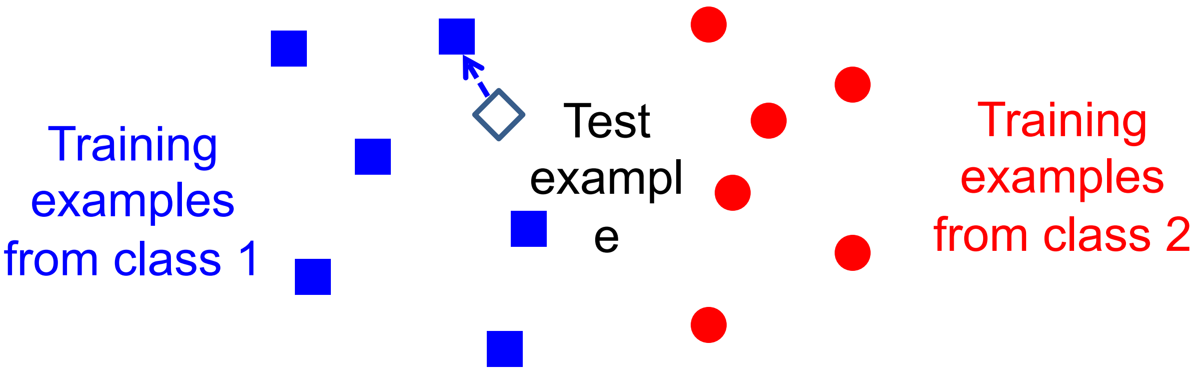


Slide credit: D. Hoiem

Supervised Learning

- Input: x (images, text, emails…)
- Output: y (spam or non-spam…)
- (Unknown) Target Function
  - f: X → Y (the "true" mapping / reality)
- Data
  - $\{ (x_1,y_1), (x_2,y_2), \ldots, (x_N,y_N) \}$

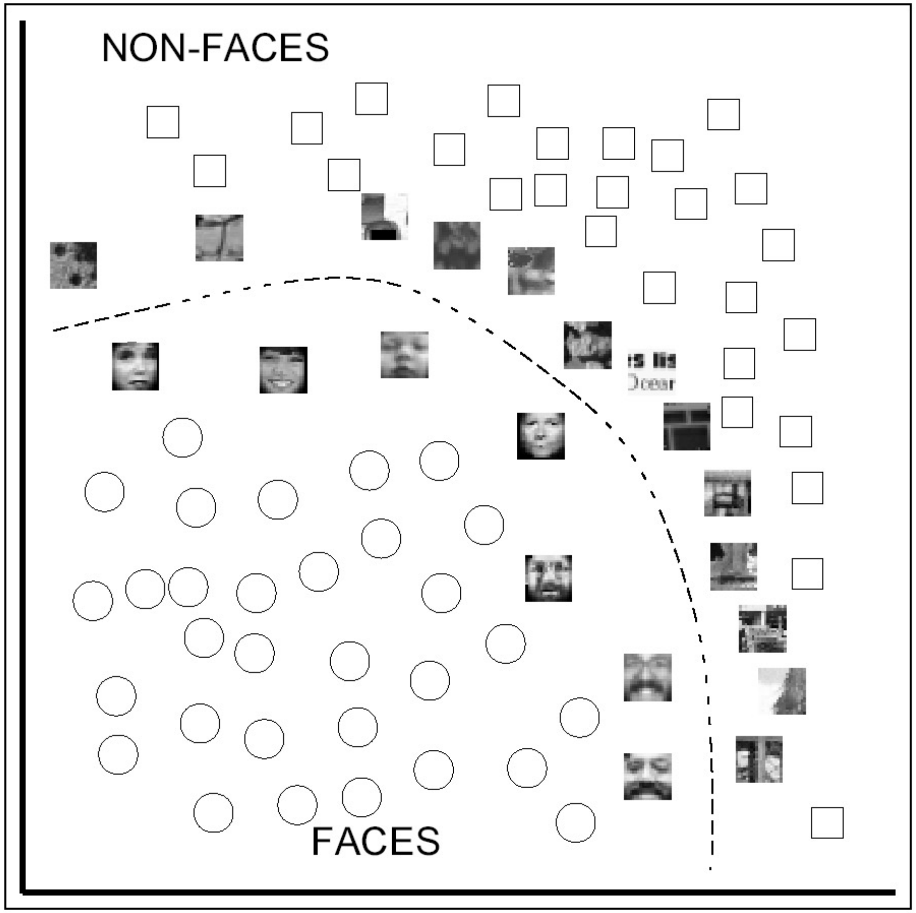
Nearest neighbor



f(**x**) = label of the training example nearest to **x**

- All we need is a distance or similarity function for our inputs
- No training required!

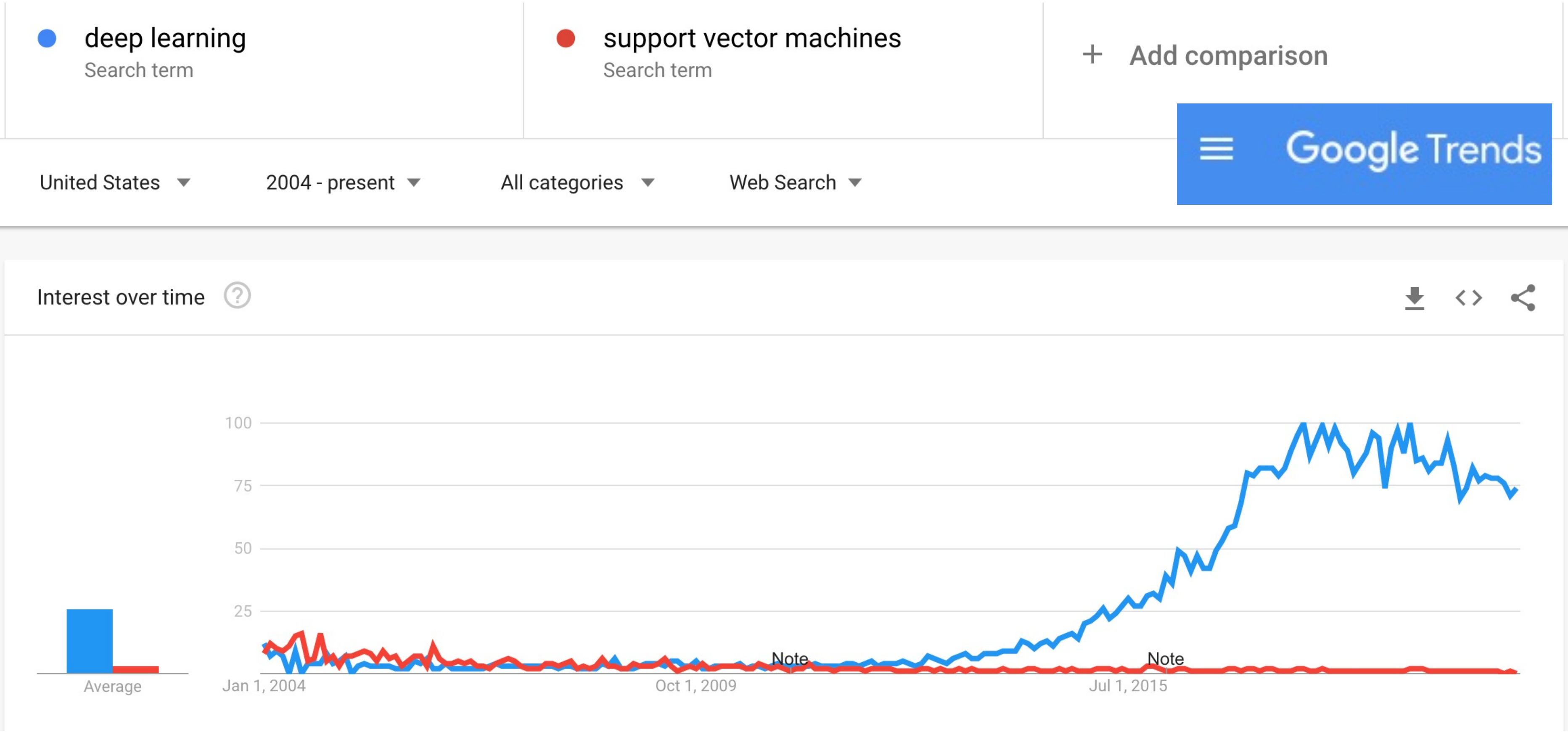
Support Vector Machines



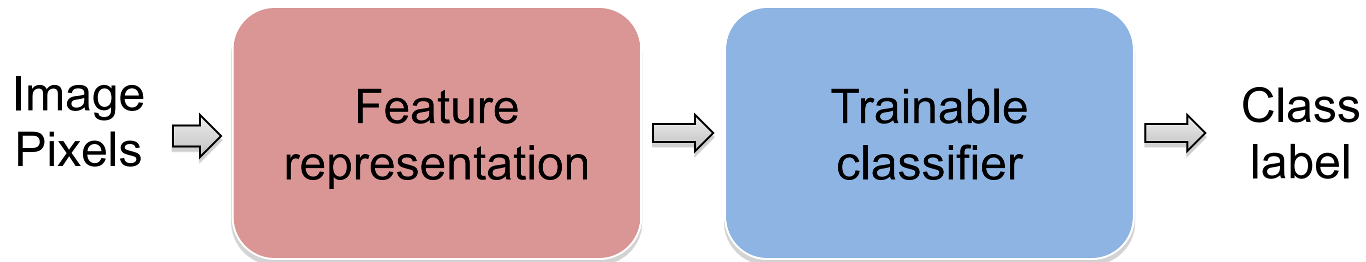
Using complex **features**, decision boundary in original space can be complex.

