# CS 3630!

## *Lecture 14: Computer Vision Fundamentals*

# Topics

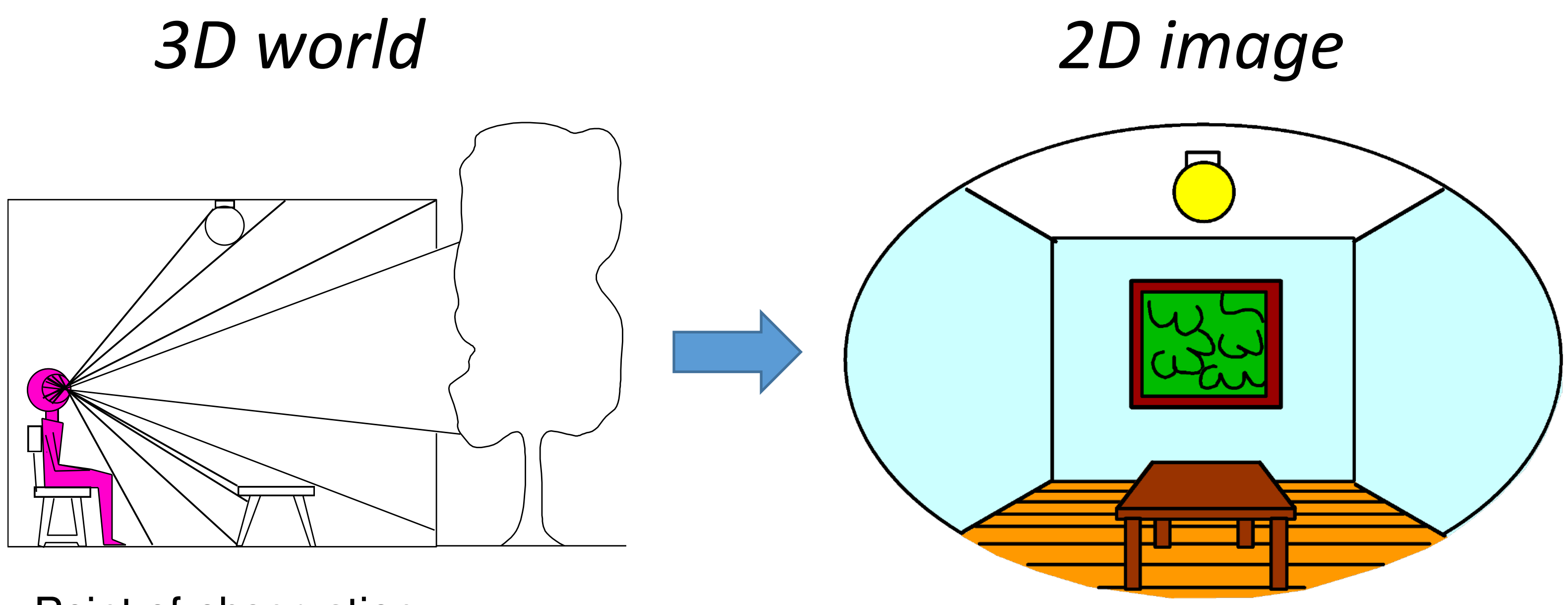
1. **Perspective Cameras**
2. **Pinhole Camera Model**
3. **Properties of projective Geometry**
4. **Stereo Vision**
5. **Stereo Geometry**
6. **Stereo Algorithms**

- Many slides borrowed from James Hays, Irfan Essa, Sing Bing Kang and others.

# Motivation

- We need to model the image formation process
- The camera can act as an (angular) measurement device
- Need a mathematical model for a simple camera
- Two cameras are better than one: metric measurements

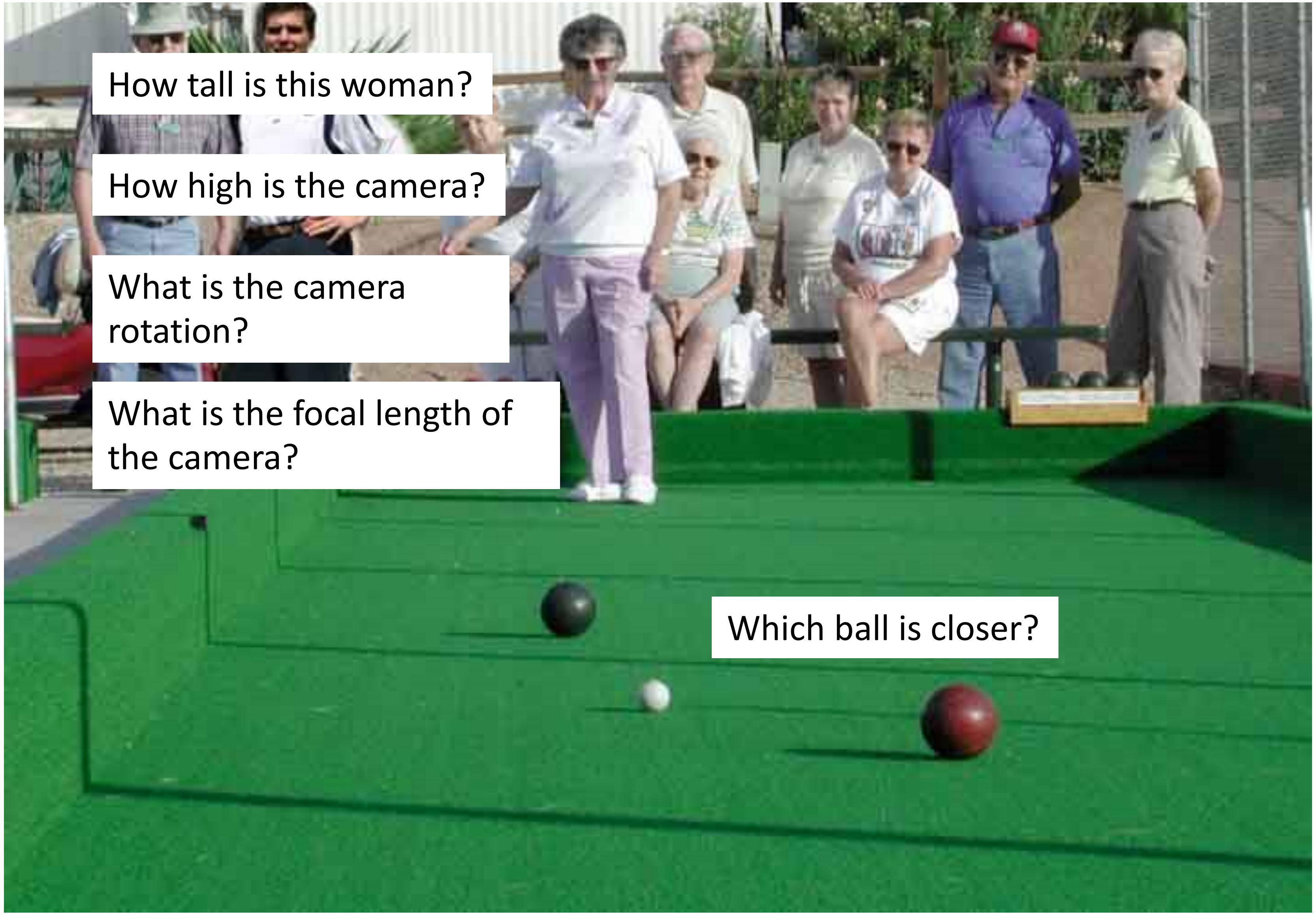
# 1. Perspective Cameras



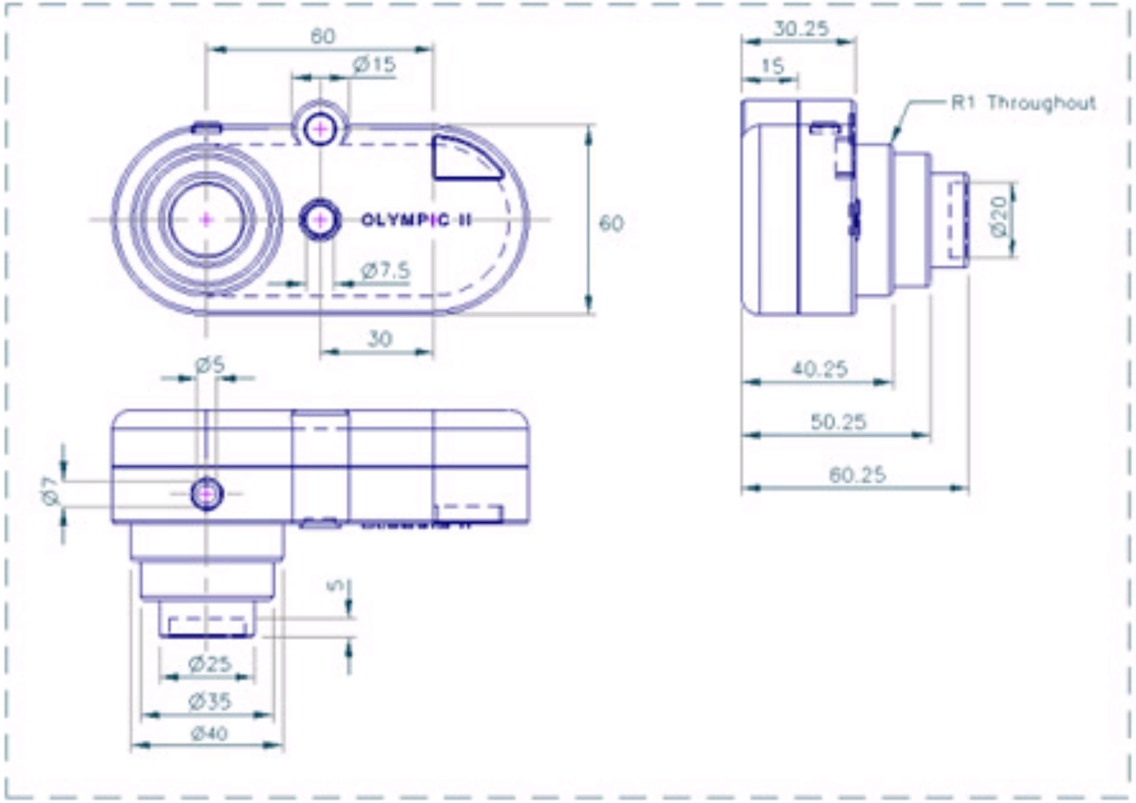

Point of observation

- Recall: Computer Vision: Images to Models
- To do this, we first need to understand the image formation process.
- We concentrate here on *geometry* (not photometry)

