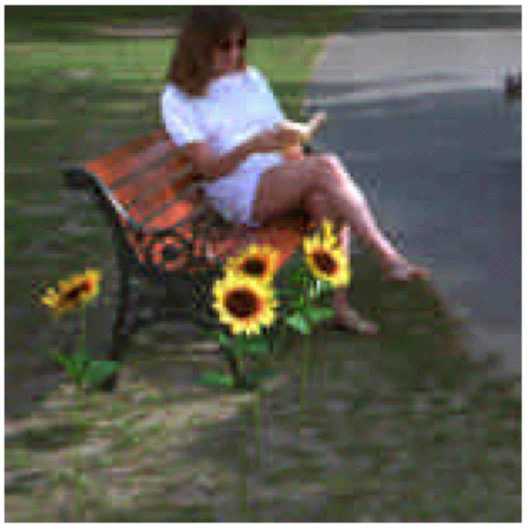
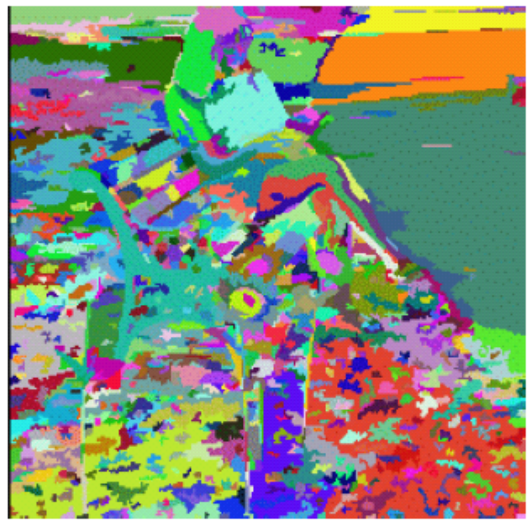
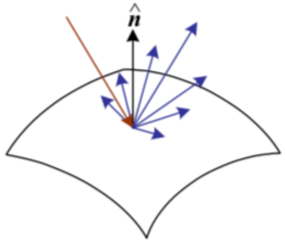
# Dense Stereo


Some Slides by Forsyth & Ponce, Jim Rehg, Sing Bing Kang

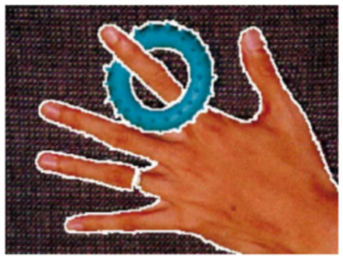
(Does not line up well with Szeliski book)