Decision Boundaries Projected back from Feature space

“Deep” vs. “shallow” (SVMs) Learning

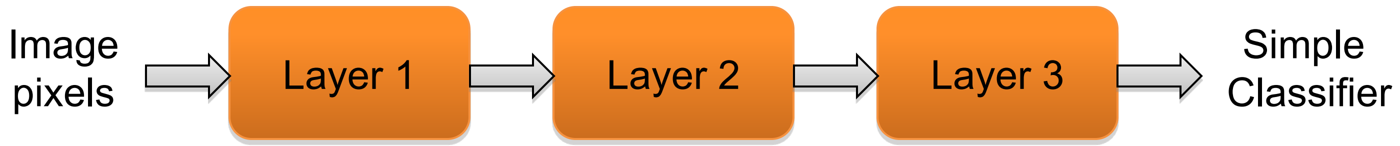


"Classic" recognition pipeline



- Hand-crafted feature representation
- Off-the-shelf trainable classifier

“Deep” recognition pipeline



- Learn a *feature hierarchy* from pixels to classifier
- Each layer extracts features from the output of previous layer
- Train all layers jointly

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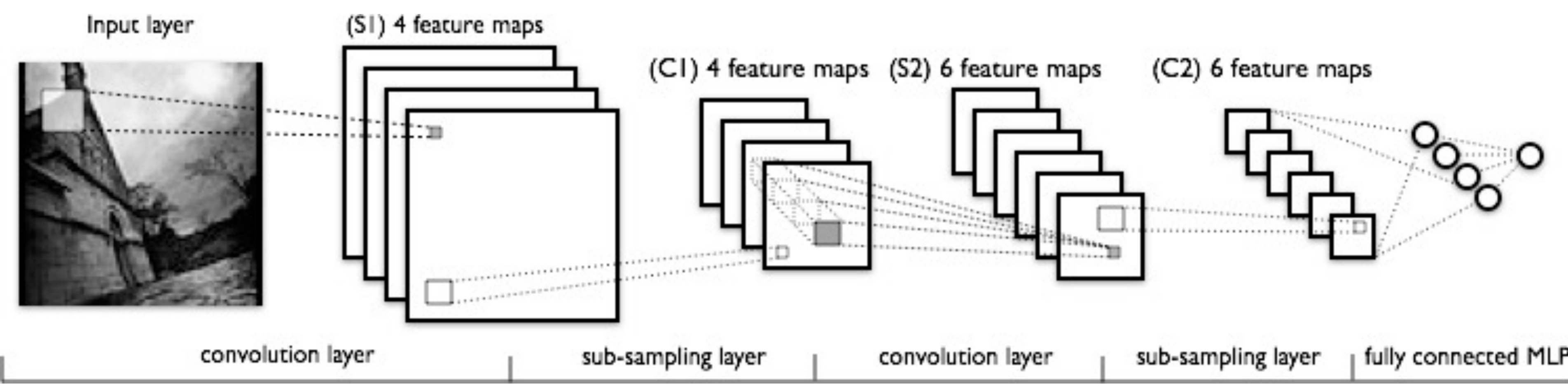
Review: CNNs



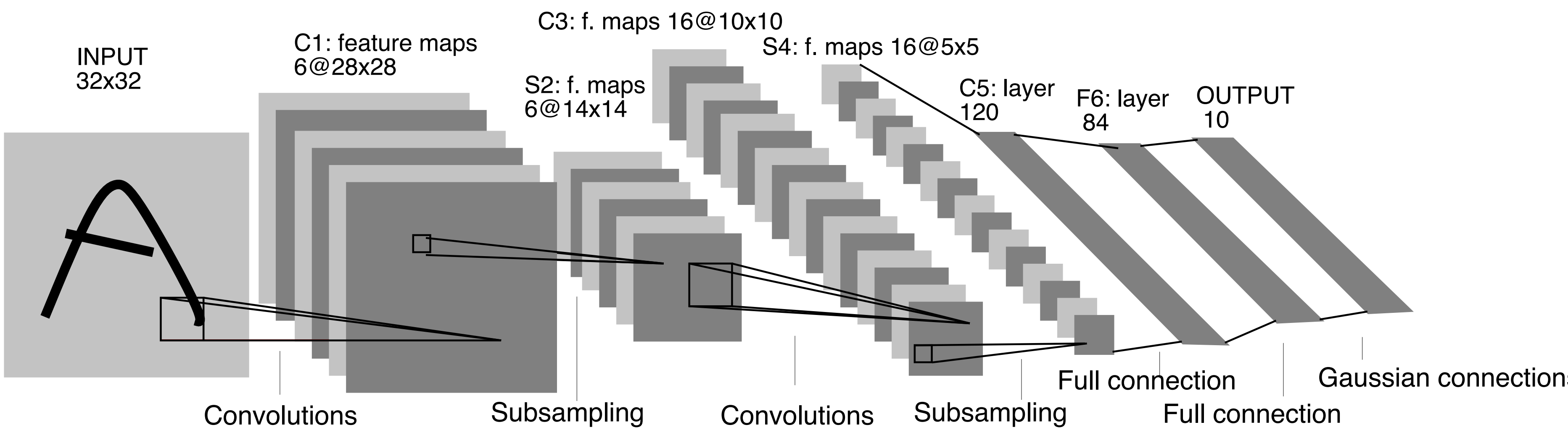
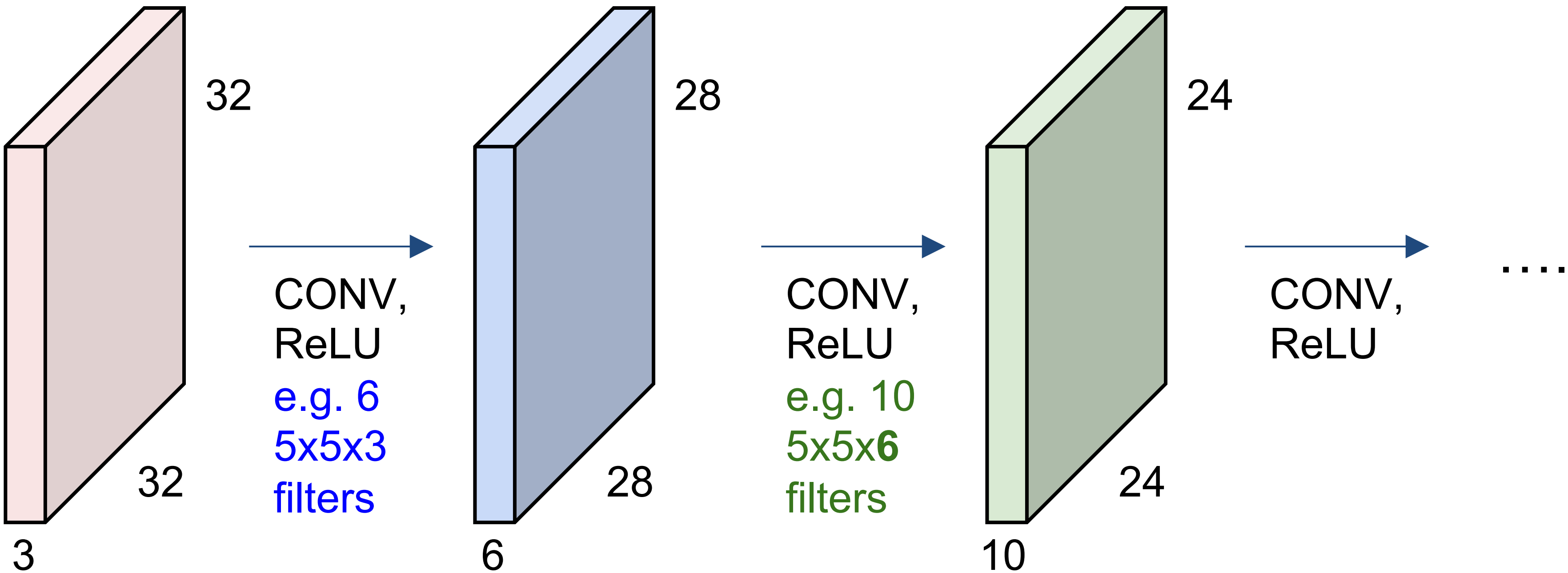
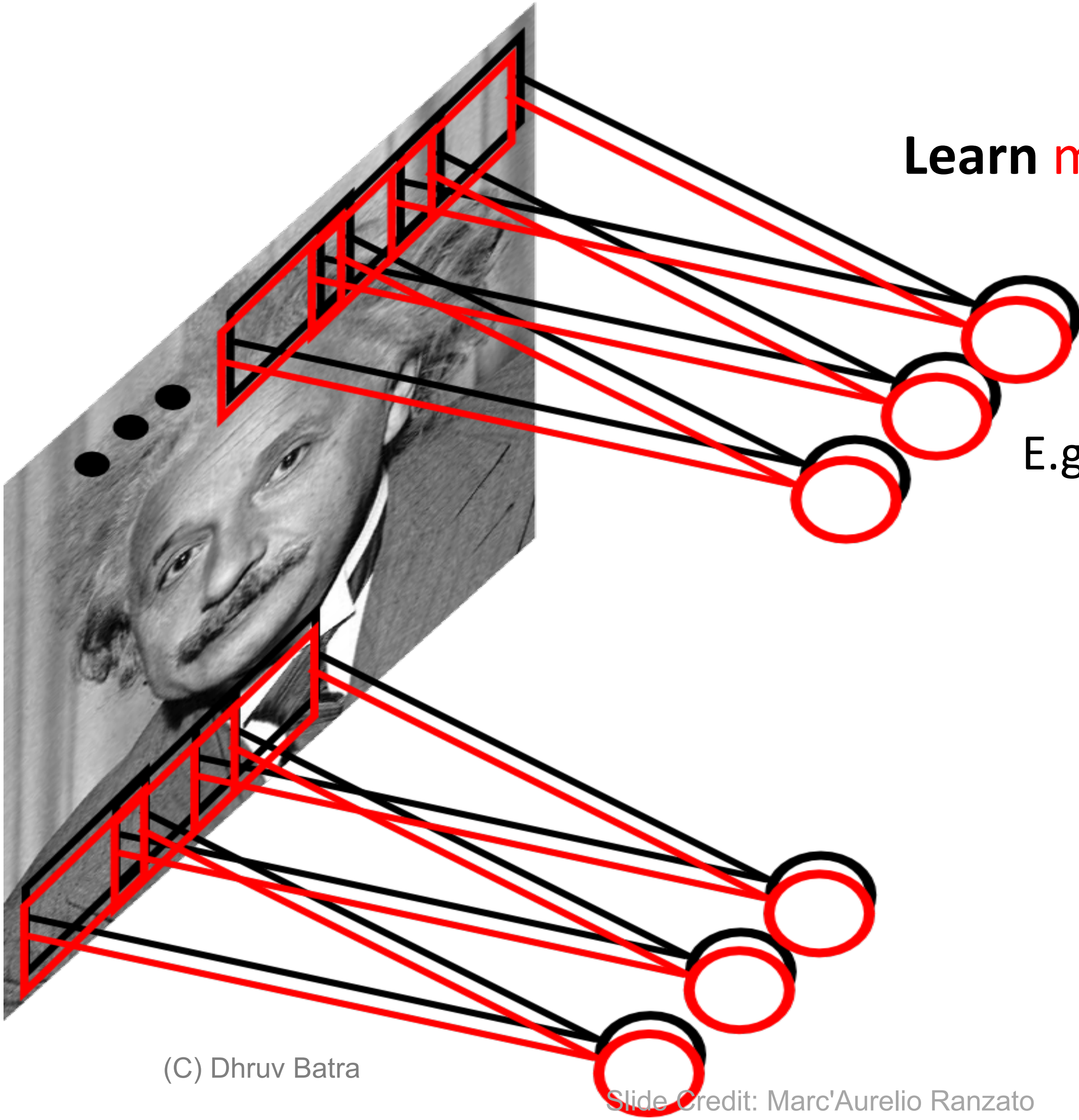