# Camera and World Geometry



# Orthographic Projection




- Might be familiar with this projection
- Most cameras behave differently

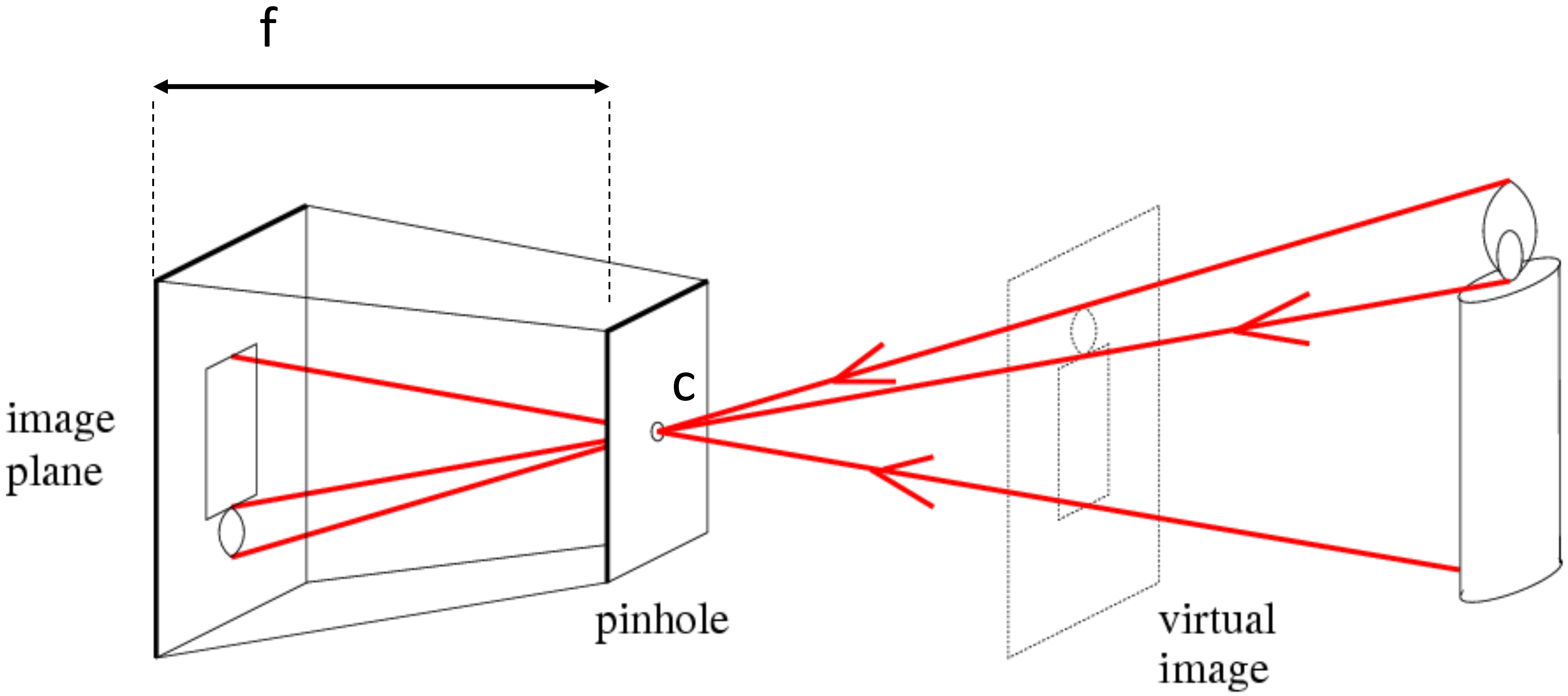
Slide source: Seitz

# Projection can be tricky…

Slide source: Seitz

# Projection can be tricky…

# 2. Pinhole camera model




f = focal length

c = center of the camera

Figure from Forsyth

# Camera obscura: the pre-camera

- Known during classical period in China and Greece (e.g. Mo-Ti, China, 470BC to 390BC)

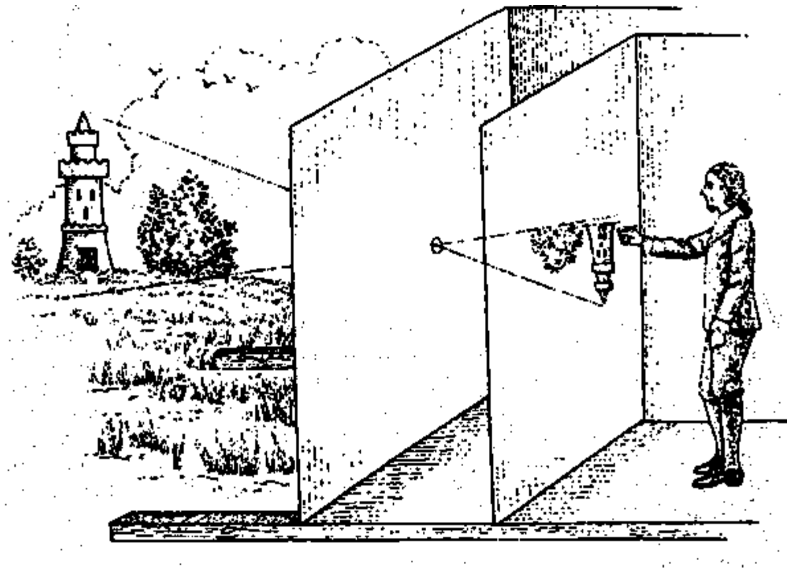
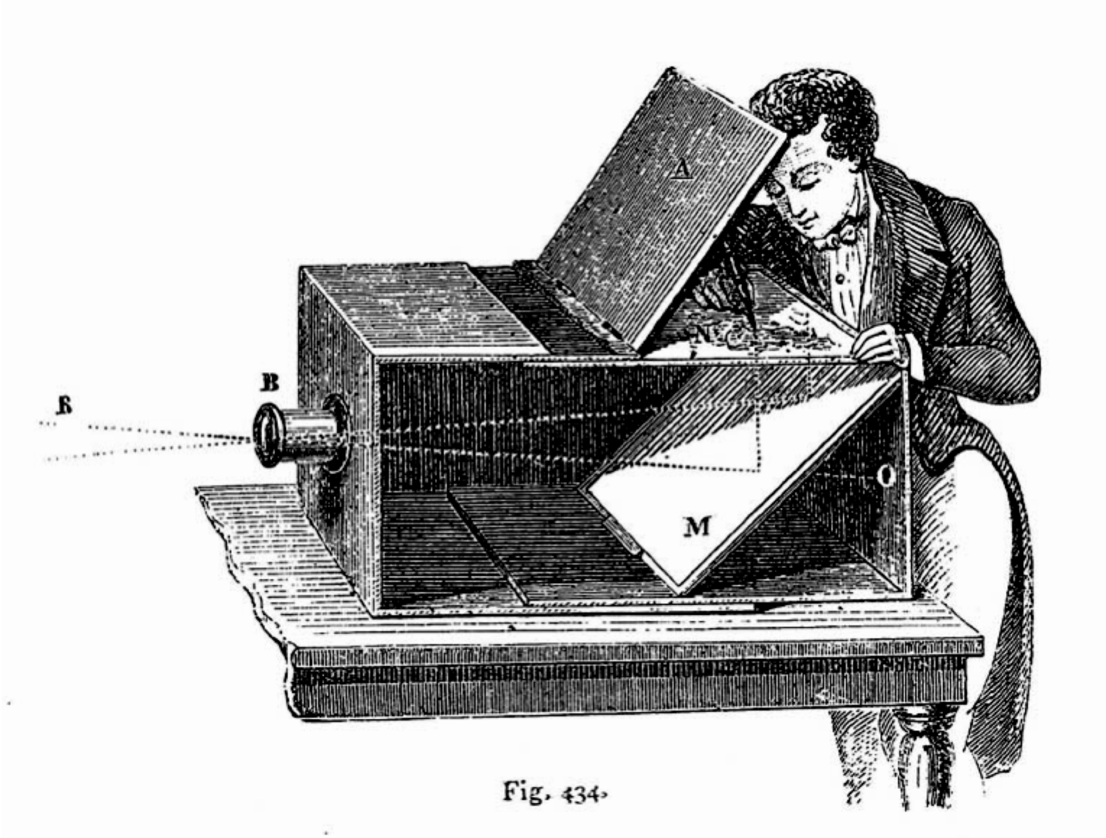


Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys

# Camera Obscura used for Tracing




Lens Based Camera Obscura, 1568

# First Photograph

Oldest surviving photograph

- Took 8 hours on pewter plate

Joseph Niepce, 1826

Photograph of the first photograph

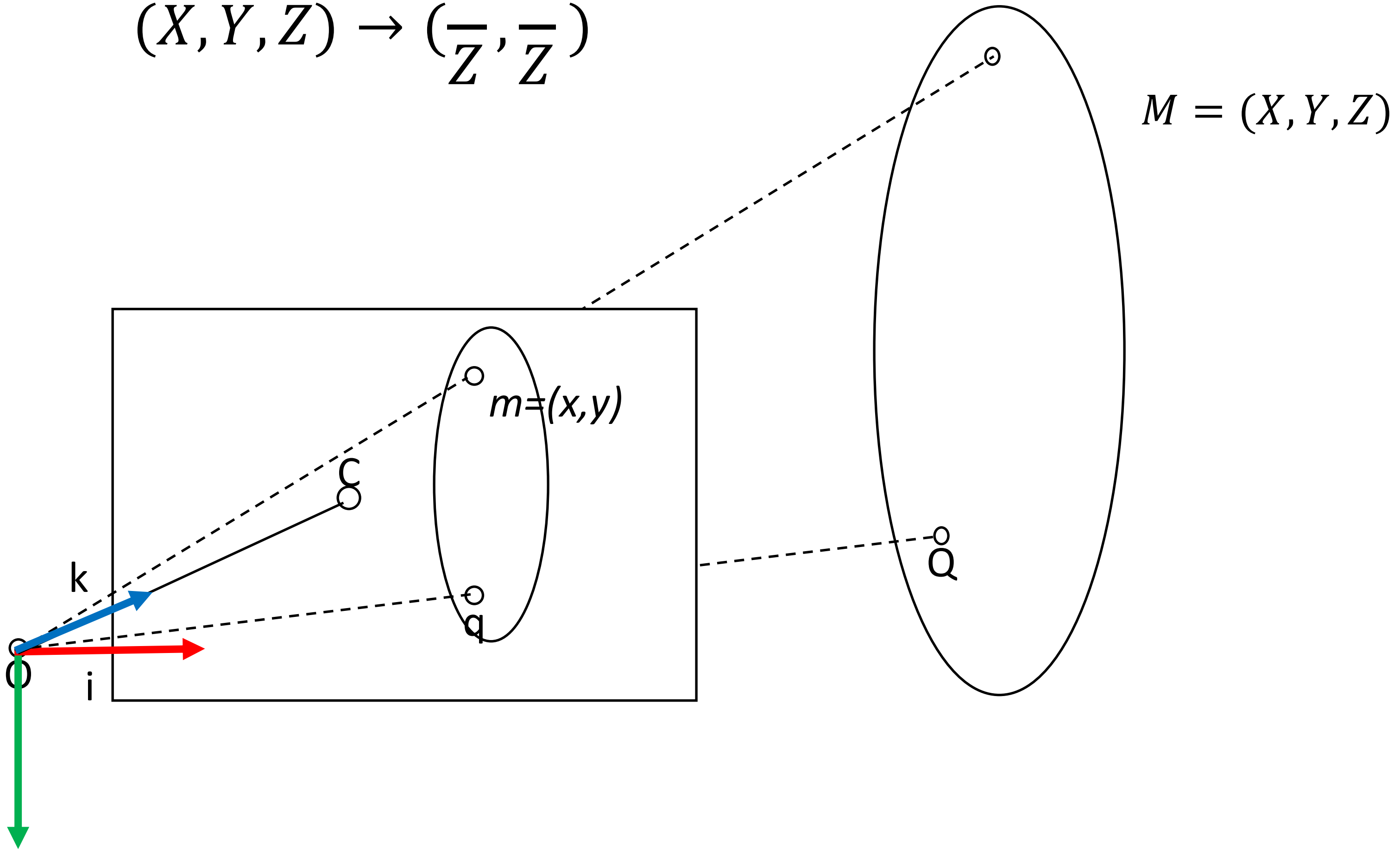
Stored at UT Austin

Niepce later teamed up with Daguerre, who eventually created Daguerrotypes

# Pinhole Camera

- Fundamental equation:

$$(X,Y,Z) \rightarrow (\frac{X}{Z},\frac{Y}{Z})$$



# Homogeneous Coordinates

Linear transformation of homogeneous (projective) coordinates

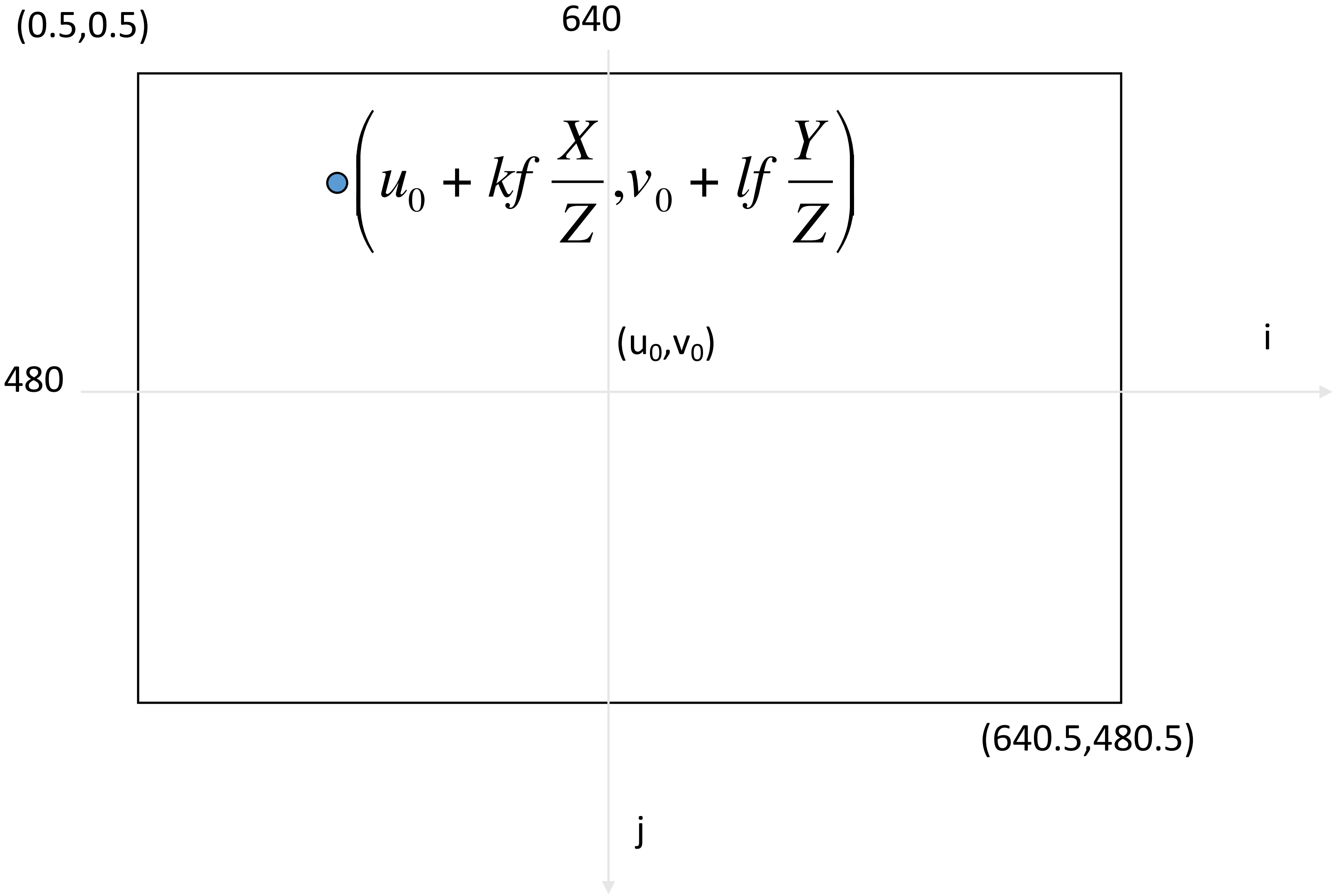
$$m = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = [\ I \quad 0\ ]M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ T \end{bmatrix}$$

Recover image (Euclidean) coordinates by normalizing:

$$x = \frac{u}{w} = \frac{X}{Z}$$

$$y = \frac{v}{w} = \frac{Y}{Z}$$

# Pixel coordinates in 2D




$$\left(u_0 + kf\frac{X}{Z}, v_0 + lf\frac{Y}{Z}\right)$$

# Intrinsic Calibration

$3 \times 3$ Calibration Matrix K

$$m = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = K[I \quad 0]M = \begin{bmatrix} \alpha & s & u_0 \\ & \beta & v_0 \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ T \end{bmatrix}$$

skew

Recover image (Euclidean) coordinates by normalizing :

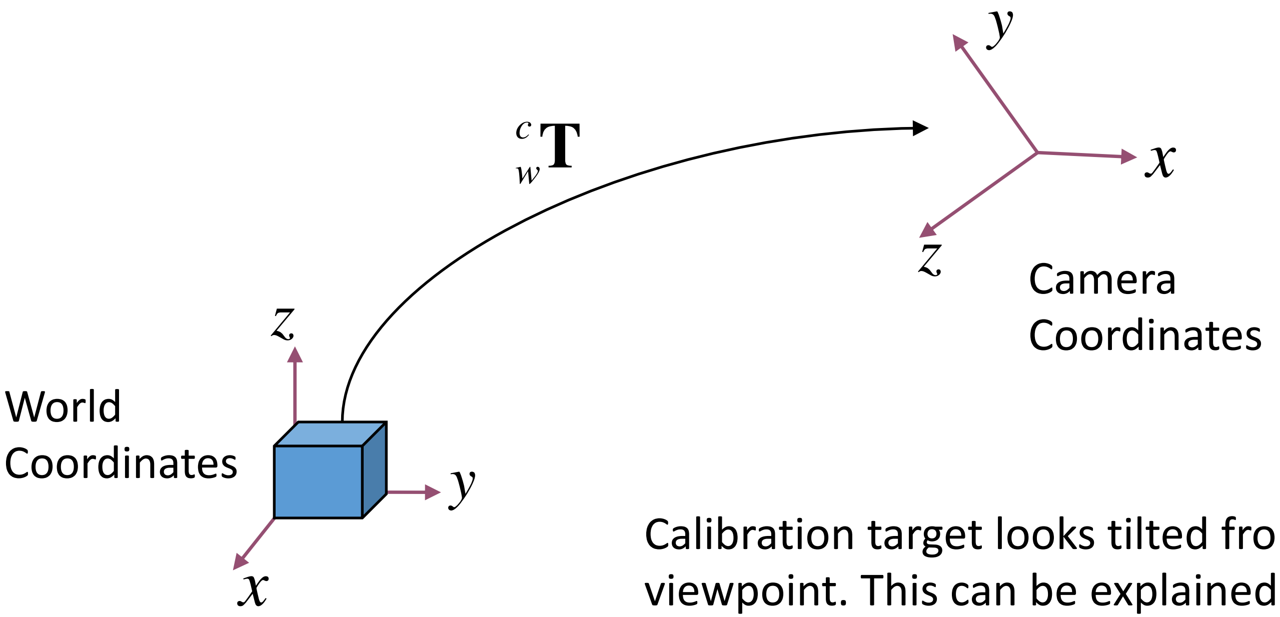
$$x = \frac{u}{w} = \frac{\alpha X + sY + u_0}{Z}$$

$$y = \frac{v}{w} = \frac{\beta Y + v_0}{Z}$$

5 Degrees of Freedom !

# Camera Pose

In order to apply the camera model, objects in the scene must be expressed in *camera coordinates*.




Calibration target looks tilted from camera viewpoint. This can be explained as a difference in coordinate systems.

# Projective Camera Matrix

$$Camera = Calibration \times \mathrm{Pr}ojection \times Extrinsics$$

$$m = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ & \beta & v_0 \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ T \end{bmatrix}$$

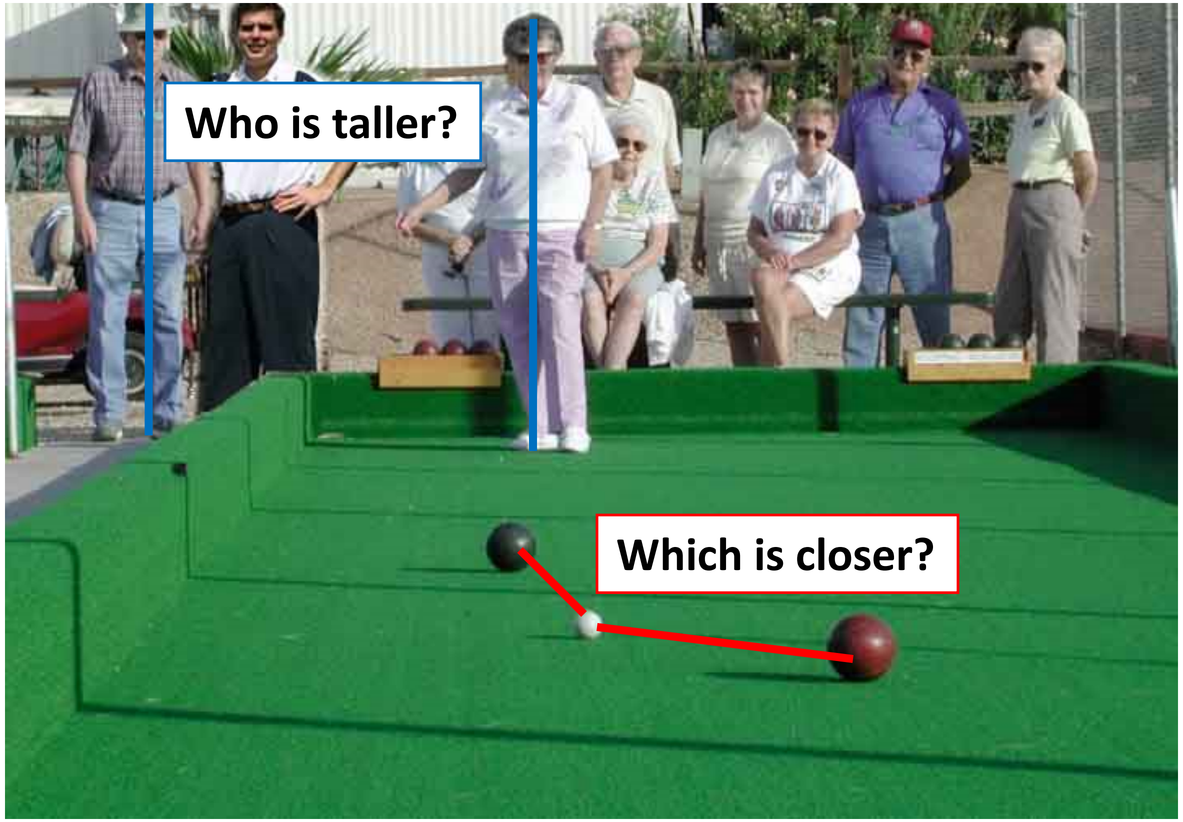
$$= K\begin{bmatrix} R & t \end{bmatrix} M = PM$$

5+6 Degrees of Freedom (DOF) = 11 !