2. Image Formation

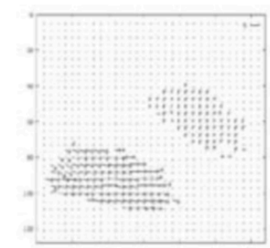
3. Image Processing

4. Features

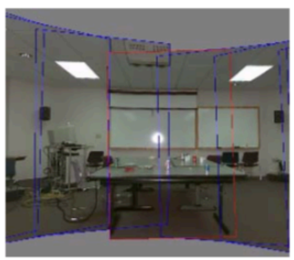


5. Segmentation

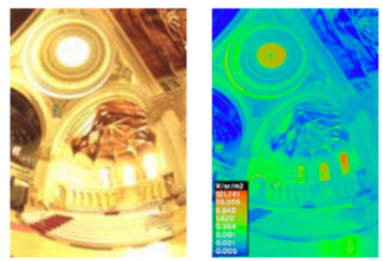
6-7. Structure from Motion



8. Motion



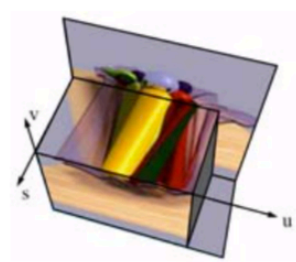
9. Stitching



10. Computational Photography

11. Stereo

12. 3D Shape




13. Image-based Rendering

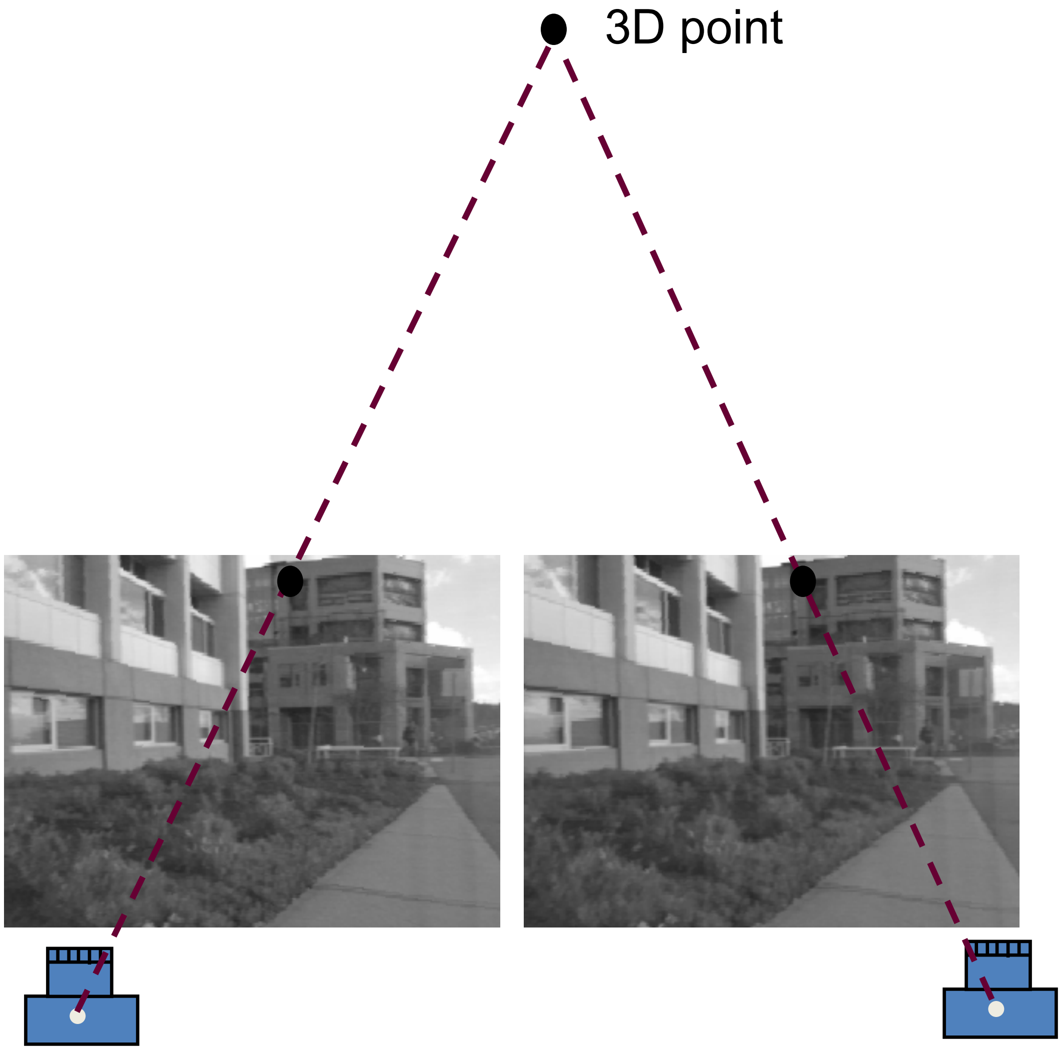
14. Recognition

# Etymology

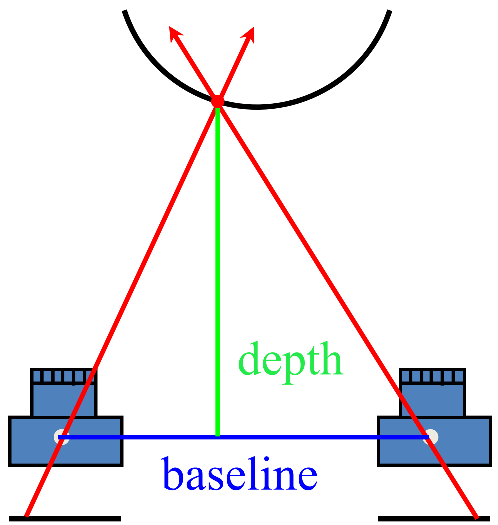
*Stereo* comes from the Greek word for *solid* (στερεο), and the term can be applied to any system using more than one channel

# Effect of Moving Camera

- As camera is shifted (viewpoint changed):
  - 3D points are projected to different 2D locations
  - Amount of shift in projected 2D location depends on depth
- 2D shifts=Parallax



# Basic Idea of Stereo



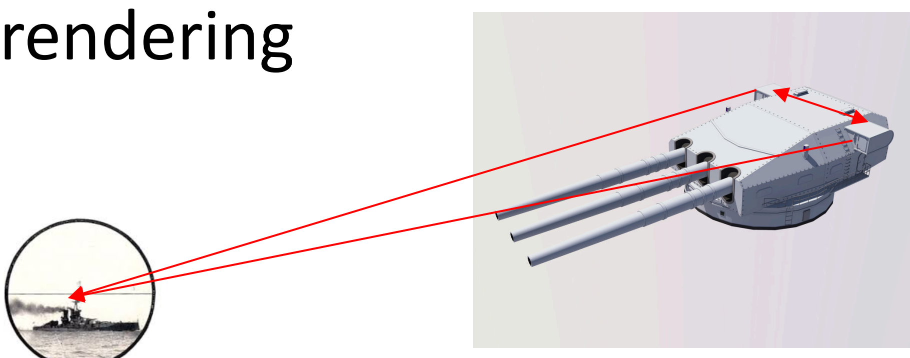

*Triangulate on two images of the same point to recover depth.*

- Feature matching across views
- Calibrated cameras


Matching correlation windows across scan lines

# Why is Stereo Useful?

- Passive and non-invasive
- Robot navigation (path planning, obstacle detection)
- 3D modeling (shap analysis, reverse engineering, visualization)
- Photorealistic rendering