Image Credit: Yann LeCun, Kevin Murphy

CNN or ConvNet is a sequence of Convolutional Layers, interspersed with activation functions



Convolutional Layer



Learn multiple filters.

E.g.: 200x200 image
100 Filters
Filter size: 10x10
10K parameters

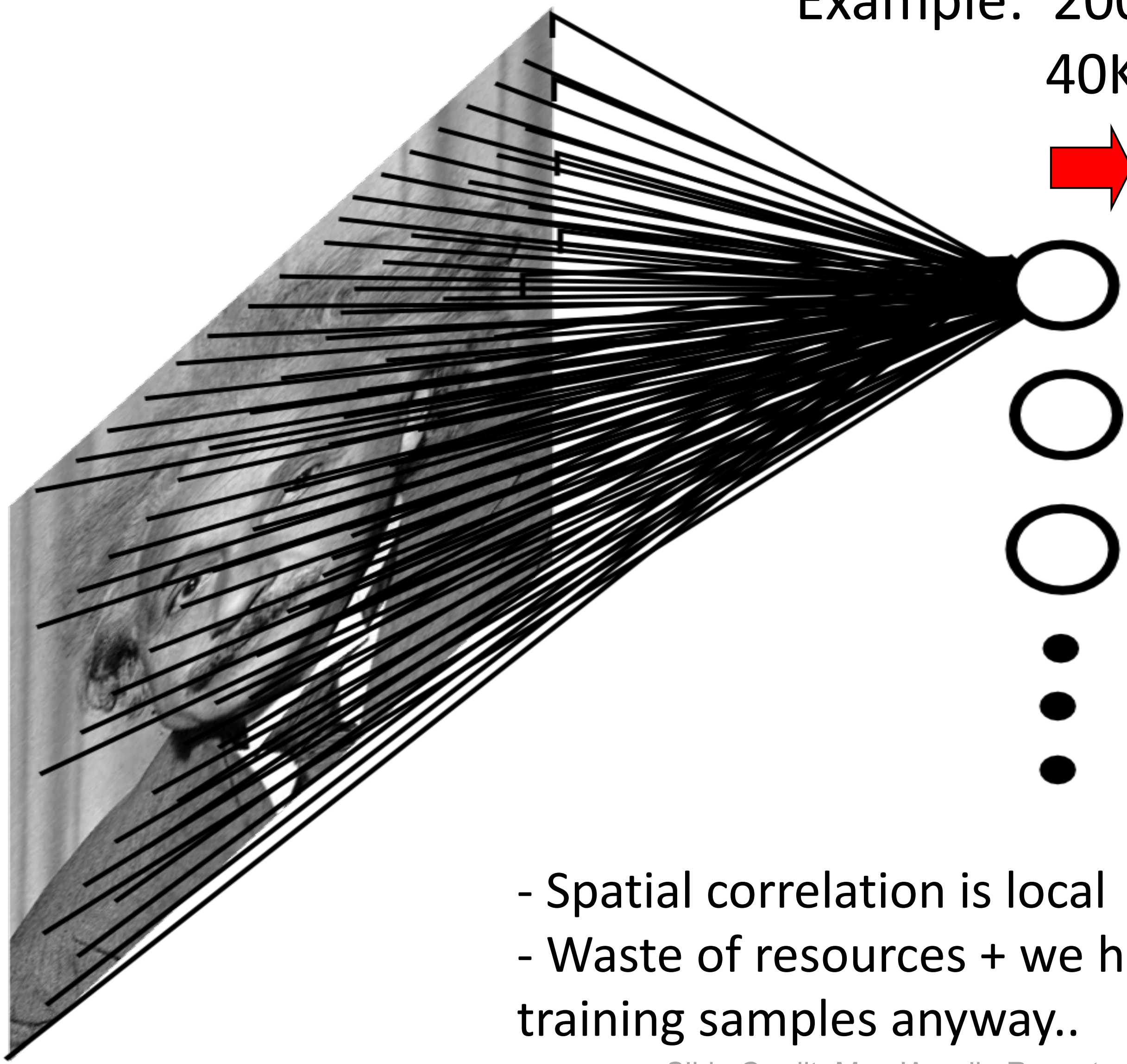
Slide Credit: Marc'Aurelio Ranzato

Fully Connected Layer

Example: 200x200 image

40K hidden units

~2B parameters!!!