# 3. Properties of projective Geometry

## What is lost?

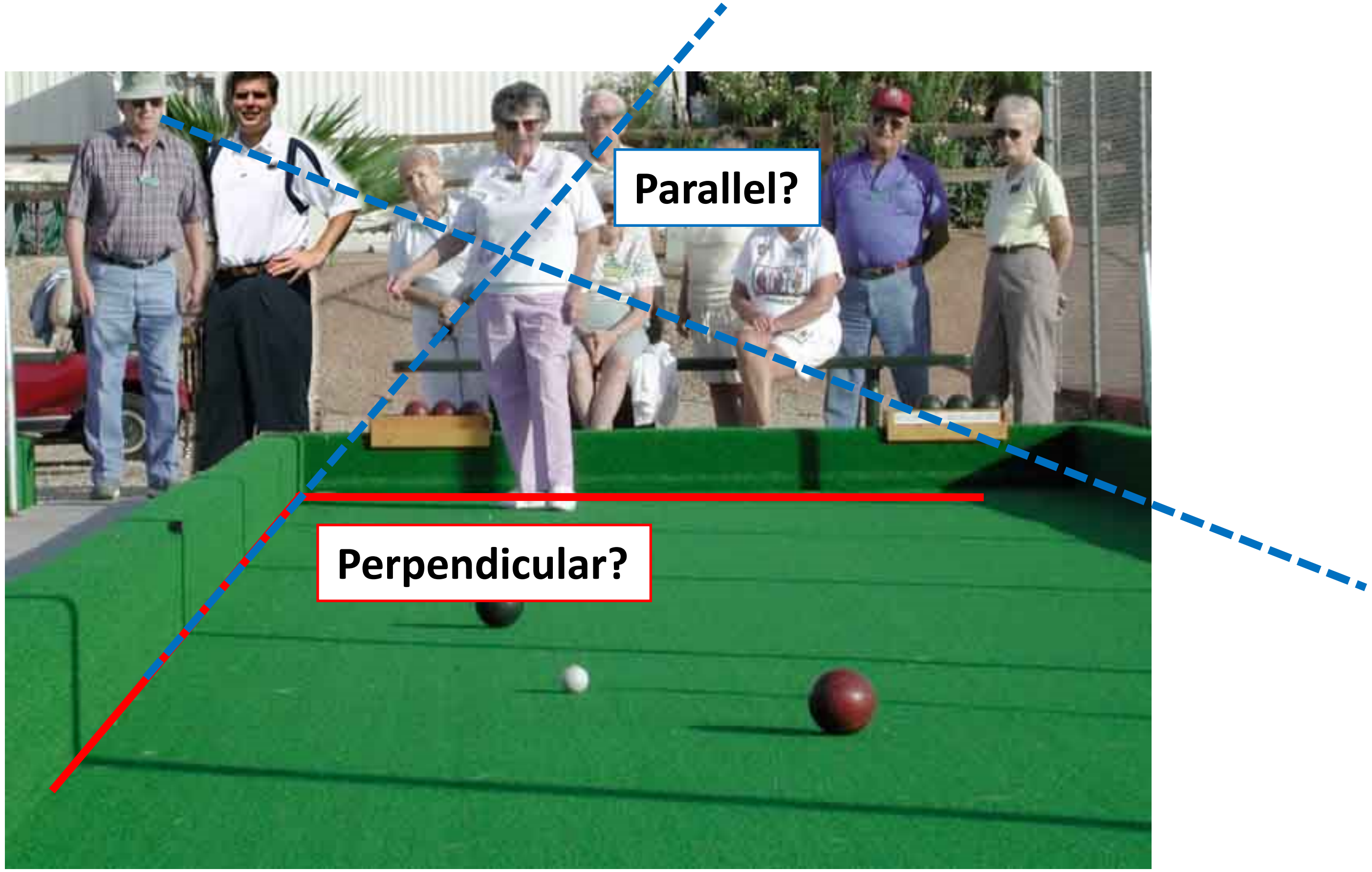
- Length



# Properties of projective Geometry

## What is lost?

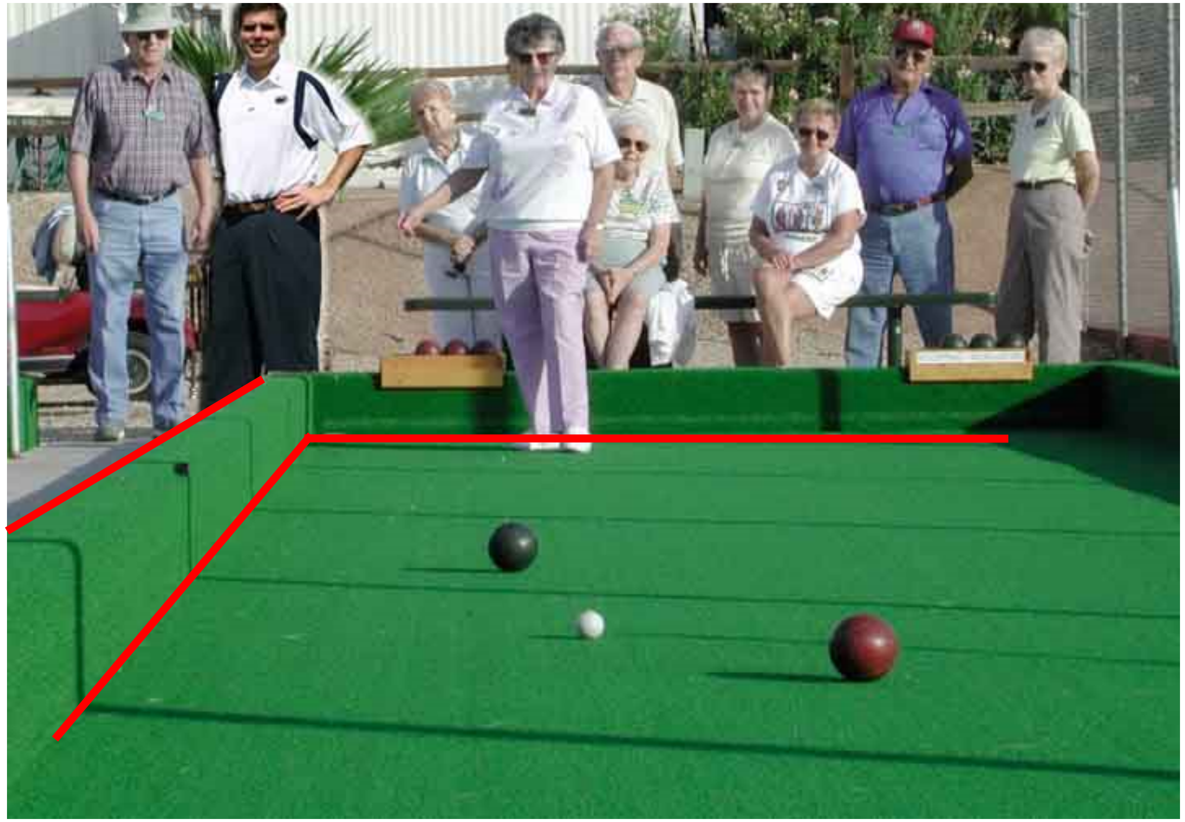
- Length
- Angles



# Properties of projective Geometry

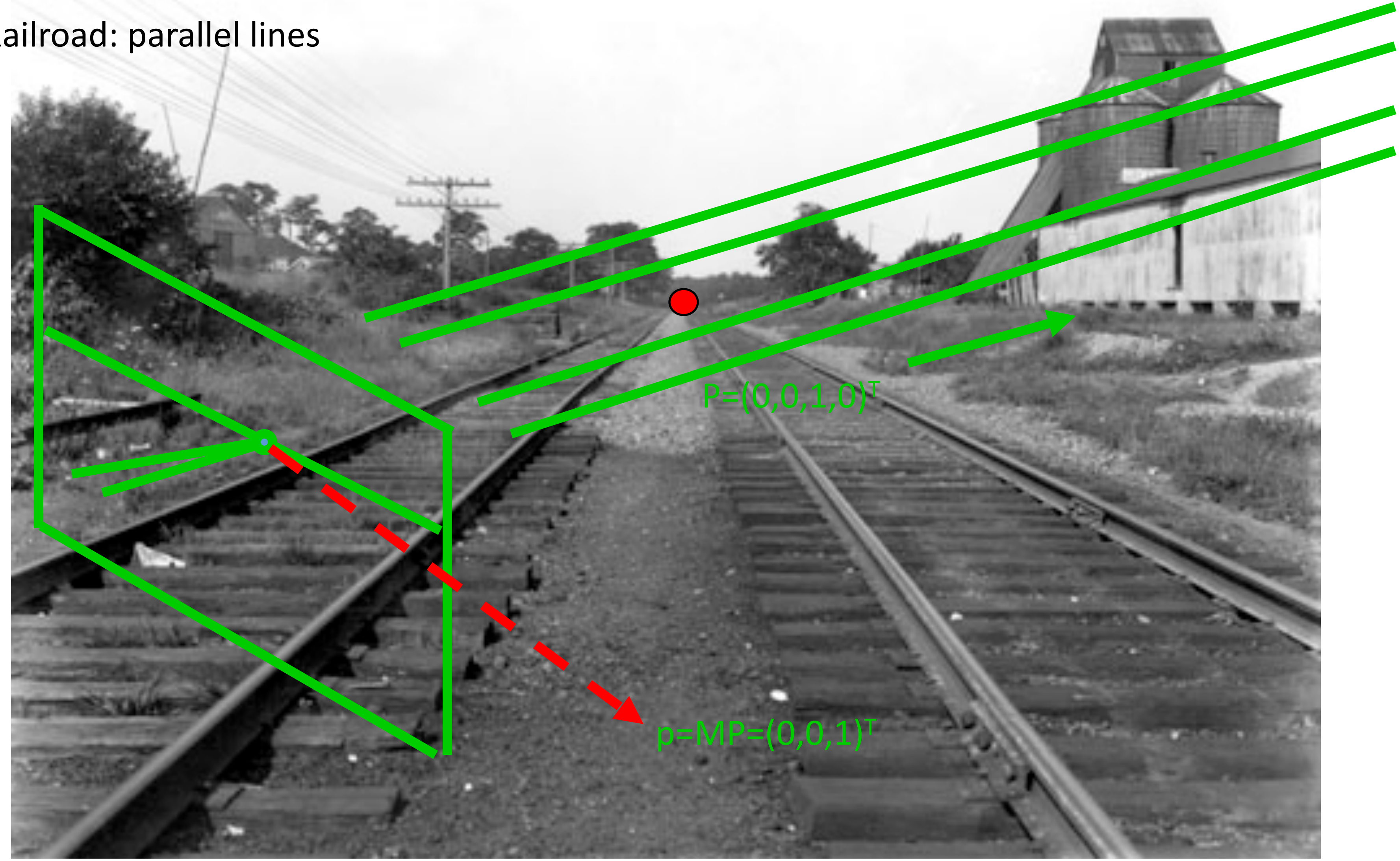
## What is preserved?