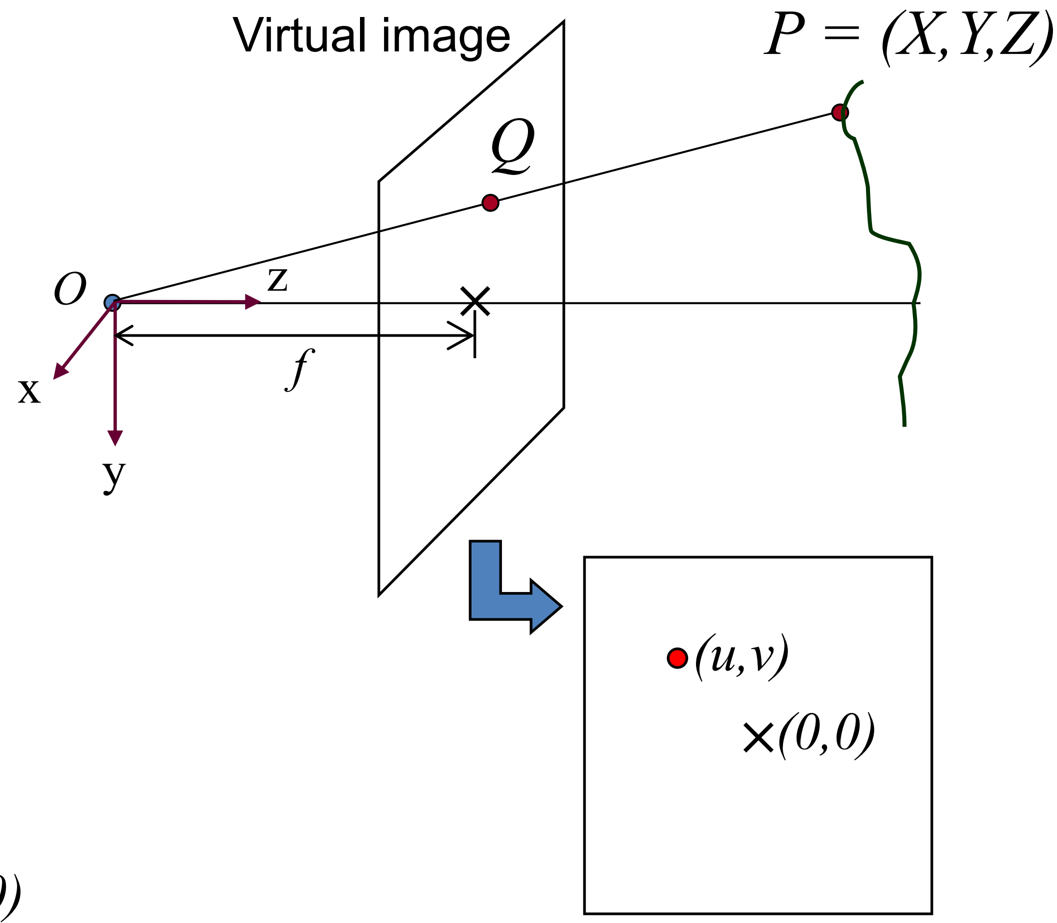


# Outline

- Pinhole camera model
- Basic (2-view) stereo algorithm
  - Equations
  - Window-based matching (SSD)
  - Dynamic programming
- Multiple view stereo