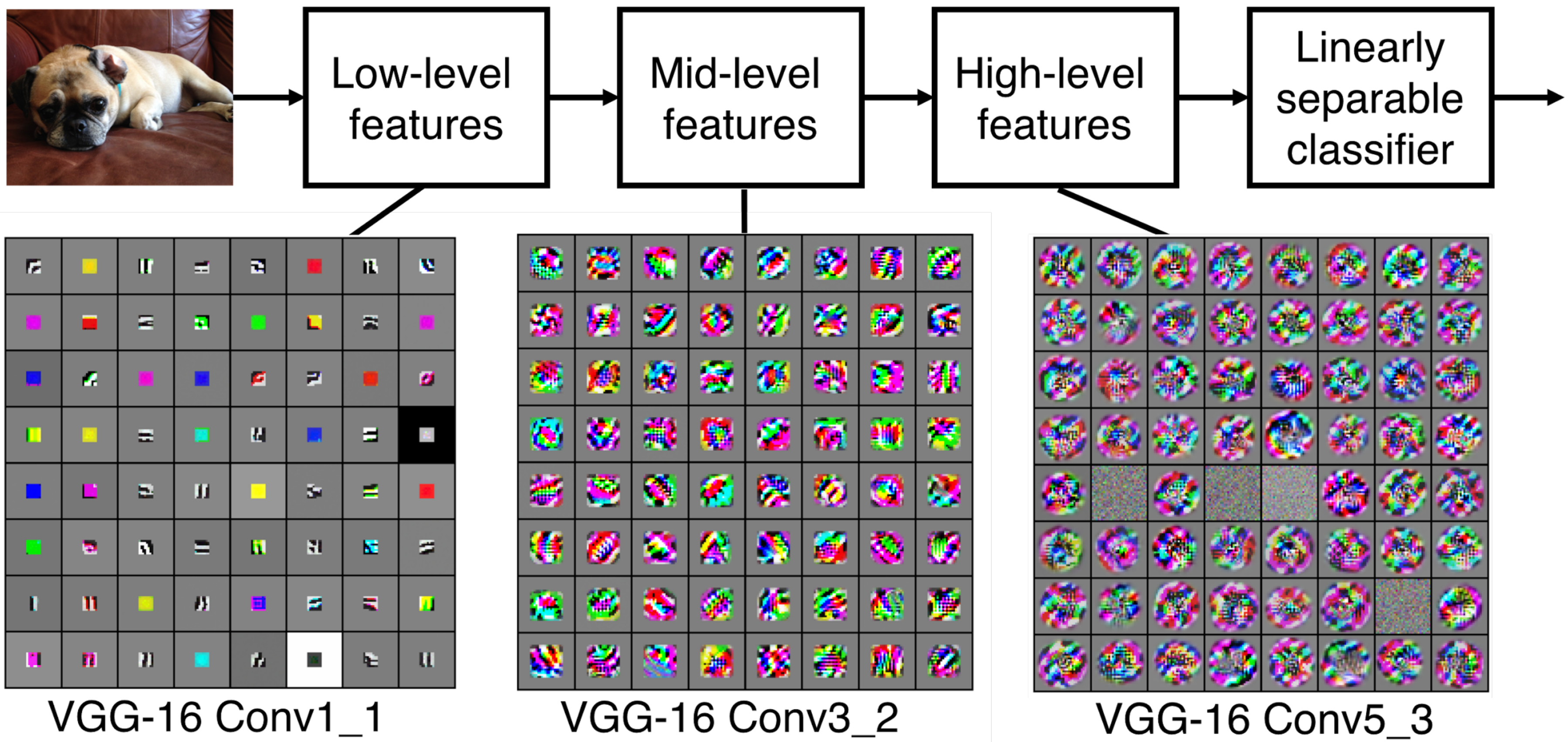
- Spatial correlation is local
- Waste of resources + we have not enough training samples anyway..

Slide Credit: Marc'Aurelio Ranzato

Preview

[Zeiler and Fergus 2013]

Visualization of VGG-16 by Lane McIntosh. VGG-16 architecture from [Simonyan and Zisserman 2014].



Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

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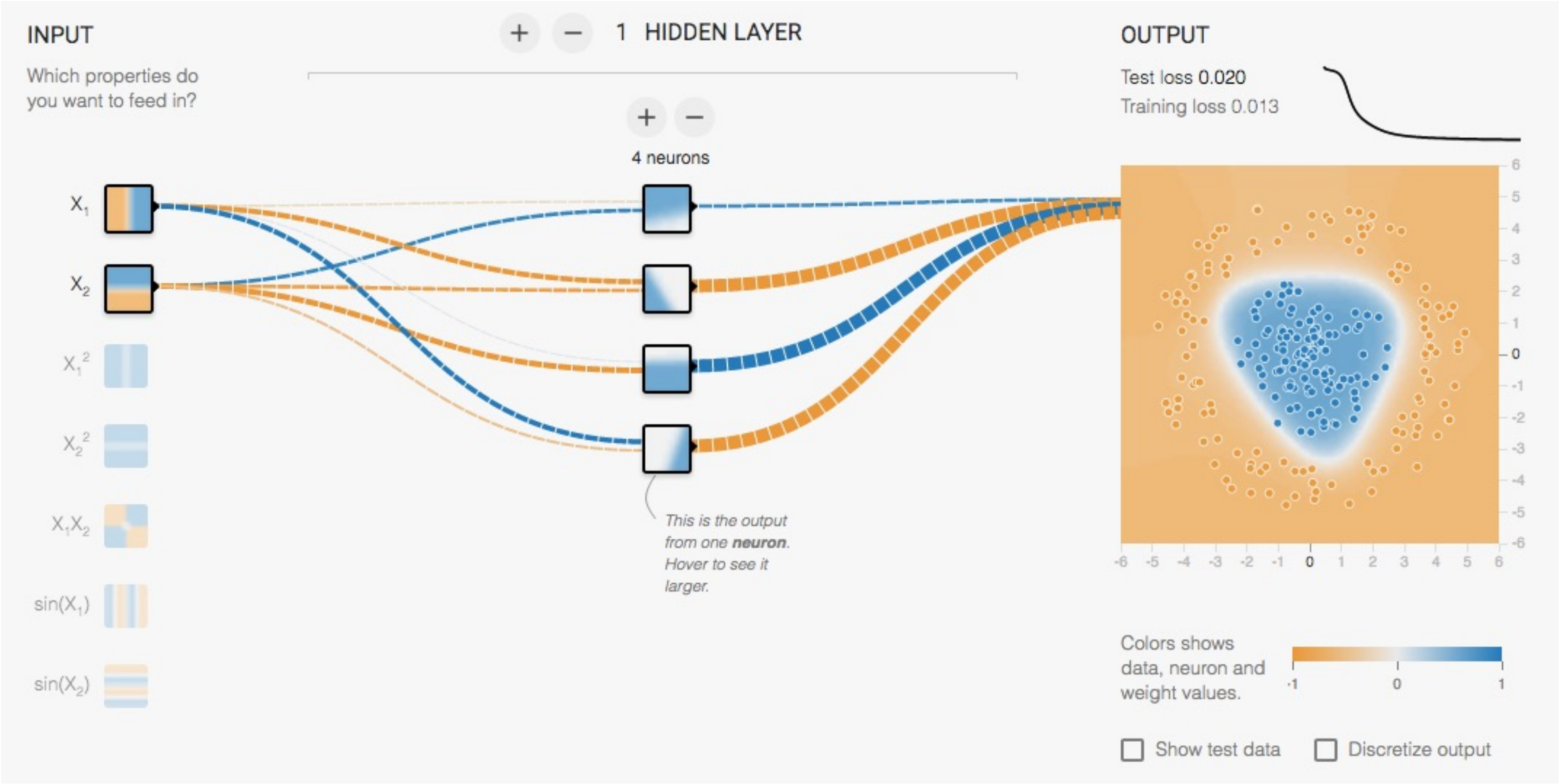
Review: Neural Networks



http://playground.tensorflow.org/

How to minimize the loss by changing the weights?
Strategy: **Follow the slope of the loss function**

Strategy: **Follow the slope**

In 1-dimension, the derivative of a function:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Strategy: **Follow the slope**