- Straight lines are still straight

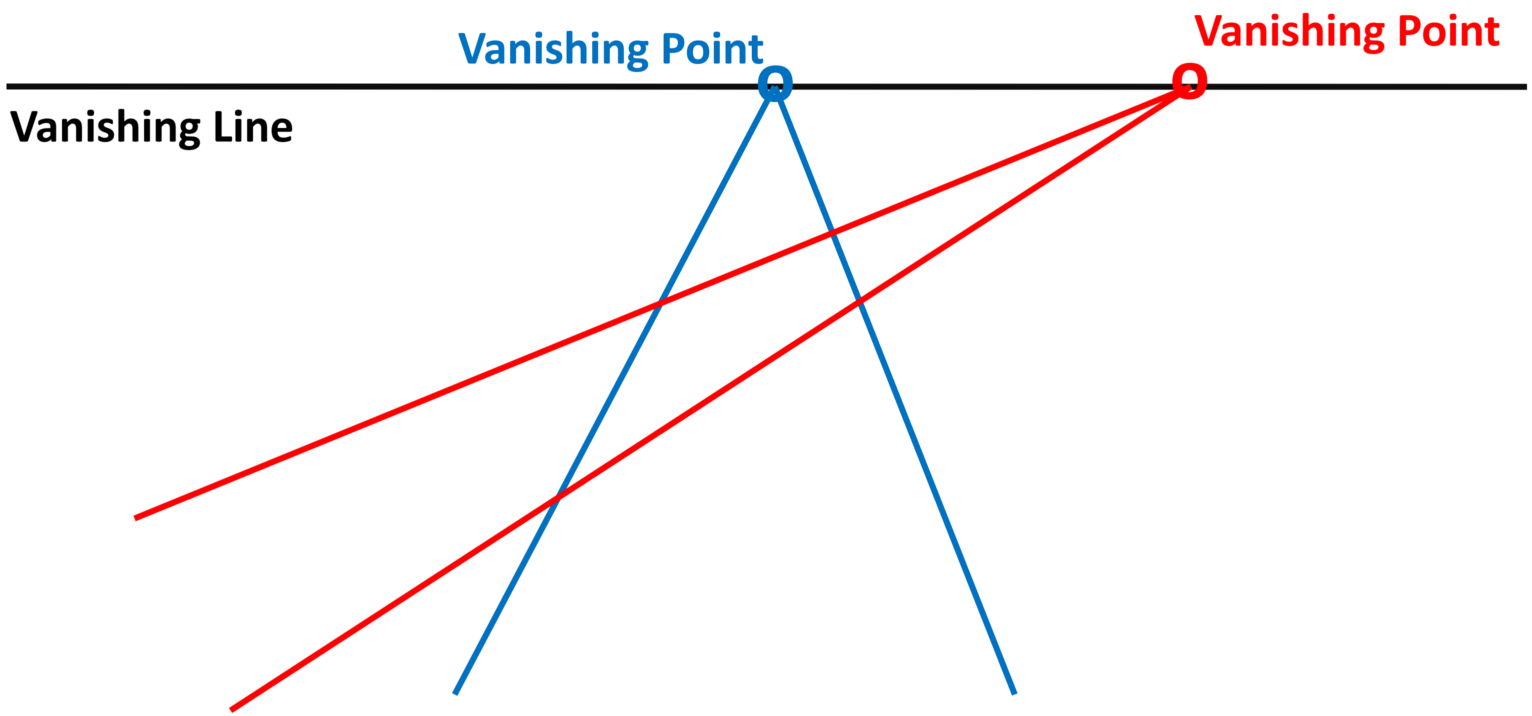


# We can see infinity !

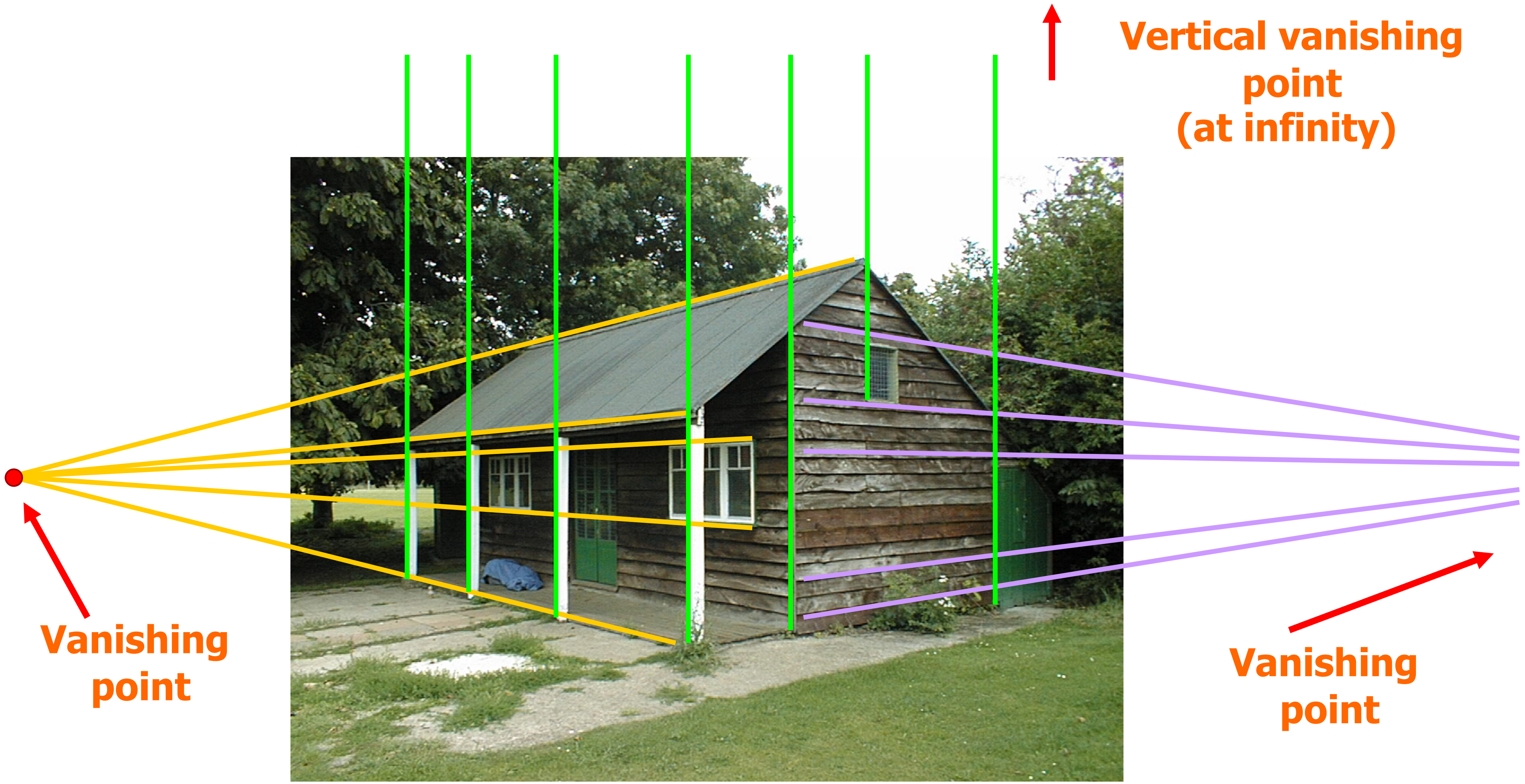
Railroad: parallel lines



# Vanishing points and lines



# Vanishing points and lines




Slide from Efros, Photo from Criminisi

# 4. Stereo Vision

- Stereo is used in the HVS
- Very useful in computer vision as well
- Eliminates scale ambiguity

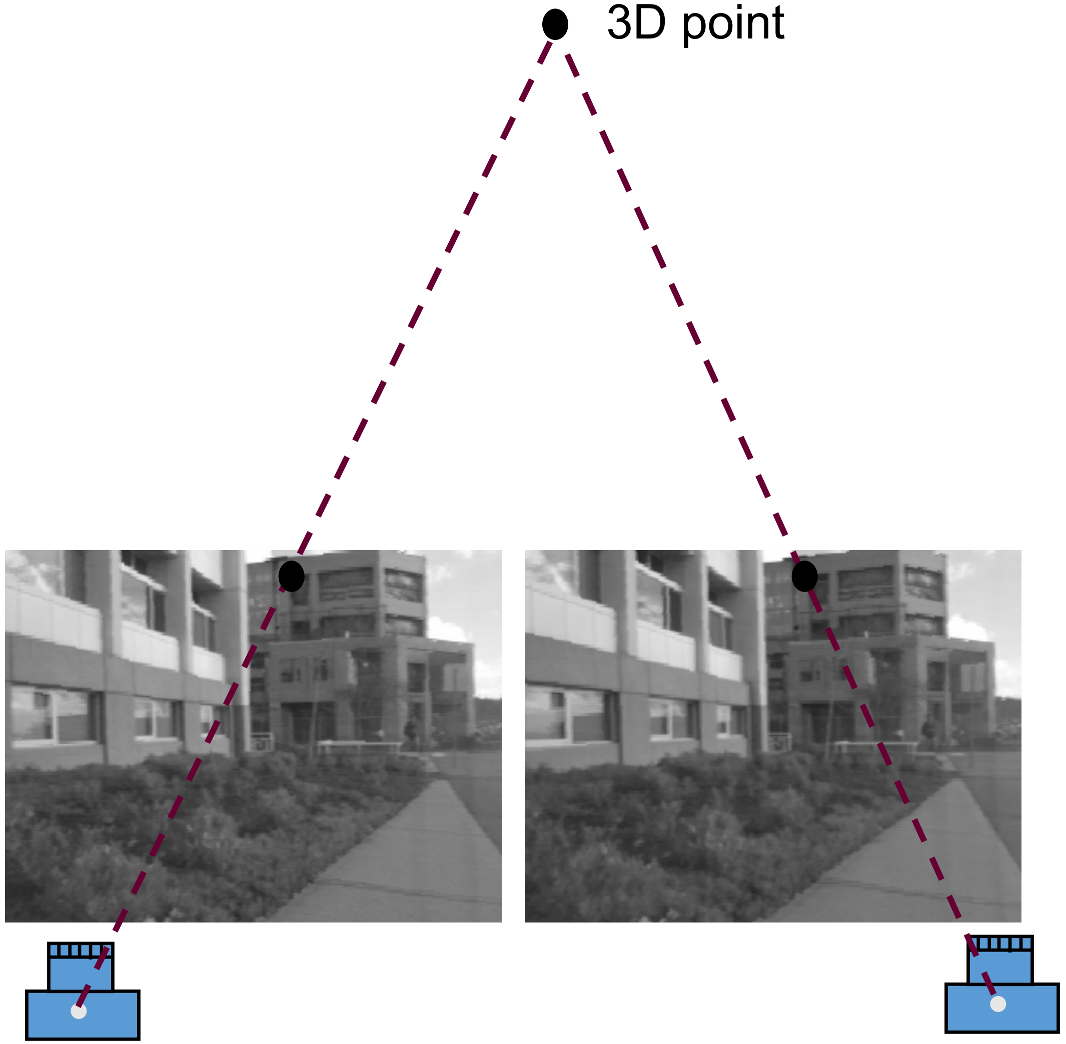
- Many slides adapted from F&P and Sing Bing Kang guest lecture

# Etymology

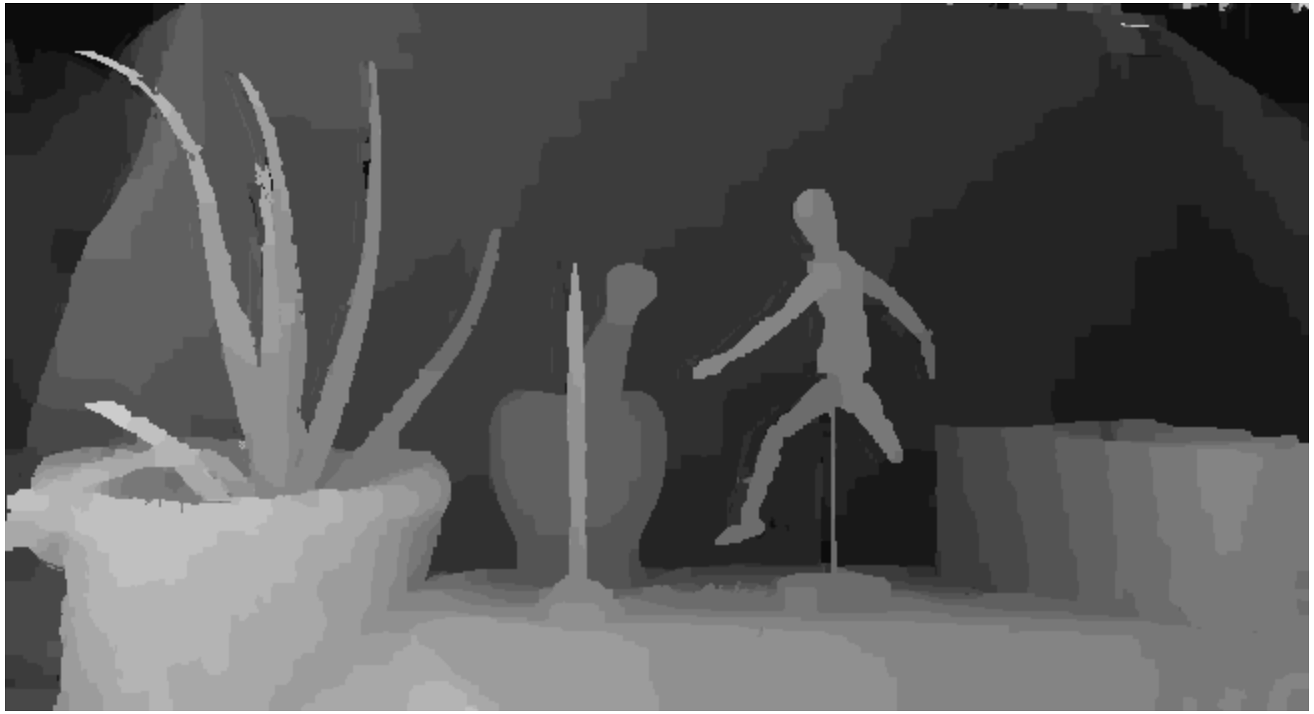
*Stereo* comes from the Greek word for *solid* (στερεο), and the term can be applied to any system using more than one channel

# Effect of Moving Camera

- As camera is shifted (viewpoint changed):
  - 3D points are projected to different 2D locations
  - Amount of shift in projected 2D location depends on depth
- 2D shifts= stereo disparity