# Review: Pinhole Camera Model

3D scene point $P$ is projected to a 2D point $Q$ in the virtual image plane

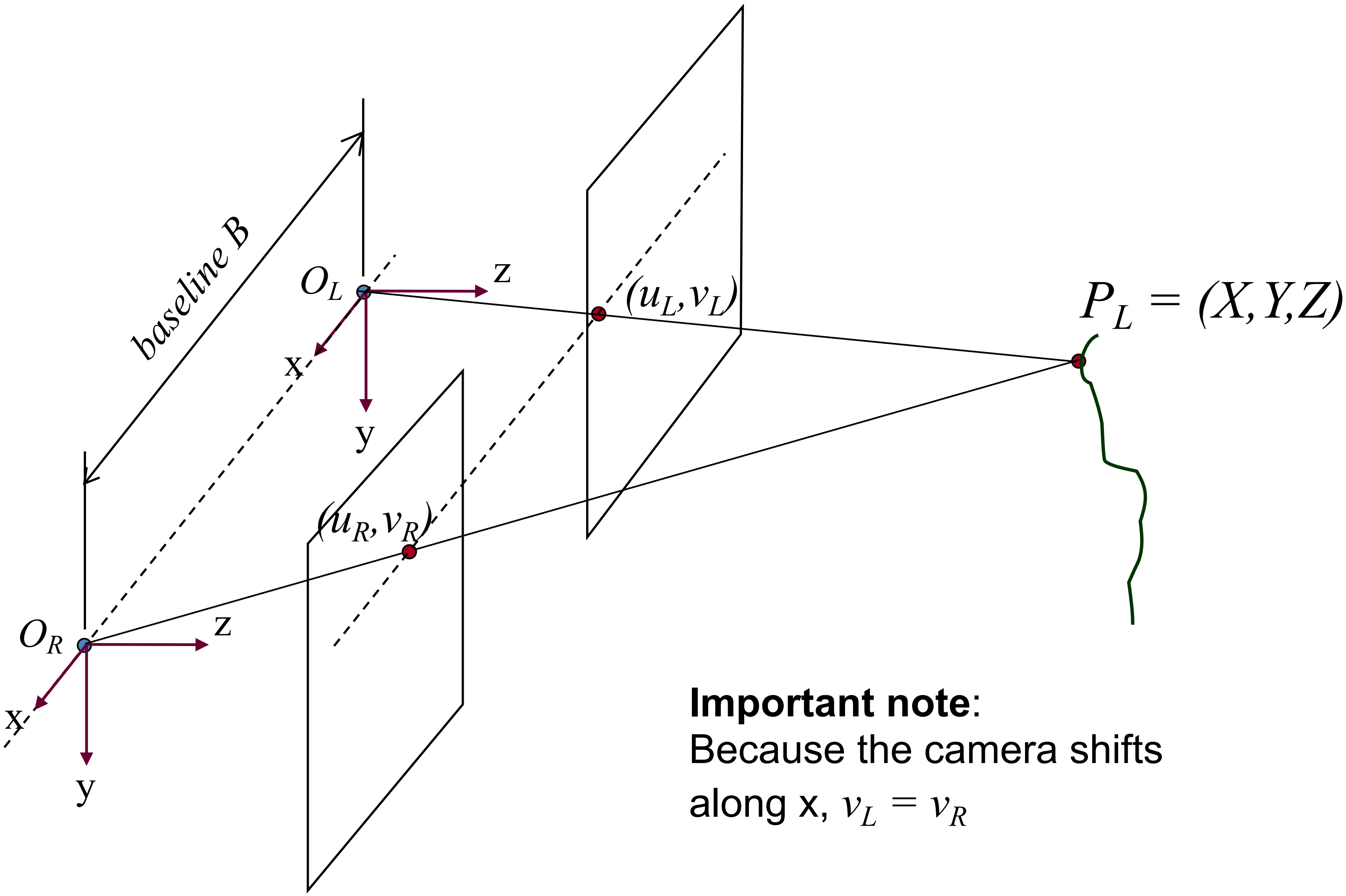



The 2D coordinates in the image are given by

$$(u,v) = \left(f\frac{X}{Z}, f\frac{Y}{Z}\right)$$

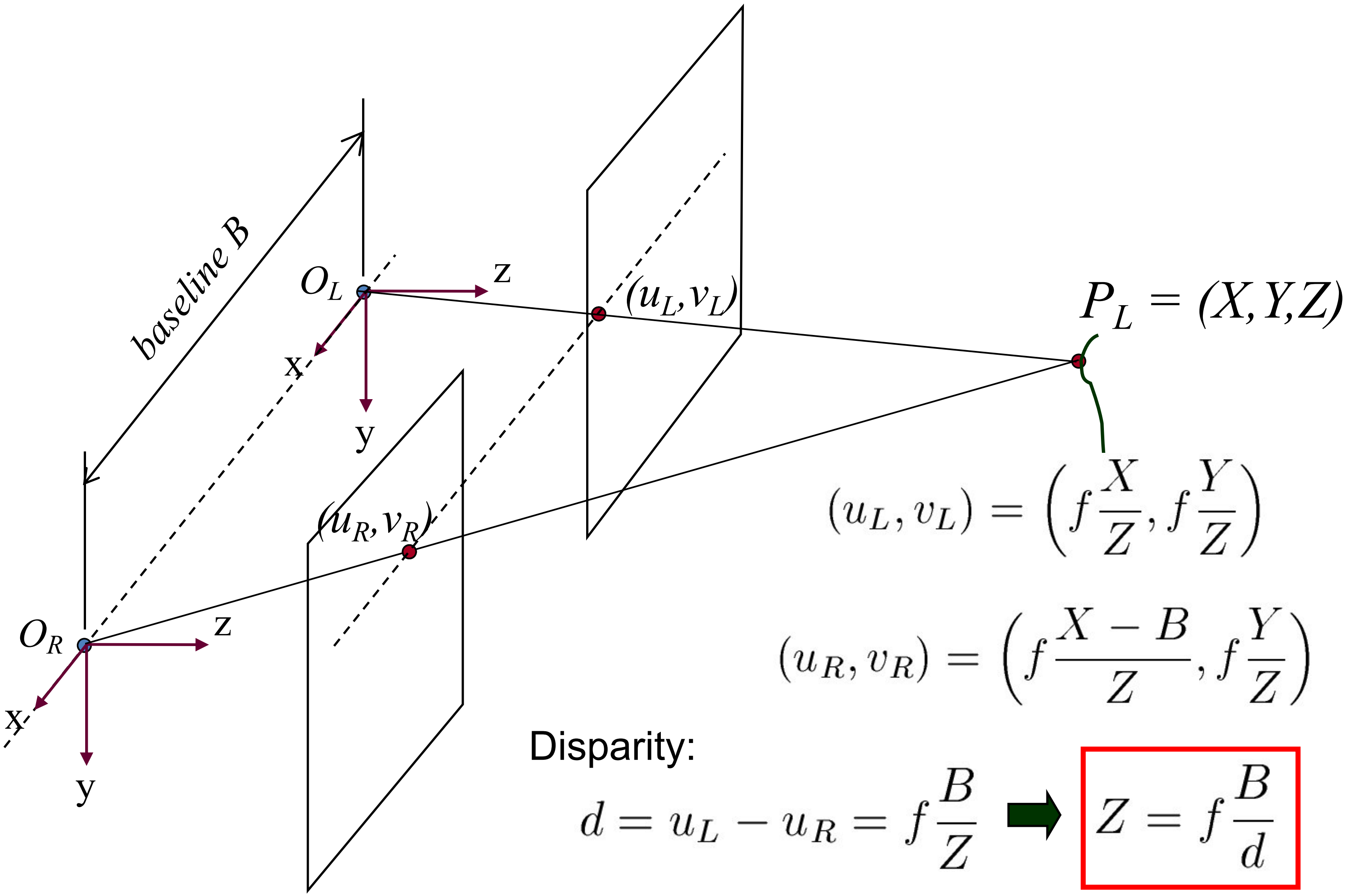
Note: image center is *(0,0)*

# Basic Stereo Derivations




**Important note**:
Because the camera shifts along x, $v_L = v_R$