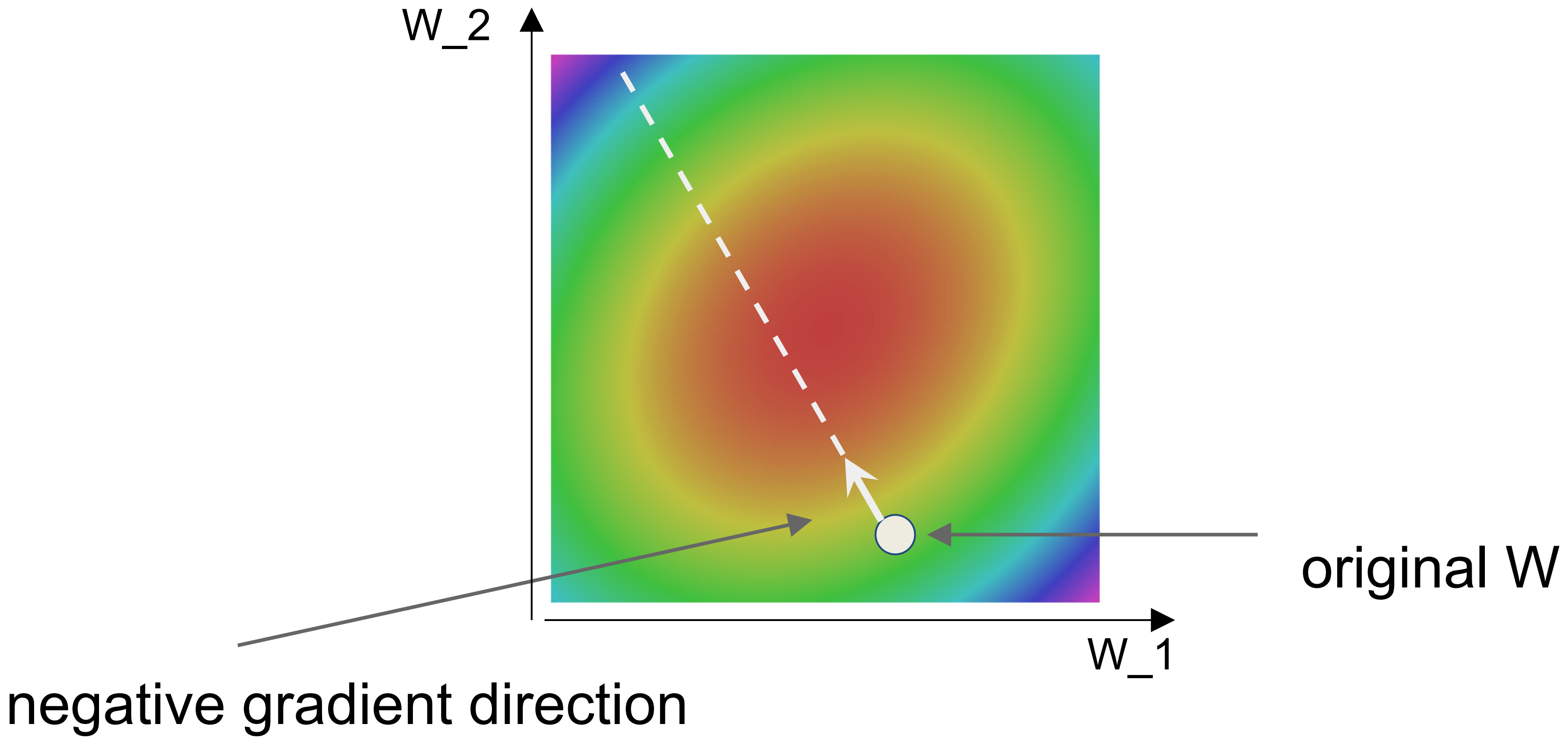
In 1-dimension, the derivative of a function:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

In multiple dimensions, the **gradient** is the vector of (partial derivatives) along each dimension

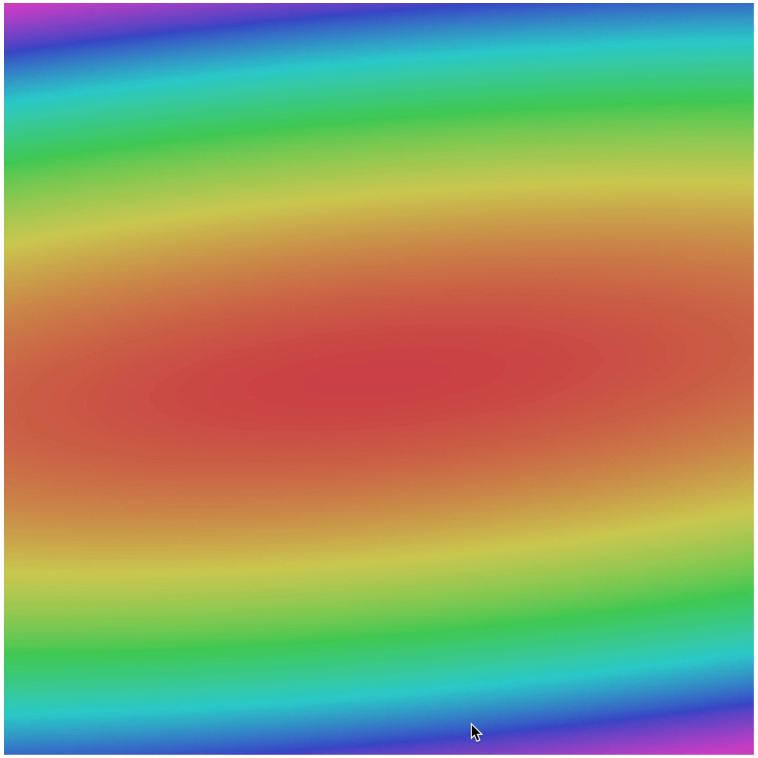
The slope in any direction is the **dot product** of the direction with the gradient
The direction of steepest descent is the **negative gradient**

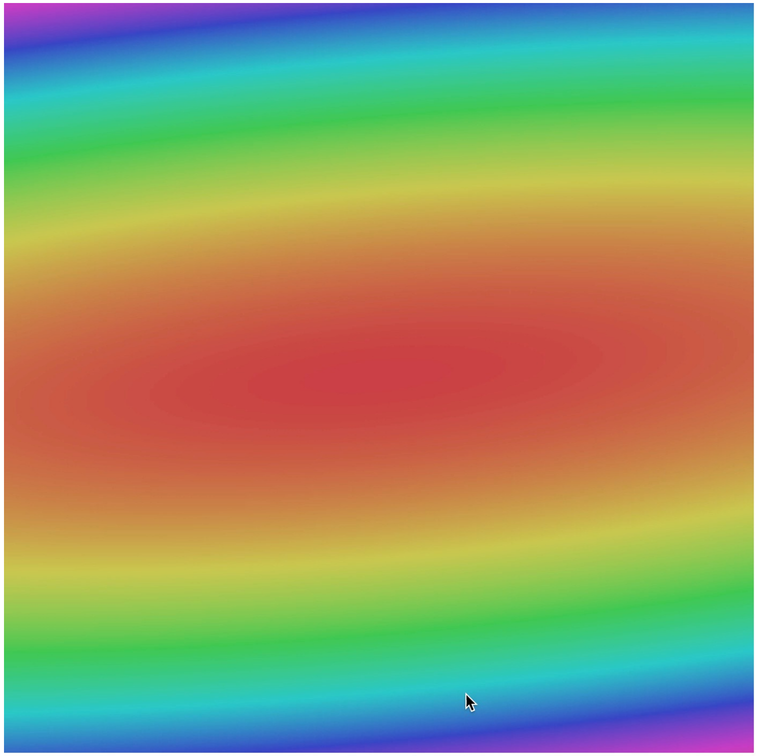
http://demonstrations.wolfram.com/VisualizingTheGradientVector/