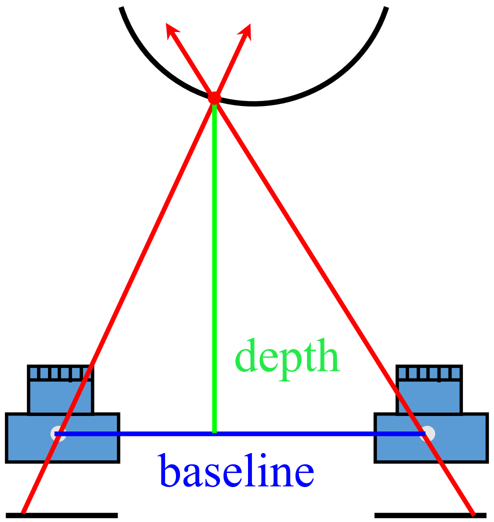
# Example



Right Image
Left Image
Disparity

# View Interpolation

# Basic Idea of Stereo



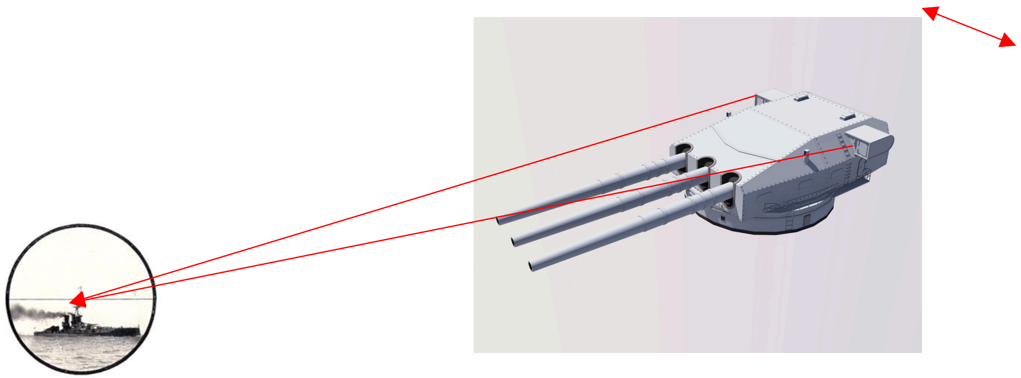

*Triangulate on two images of the same point to recover depth.*

- Feature matching across views
- Calibrated cameras


Matching correlation windows across scan lines

# Why is Stereo Useful?

- Passive and non-invasive
- Robot navigation (path planning, obstacle detection)
- 3D modeling (shape analysis, reverse engineering, visualization)
- Photorealistic rendering

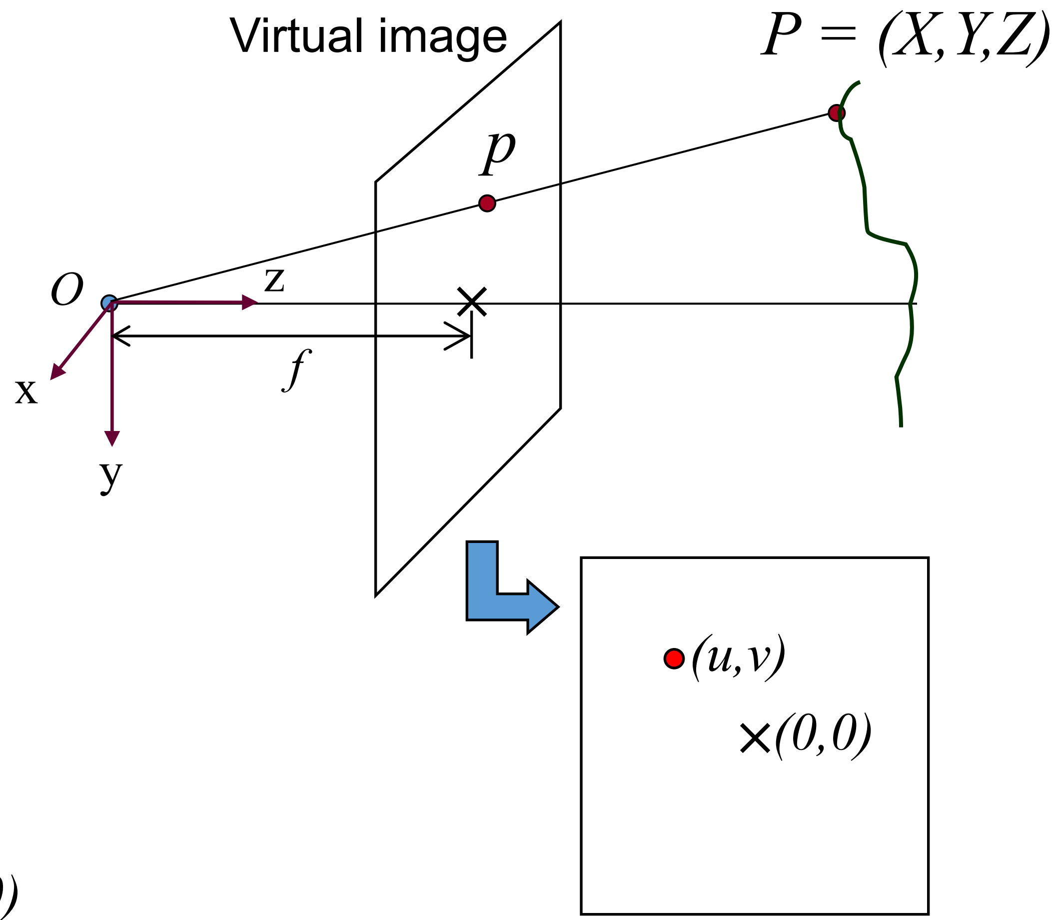


# 5. Stereo Geometry

- Recall: Pinhole model
- Now we have two !
- How to recover depth from two measurements?