# Basic Stereo Derivations



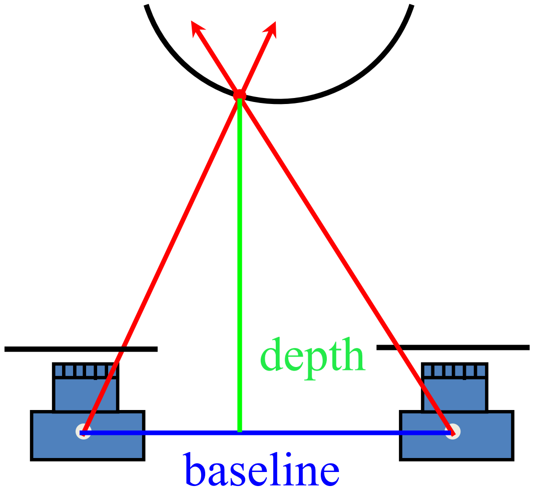

$$(u_L, v_L) = \left(f\frac{X}{Z}, f\frac{Y}{Z}\right)$$

$$(u_R, v_R) = \left(f\frac{X-B}{Z}, f\frac{Y}{Z}\right)$$

Disparity:

$$d = u_L - u_R = f\frac{B}{Z} \Rightarrow \boxed{Z = f\frac{B}{d}}$$

# Stereo Vision




$$Z(x,y) = \frac{f\,B}{d(x,y)}$$

$Z(x, y)$ is depth at pixel $(x, y)$
$d(x, y)$ is disparity

Left

Right

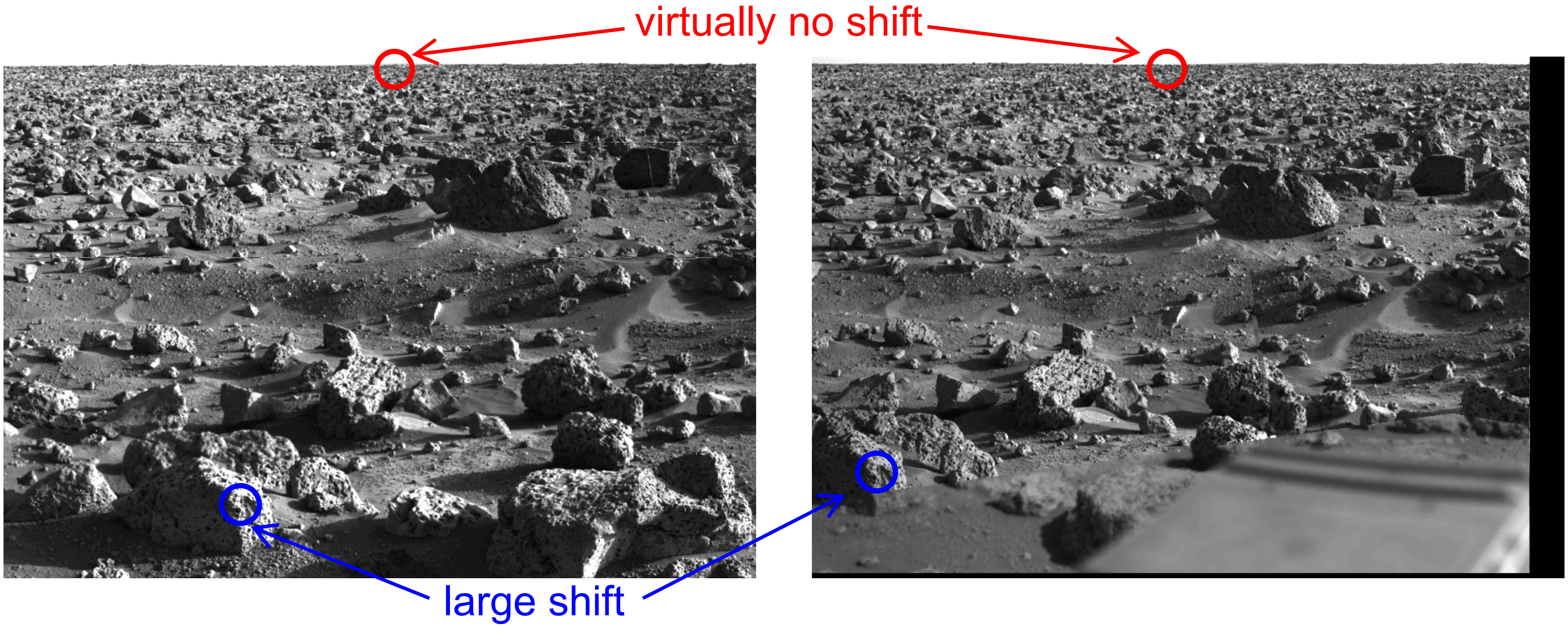
Matching correlation windows across scan lines