Gradient Descent

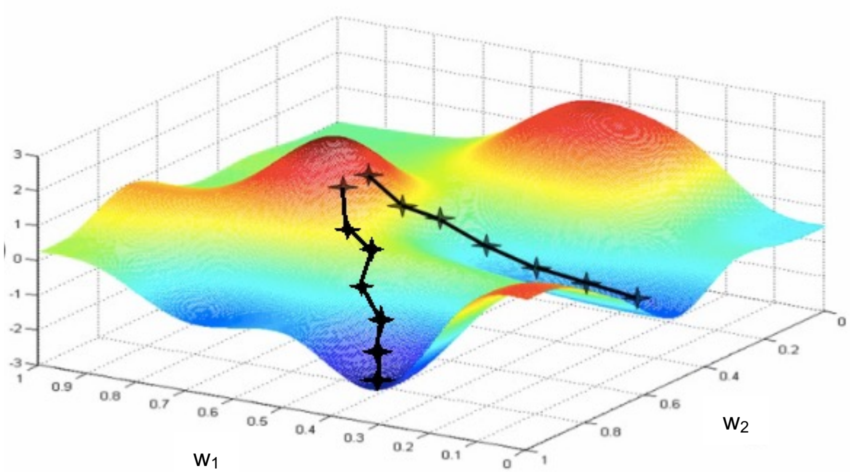
```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```





Training of multi-layer networks

- Find network weights to minimize the prediction loss between true and estimated labels of training examples:
- $E(\mathbf{w}) = \sum_i l(\mathbf{x}_i, y_i; \mathbf{w})$
- Update weights by **gradient descent:** $\mathbf{w} \leftarrow \mathbf{w} - \alpha \frac{\partial E}{\partial \mathbf{w}}$



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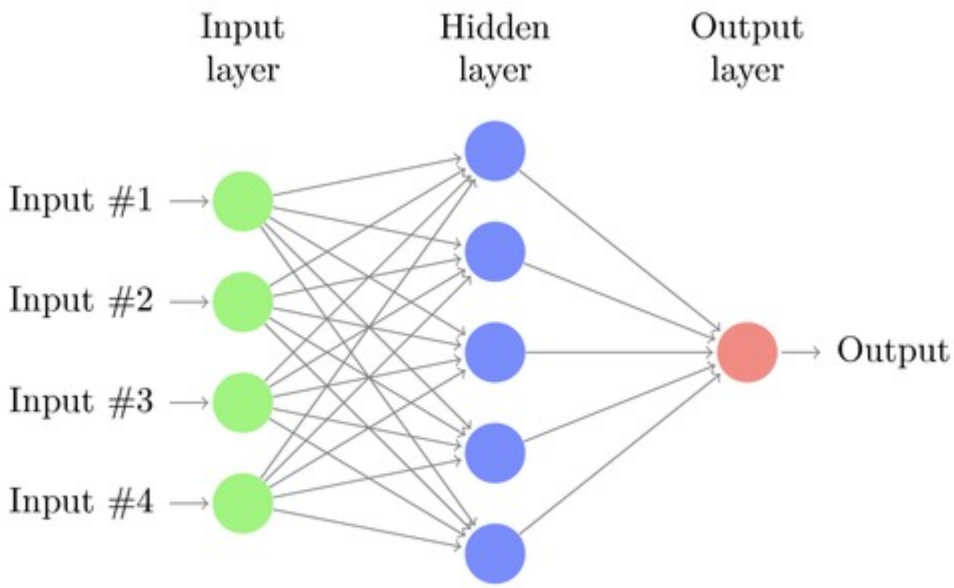
- Update weights by **gradient descent:** $\mathbf{w} \leftarrow \mathbf{w} - \alpha \frac{\partial E}{\partial \mathbf{w}}$
- **Back-propagation:** gradients are computed in the direction from output to input layers and combined using chain rule
- **Stochastic gradient descent:** compute the weight update w.r.t. one training example (or a small batch of examples) at a time, cycle through training examples in random order in multiple epochs

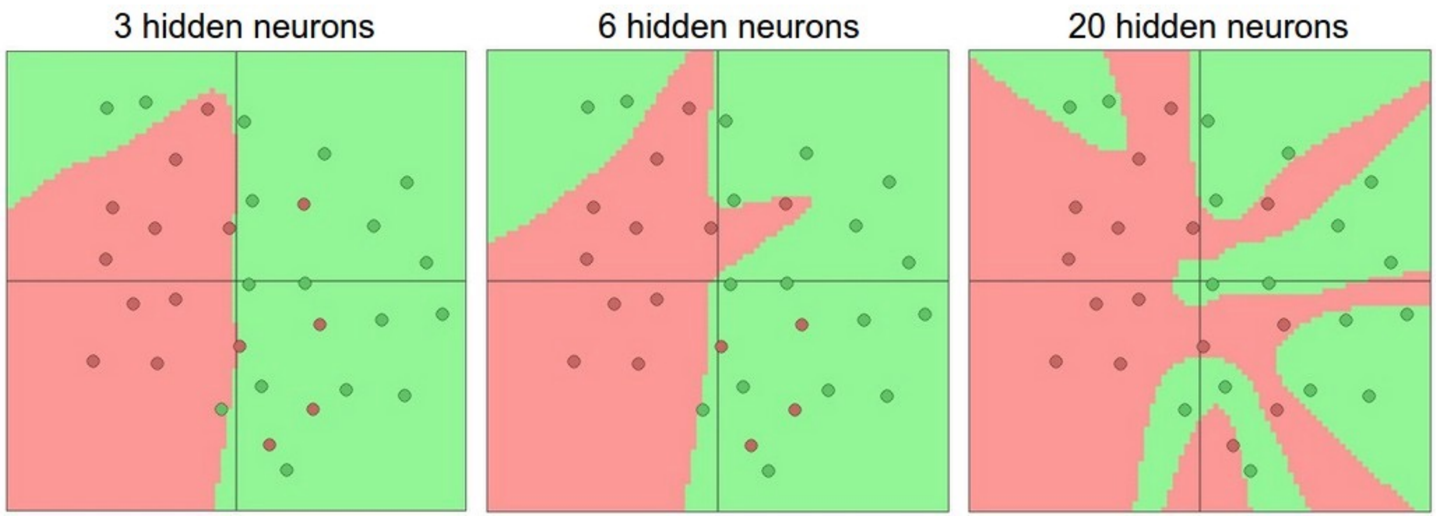
Network with a single hidden layer

- Neural networks with at least one hidden layer are *universal function approximators*



Network with a single hidden layer

Hidden layer size and *network capacity*:



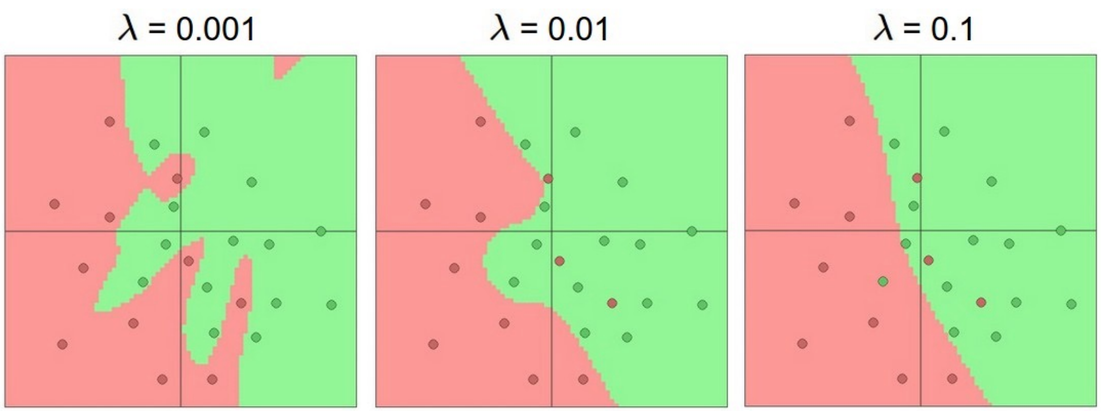
Source: http://cs231n.github.io/neural-networks-1/

Regularization

- It is common to add a penalty (e.g., quadratic) on weight magnitudes to the objective function:

$$E(\mathbf{w}) = \sum_i l(\mathbf{x}_i, y_i; \mathbf{w}) + \lambda \|\mathbf{w}\|^2$$

  – Quadratic penalty encourages network to use all of its inputs “a little” rather than a few inputs “a lot”



Source: http://cs231n.github.io/neural-networks-1/

Neural networks: Pros and cons

- Pros
  - Flexible and general function approximation framework
  - Can build extremely powerful models by adding more layers
- Cons
  - Hard to analyze theoretically (e.g., training is prone to local optima)
  - Huge amount of training data, computing power may be required to get good performance
  - The space of implementation choices is huge (network architectures, parameters)

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Suppose: 3 training examples, 3 classes.
With some W the scores $f(x,W) = Wx$ are:

  

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

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car	5.1	**4.9**	2.5
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A **loss function** tells how good our current classifier is

Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and y_i is (integer) label

Loss over the dataset is a sum of loss over examples:

$$L = \frac{1}{N} \sum_i L_i(f(x_i, W), y_i)$$

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**
car	5.1
frog	-1.7

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

$$L_i = \sum_{j \neq y_i} \begin{cases} 0 & \text{if } s_{y_i} \geq s_j + 1 \\ s_j - s_{y_i} + 1 & \text{otherwise} \end{cases}$$

$$= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Suppose: 3 training examples, 3 classes.
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cat	**3.2**	1.3	2.2
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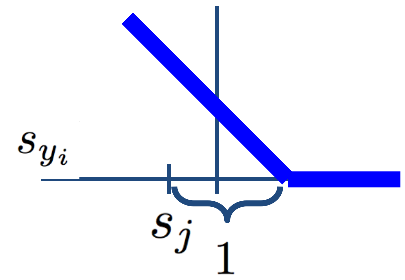
Multiclass SVM loss:

"Hinge loss"



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With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9		

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 5.1 - 3.2 + 1)
+max(0, -1.7 - 3.2 + 1)
= max(0, 2.9) + max(0, -3.9)
= 2.9 + 0
= 2.9

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	

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Given an example (x_i, y_i)
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where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

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the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 2.2 - (-3.1) + 1)
+max(0, 2.5 - (-3.1) + 1)
= max(0, 6.3) + max(0, 6.6)
= 6.3 + 6.6
= 12.9

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
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frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the
scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

Loss over full dataset is average:

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i$$

L = (2.9 + 0 + 12.9)/3
= **5.27**

Softmax approach to dealing with multiple classes

- If we need to classify inputs into C different classes, we put C units in the last layer to produce C *one-vs.-others* scores $f_1, f_2, \ldots, f_C$
- Apply *softmax* function to convert these scores to probabilities:

$$\text{softmax}(f_1, \ldots, f_c) = \left(\frac{\exp(f_1)}{\sum_j \exp(f_j)}, \ldots, \frac{\exp(f_C)}{\sum_j \exp(f_j)}\right)$$

If one of the inputs is much larger than the others, then the corresponding softmax value will be close to 1 and others will be close to 0

- Use log likelihood (*cross-entropy*) loss:
- $l(\mathbf{x}_i, y_i; \mathbf{w}) = -\log P_{\mathbf{w}}(y_i \mid \mathbf{x}_i)$

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$\boxed{\text{Stochastic Gradient Descent}}$

Computing Gradients

Gradient Descent has a problem

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(x_i, y_i, W) + \lambda R(W)$$

$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W) + \lambda \nabla_W R(W)$$

Full sum expensive when N is large!

Stochastic Gradient Descent (SGD)

$$L(W) = \frac{1}{N}\sum_{i=1}^{N} L_i(x_i, y_i, W) + \lambda R(W)$$

$$\nabla_W L(W) = \frac{1}{N}\sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W) + \lambda \nabla_W R(W)$$

Full sum expensive when N is large!

Approximate sum using a **minibatch** of examples
32 / 64 / 128 common