# Review: Pinhole Camera Model

3D scene point $P$ is projected to a 2D point $Q$ in the virtual image plane

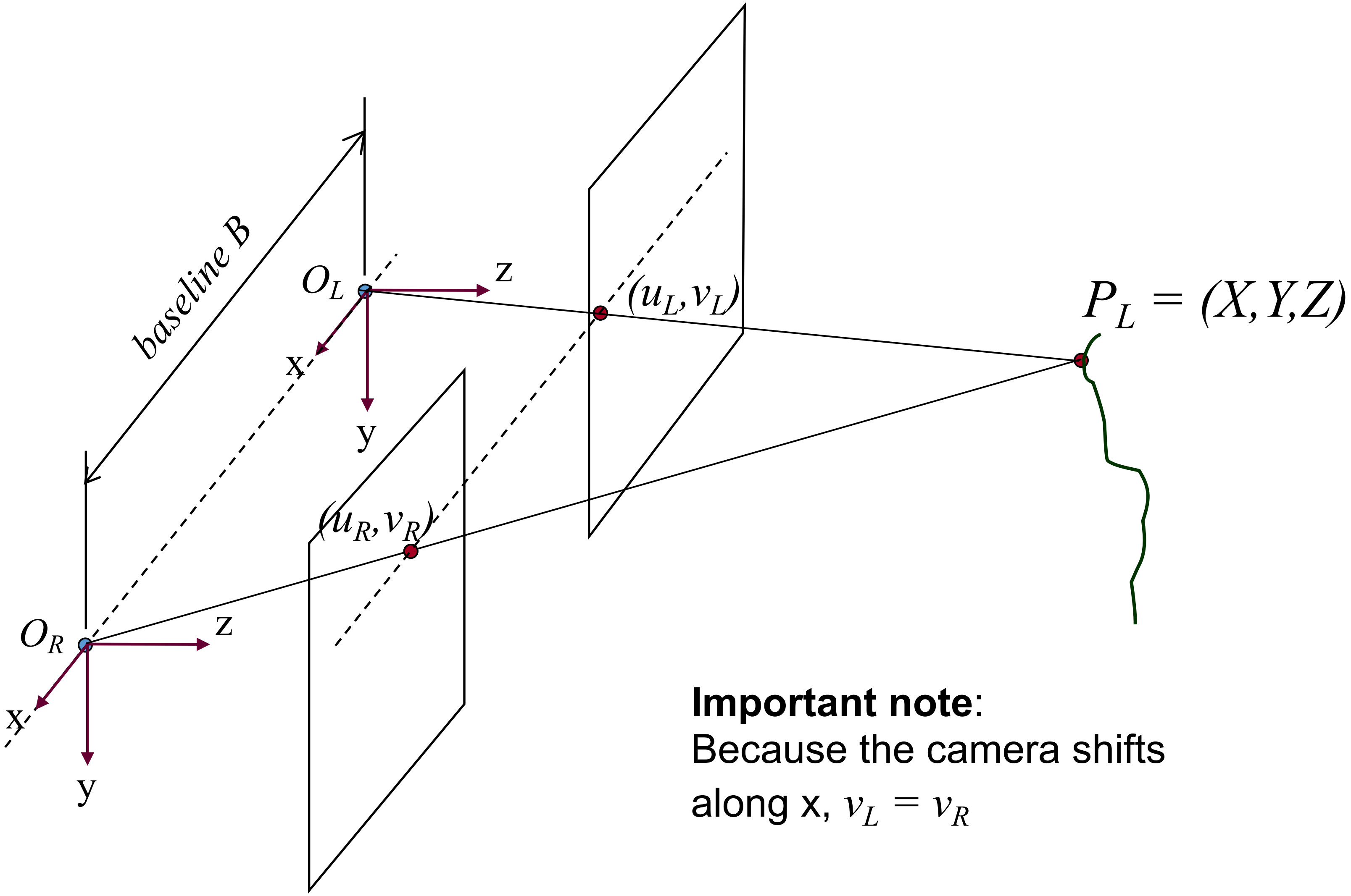



The 2D coordinates in the image are given by

$$(u, v) = \left(f\frac{X}{Z}, f\frac{Y}{Z}\right)$$

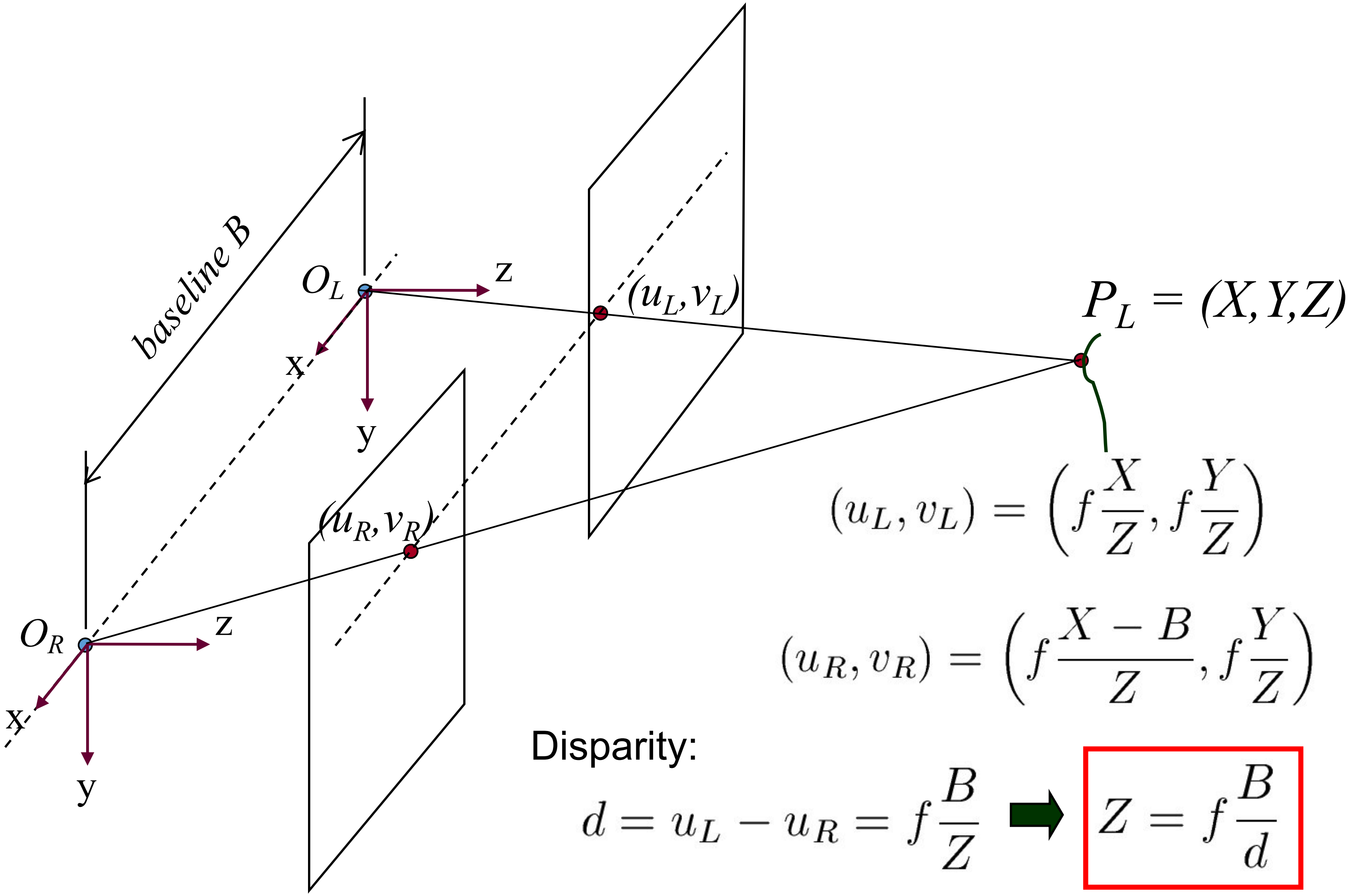
Note: image center is *(0,0)*

# Basic Stereo Derivations




**Important note**:
Because the camera shifts along x, $v_L = v_R$

# Basic Stereo Formula



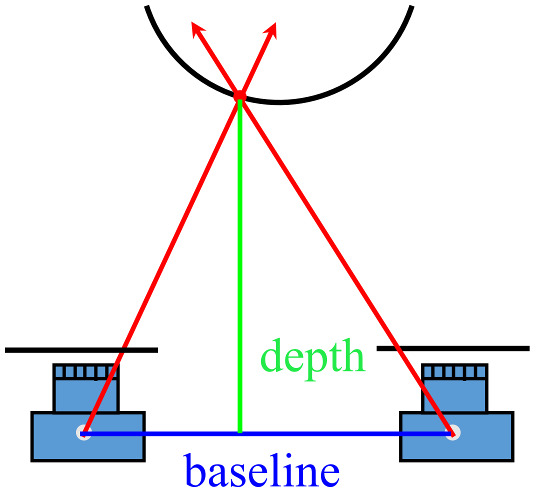

$$(u_L, v_L) = \left(f\frac{X}{Z}, f\frac{Y}{Z}\right)$$

$$(u_R, v_R) = \left(f\frac{X-B}{Z}, f\frac{Y}{Z}\right)$$

Disparity:

$$d = u_L - u_R = f\frac{B}{Z} \Rightarrow Z = f\frac{B}{d}$$

# 6. Stereo Algorithm




$$Z(x,y) = \frac{f\,B}{d(x,y)}$$

$Z(x, y)$ is depth at pixel $(x, y)$
$d(x, y)$ is disparity

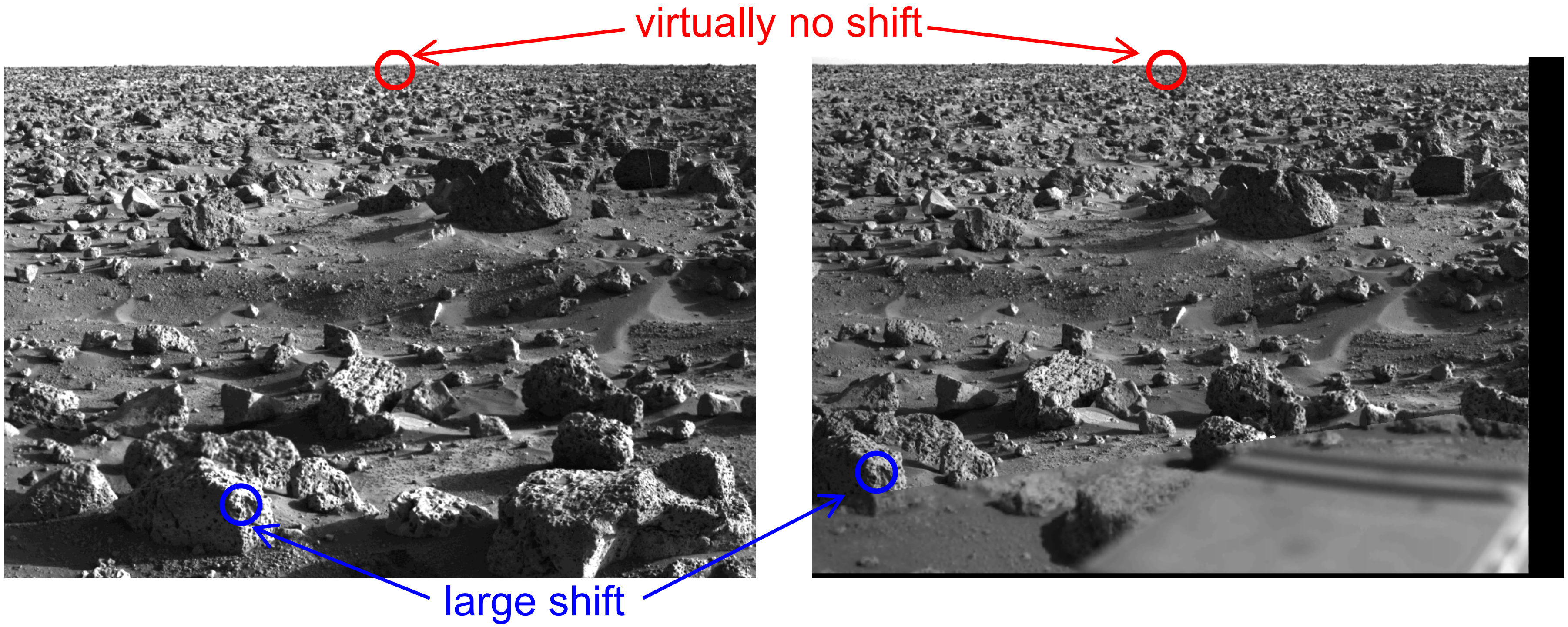

Matching correlation windows across scan lines

# Components of Stereo Algorithms

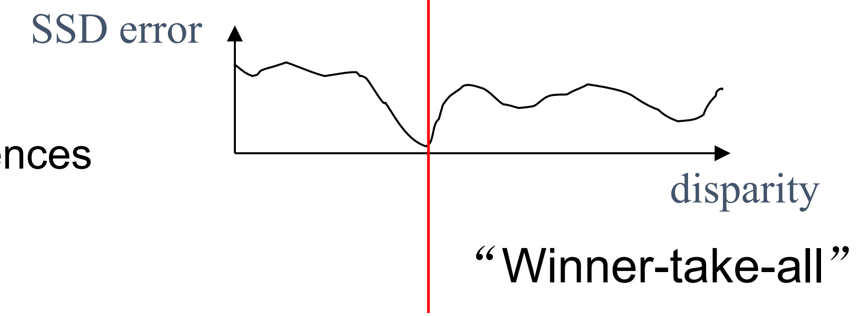
- Matching criterion (error function)
  - Quantify similarity of pixels
  - Most common: direct intensity difference
- Aggregation method
  - How error function is accumulated
  - Options: Pixel, edge, window, or segmented regions
- Optimization and winner selection
  - Examples: Winner-take-all, dynamic programming, graph cuts, belief propagation

# Stereo Correspondence

- Search over disparity to find correspondences
- Range of disparities can be large



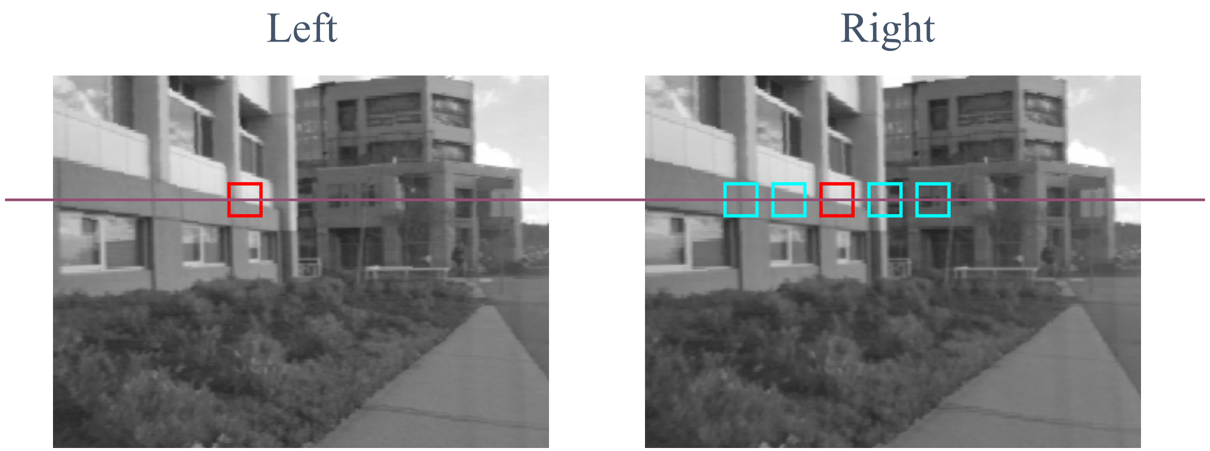
# Correspondence Using Window-based Correlation


Matching criterion = Sum-of-squared differences

Aggregation method = Fixed window size

# Sum of Squared (Intensity) Differences




$w_L$ and $w_R$ are corresponding $m$ by $m$ windows of pixels.

We define the window function:

$$W_m(x,y) = \{u,v \mid x - \tfrac{m}{2} \le u \le x + \tfrac{m}{2}, y - \tfrac{m}{2} \le v \le y + \tfrac{m}{2}\}$$

The SSD cost measures the intensity difference as a function of disparity:

$$C_r(x,y,d) = \sum_{(u,v)\in W_m(x,y)} [I_L(u,v) - I_R(u-d,v)]^2$$

# Correspondence Using Correlation

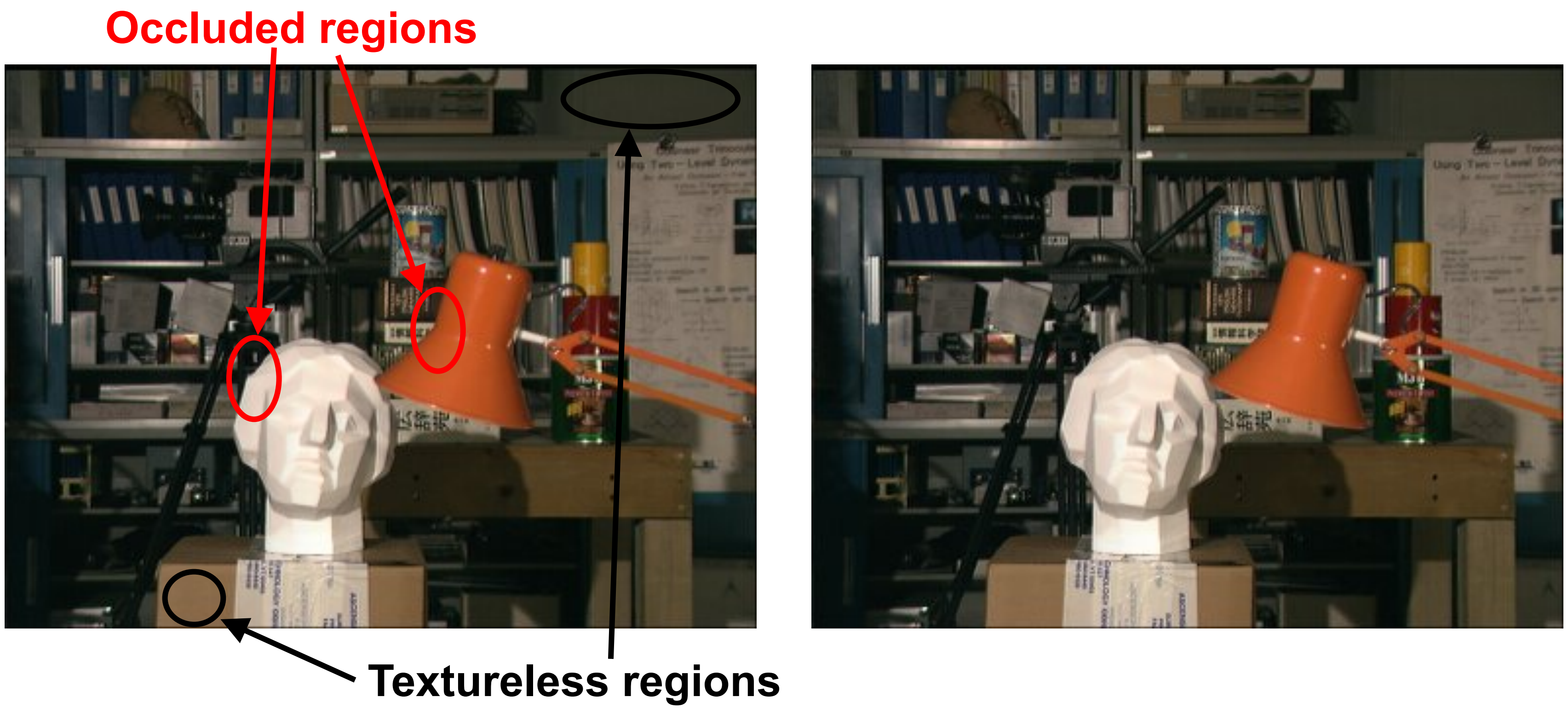

Left

Disparity Map

Images courtesy of Point Grey Research

# Two major roadblocks

- Textureless regions create ambiguities
- Occlusions result in missing data

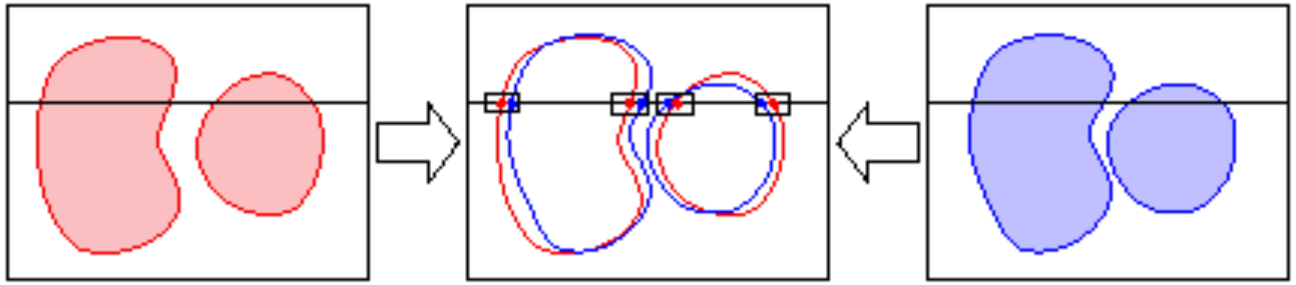


# Dealing with ambiguities and occlusion

- Ordering constraint:
  - Impose same matching order along scanlines
- Uniqueness constraint:
  - Each pixel in one image maps to unique pixel in other
- Can encode these constraints easily in dynamic programming

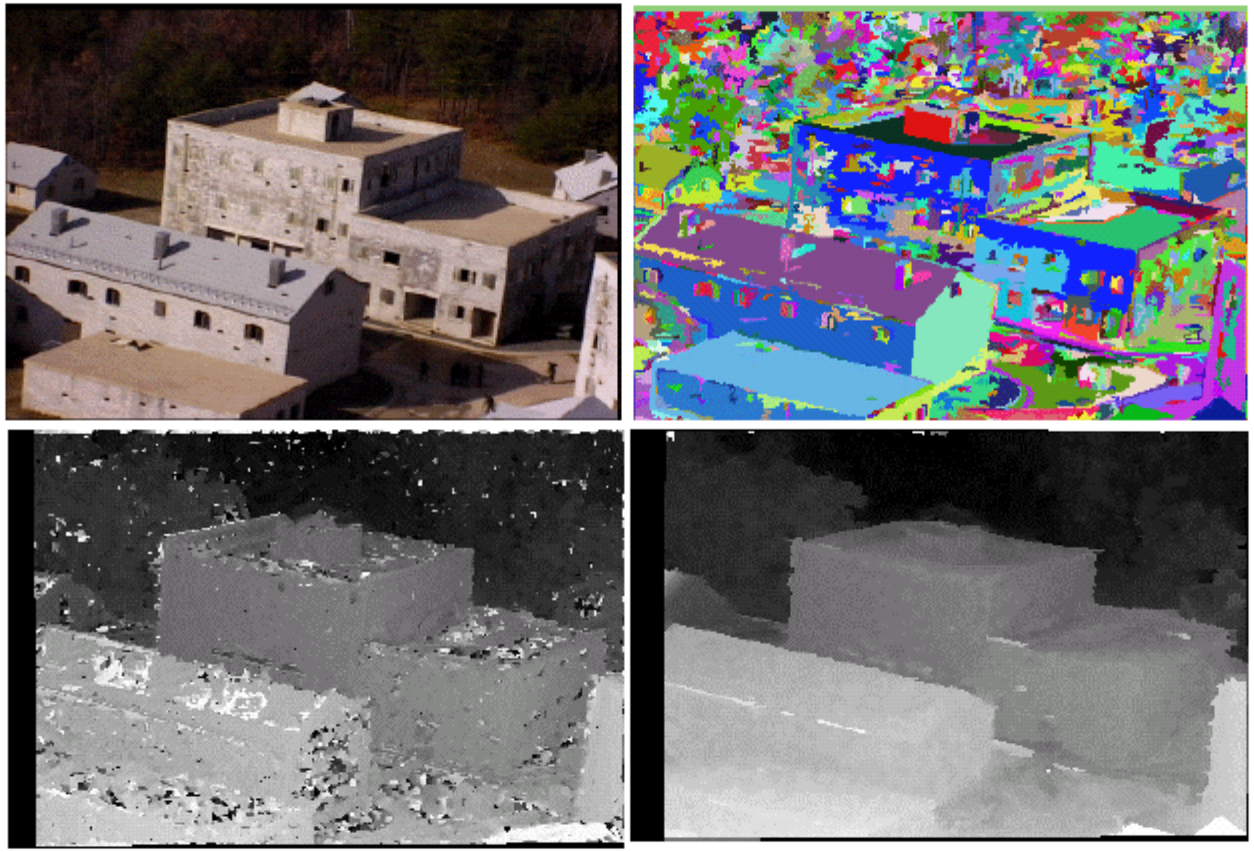
# Edge-based Stereo

- Another approach is to match *edges* rather than windows of pixels:



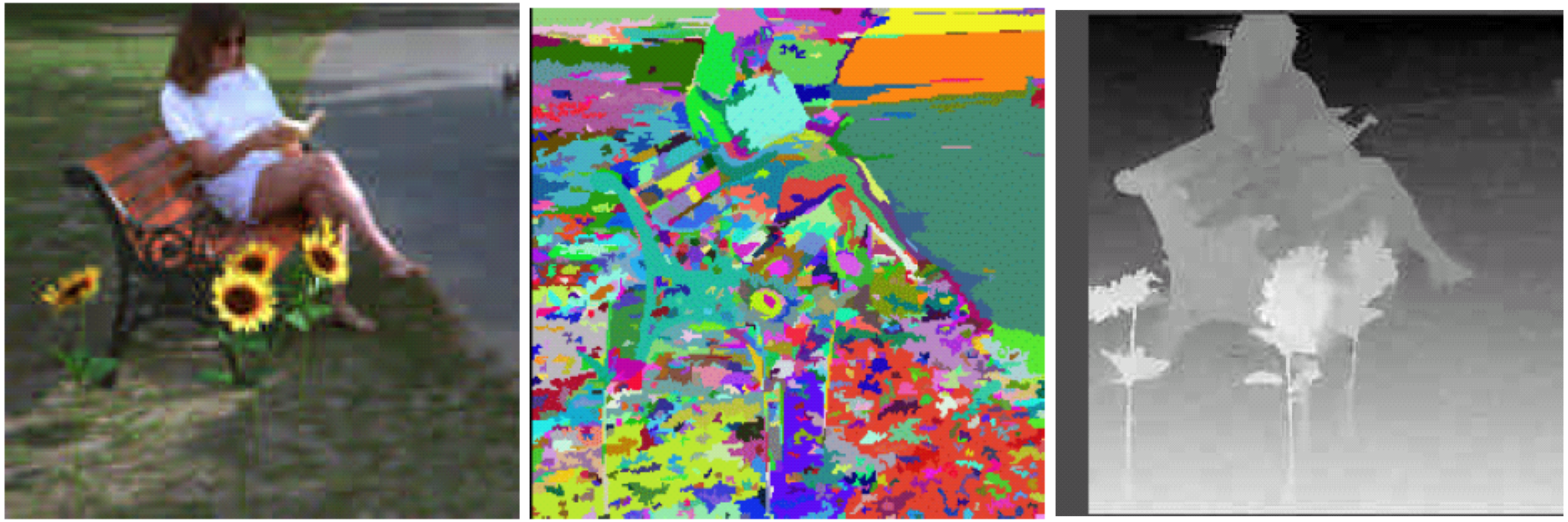
- Which method is better?
  - Edges tend to fail in dense texture (outdoors)
  - Correlation tends to fail in smooth featureless areas
  - Sparse correspondences

# Segmentation-based Stereo



**Hai Tao and Harpreet W. Sawhney**

# Another Example



# Stereo is Still Unresolved

- Depth discontinuities
- Lack of texture (depth ambiguity)
- Non-rigid effects (highlights, reflection, translucency)

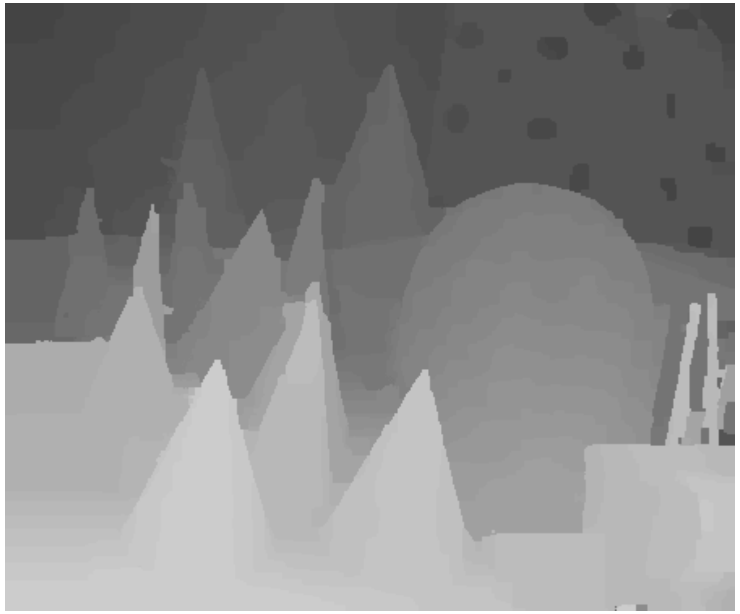
# Hallmarks of
# A Good Stereo Technique

- Should account for occlusions
- Should account for depth discontinuity
- Should have reasonable shape priors to handle textureless regions (e.g., planar or smooth surfaces)

- Advanced: account for non-Lambertian surfaces


Left


Right



Disparity Map

Result of using a more sophisticated stereo algorithm

# View Interpolation

# Summary

1. Perspective Cameras Intro
2. Pinhole Camera Model defined
3. Properties of Projective Geometry
4. Stereo Vision can recover metric structure
5. Stereo Geometry is simply $Z = f\ B/d$
6. Amazing Stereo Algorithms are still elusive