# Components of Stereo

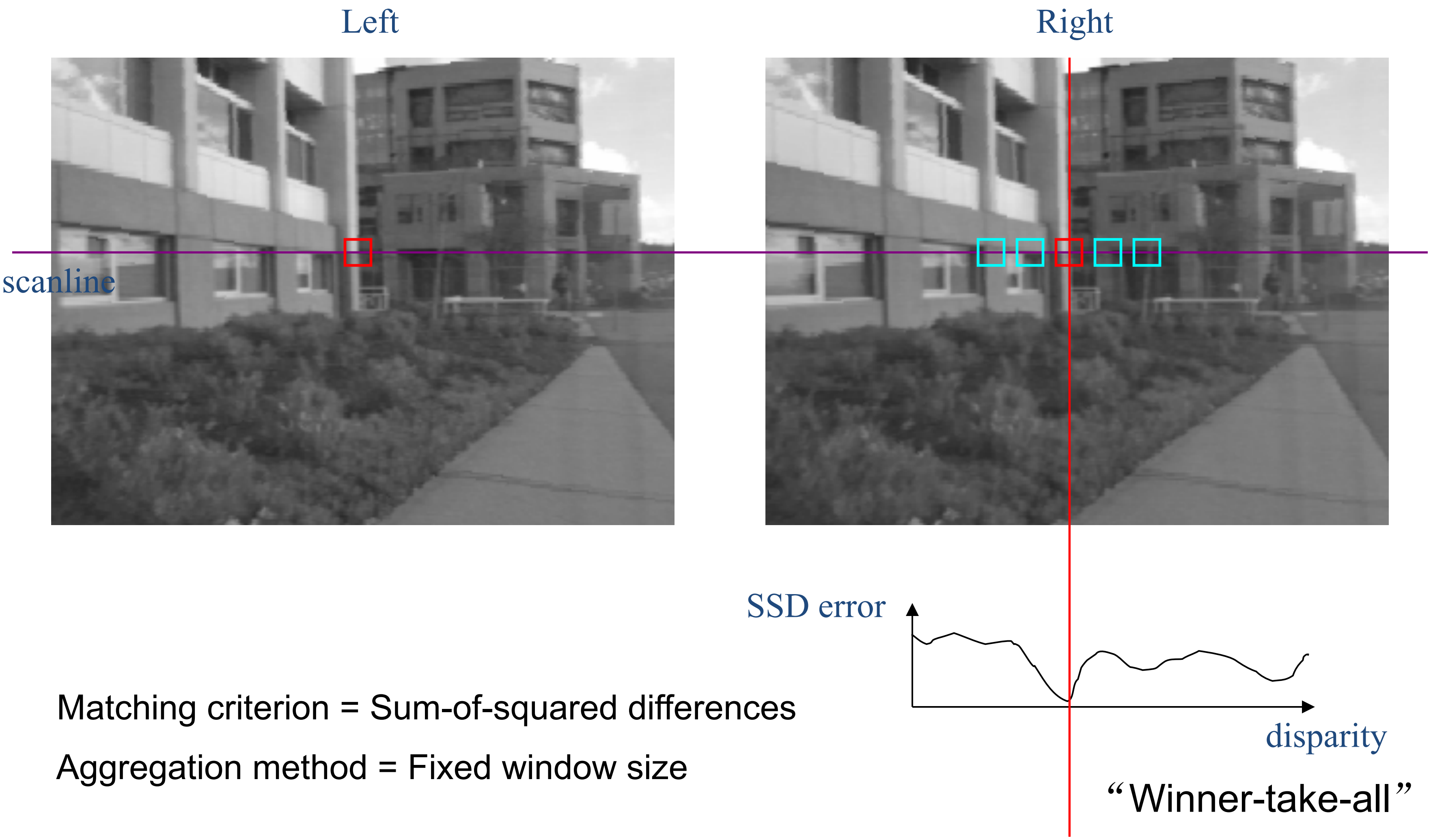
- Matching criterion (error function)
  - Quantify similarity of pixels
  - Most common: direct intensity difference
- Aggregation method
  - How error function is accumulated
  - Options: Pixel, edge, window, or segmented regions
- Optimization and winner selection
  - Examples: Winner-take-all, dynamic programming, graph cuts, belief propagation