```
# Vanilla Minibatch Gradient Descent

while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

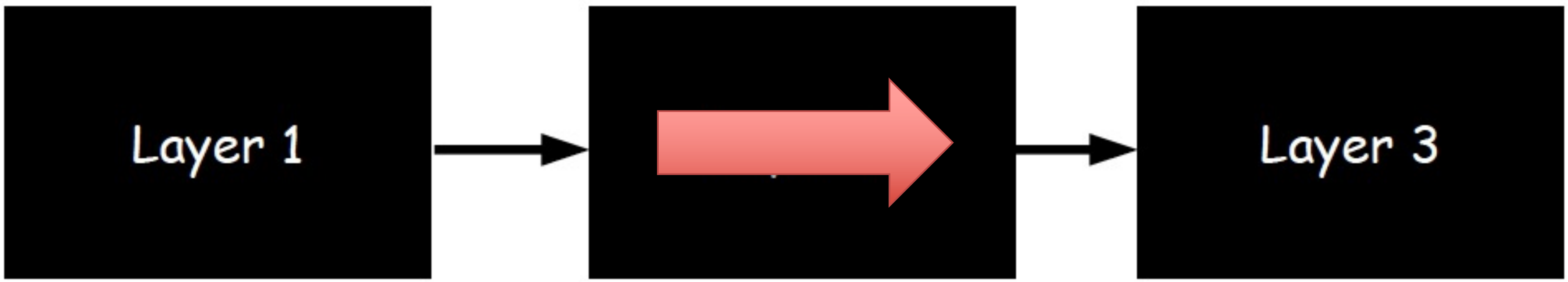
Neural Network Training

- Step 1: Compute Loss on mini-batch [F-Pass]

Slide Credit: Marc'Aurelio Ranzato, Yann LeCun

Neural Network Training

- Step 1: Compute Loss on mini-batch [F-Pass]

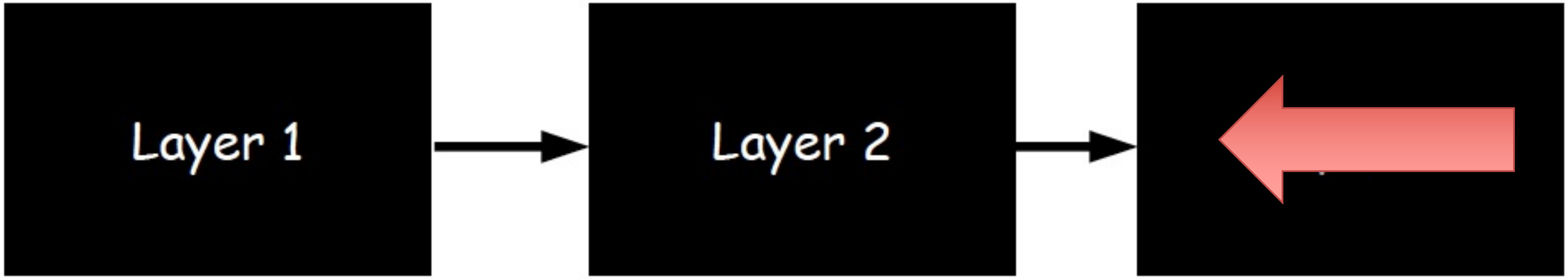


Neural Network Training

- Step 1: Compute Loss on mini-batch [F-Pass]

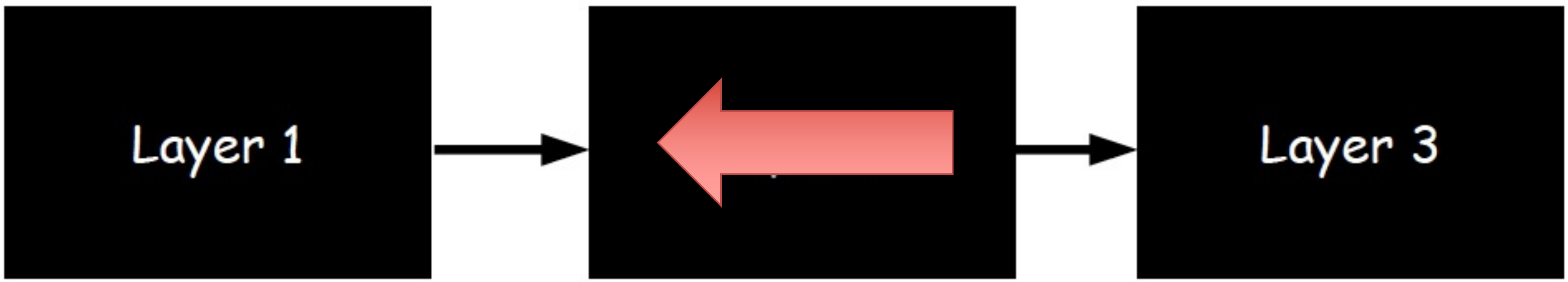
Neural Network Training

- Step 1: Compute Loss on mini-batch [F-Pass]
- Step 2: Compute gradients wrt parameters [B-Pass]



Neural Network Training

- Step 1: Compute Loss on mini-batch [F-Pass]
- Step 2: Compute gradients wrt parameters [B-Pass]



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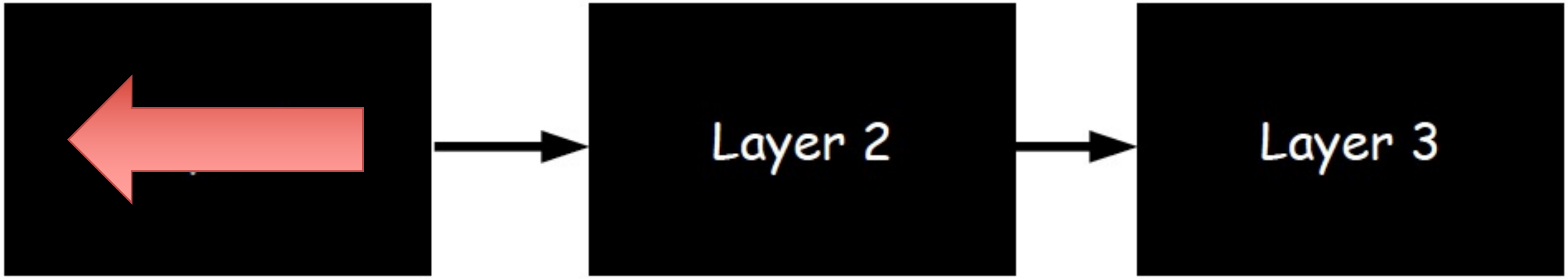


Neural Network Training

- Step 1: Compute Loss on mini-batch [F-Pass]
- Step 2: Compute gradients wrt parameters [B-Pass]
- Step 3: Use gradient to update parameters

$$\theta \leftarrow \theta - \eta \frac{dL}{d\theta}$$

Slide Credit: Marc'Aurelio Ranzato, Yann LeCun

Image Classification

Supervised Learning

CNN Review

Training CNNs

Loss Functions

Stochastic Gradient Descent

$\boxed{\text{Computing Gradients}}$

How do we compute gradients?