# Stereo Correspondence

- Search over disparity to find correspondences
- Range of disparities can be large



# Correspondence Using Window-based Correlation

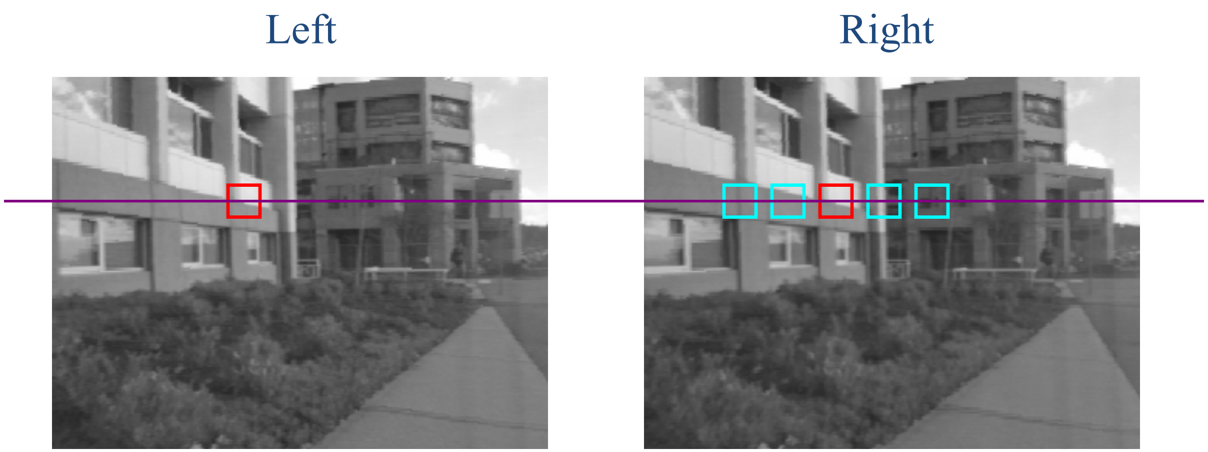



Matching criterion = Sum-of-squared differences

Aggregation method = Fixed window size

"Winner-take-all"

# Sum of Squared (Intensity) Differences




$w_L$ and $w_R$ are corresponding $m$ by $m$ windows of pixels.

We define the window function:

$$W_m(x,y) = \{u,v \mid x - \tfrac{m}{2} \le u \le x + \tfrac{m}{2}, y - \tfrac{m}{2} \le v \le y + \tfrac{m}{2}\}$$

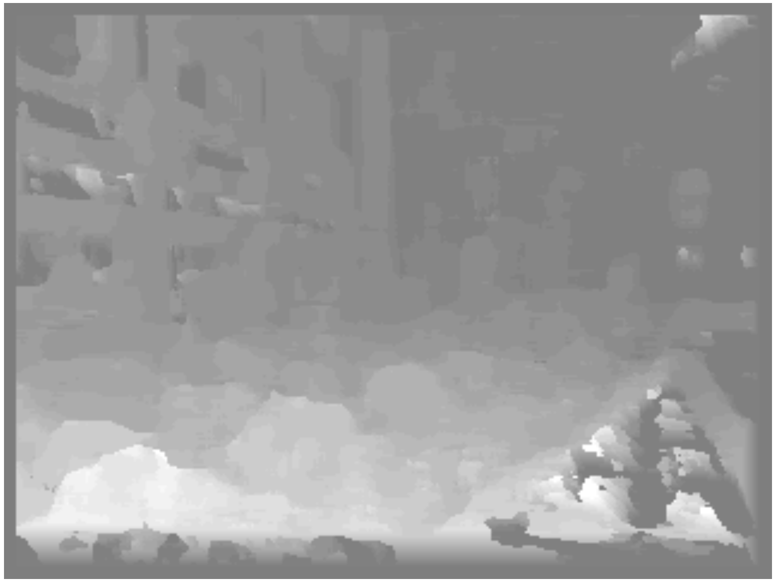
The SSD cost measures the intensity difference as a function of disparity:

$$C_r(x,y,d) = \sum_{(u,v)\in W_m(x,y)} [I_L(u,v) - I_R(u-d,v)]^2$$

# Correspondence Using Correlation


Left

Disparity Map



Images courtesy of Point Grey Research

# Image Normalization

- Images may be captured under different exposures (gain and aperture)
- Cameras may have different radiometric characteristics
- Surfaces may not be Lambertian
- Hence, it is reasonable to normalize pixel intensity in each window (to remove bias and scale):