- Analytic or “Manual” Differentiation
- Symbolic Differentiation
- Numerical Differentiation
- Automatic Differentiation
  - Forward mode AD
  - Reverse mode AD
    - aka “backprop”

Numerical vs Analytic Gradients

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Numerical gradient: slow :(, approximate :(, easy to write :)
Analytic gradient: fast :), exact :), error-prone :(

In practice: Derive analytic gradient, check your implementation with numerical gradient.
This is called a **gradient check.**

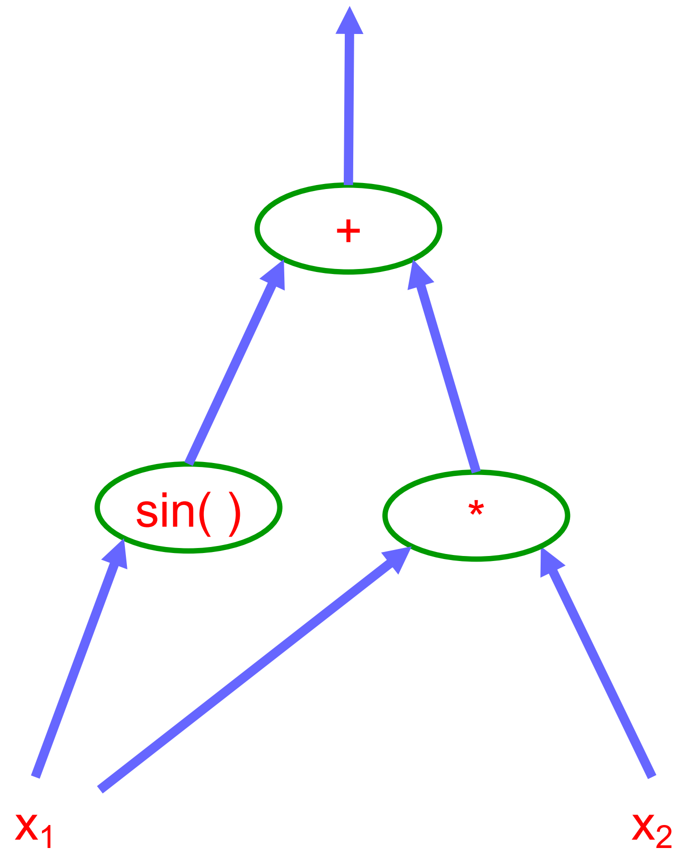
Automatic Differentiation

- Notation

$$f(x_1, x_2) = x_1 x_2 + \sin(x_1)$$

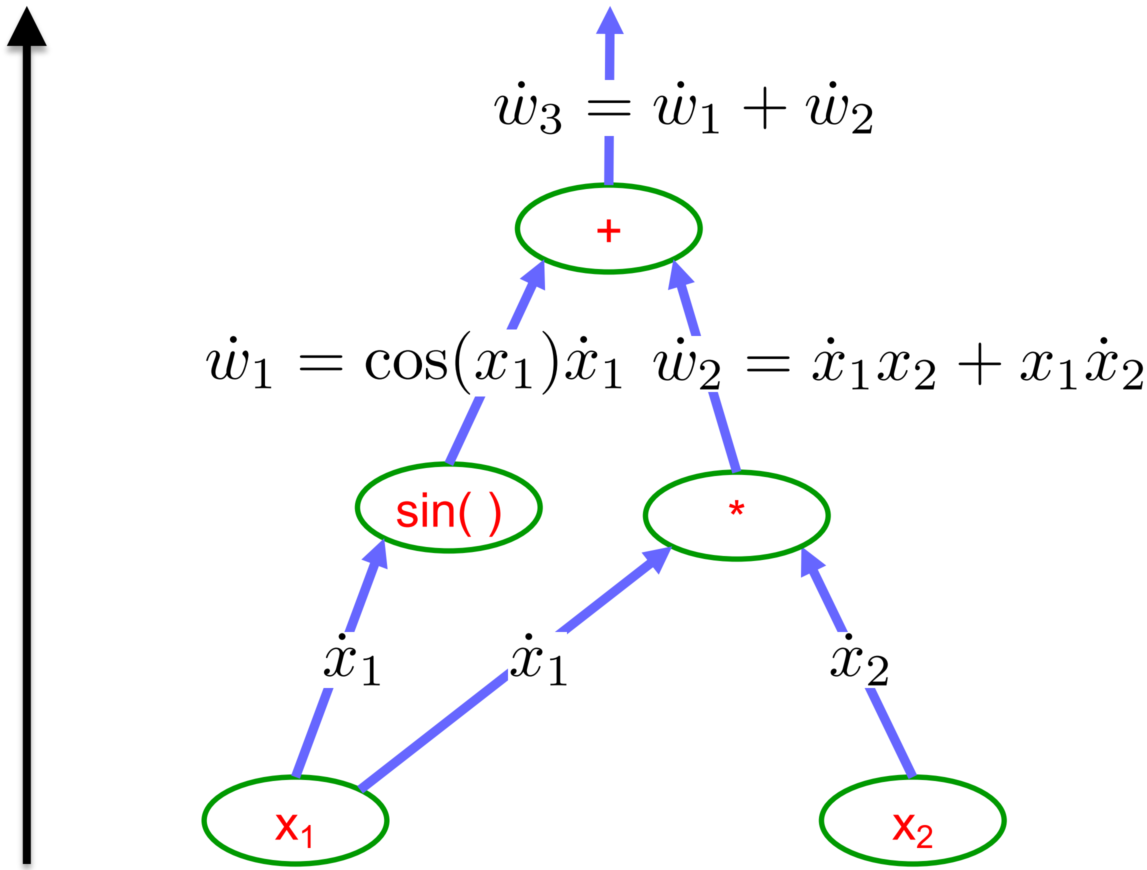
Computational Graphs

$$f(x_1, x_2) = x_1 x_2 + \sin(x_1)$$



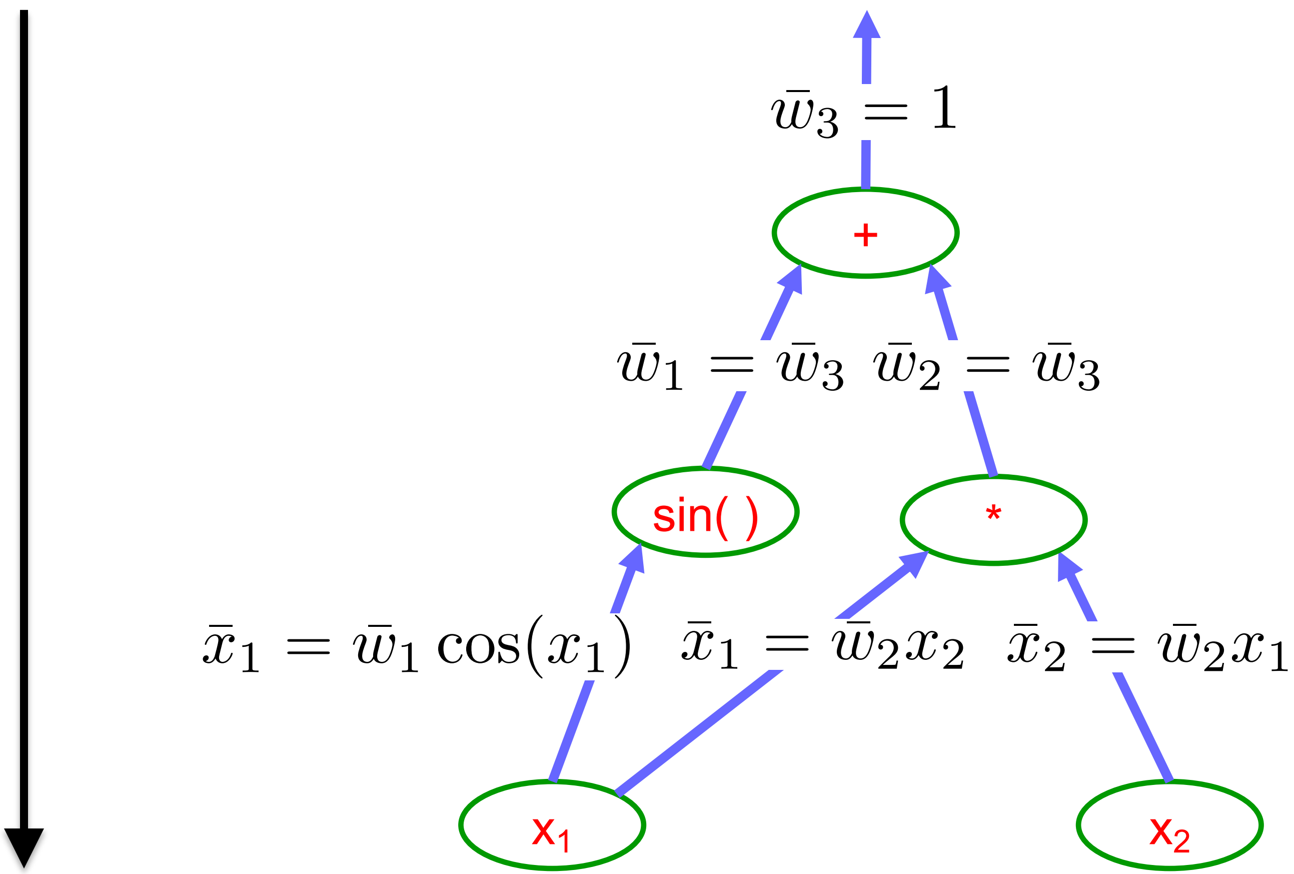
Example: Forward mode AD

$$f(x_1, x_2) = x_1 x_2 + \sin(x_1)$$



Example: Reverse mode AD

$$f(x_1, x_2) = x_1 x_2 + \sin(x_1)$$



Forward Pass vs
Forward mode AD vs Reverse Mode AD