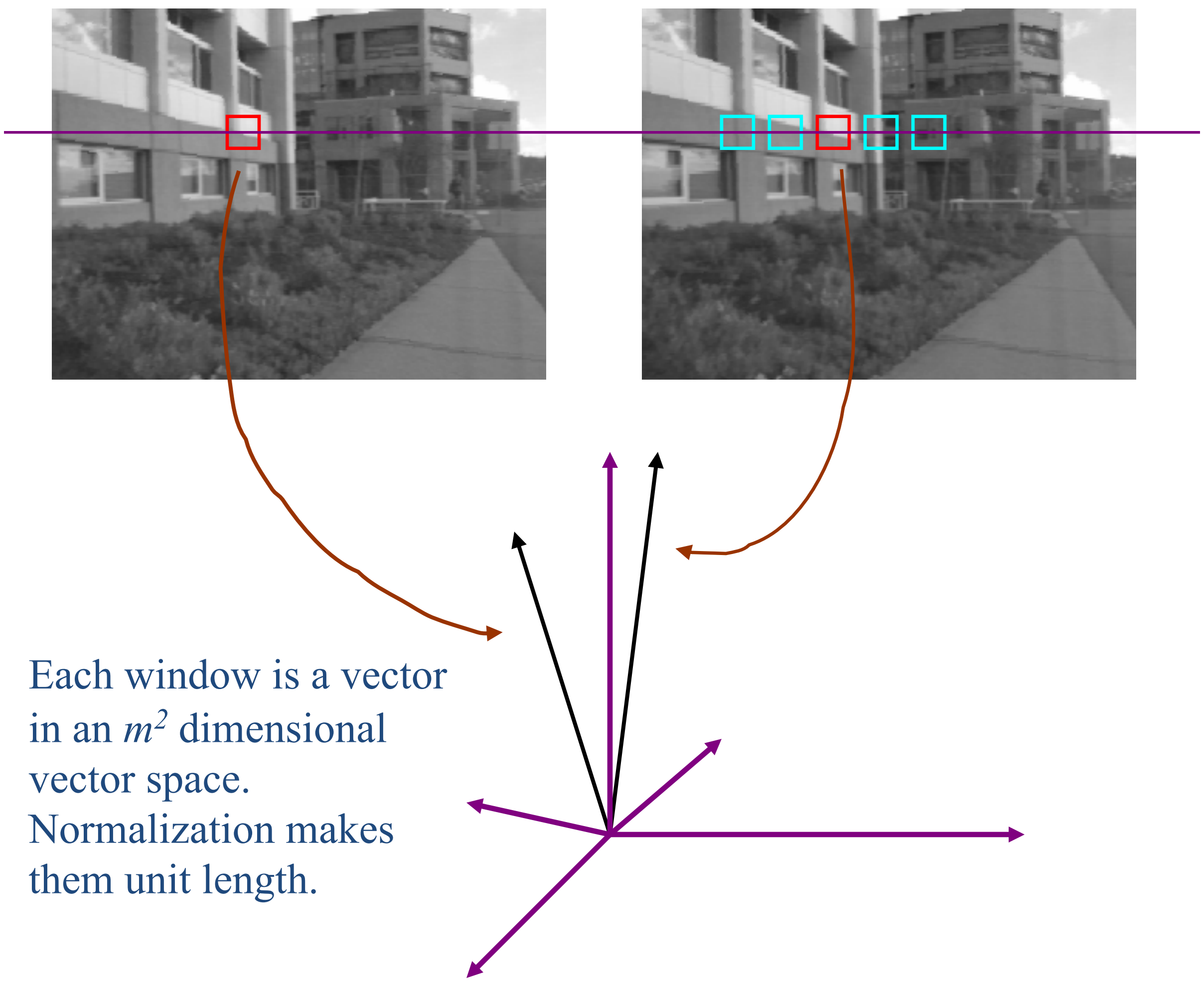
$$\bar{I} = \frac{1}{|W_m(x,y)|} \sum_{(u,v) \in W_m(x,y)} I(u,v)$$ Average pixel

$$\|I\|_{W_m(x,y)} = \sqrt{\sum_{(u,v) \in W_m(x,y)} [I(u,v)]^2}$$ Window magnitude

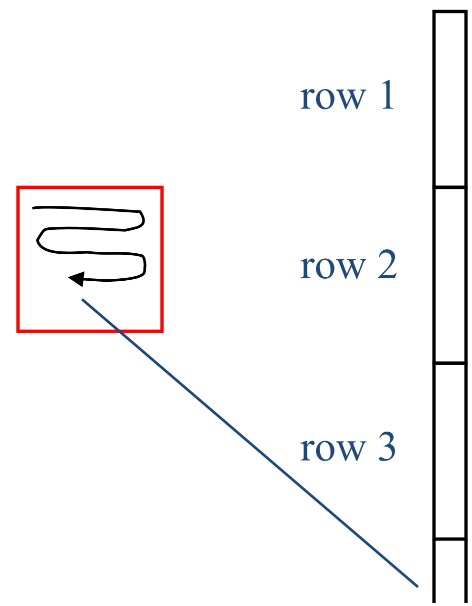
$$\hat{I}(x,y) = \frac{I(x,y) - \bar{I}}{\|I - \bar{I}\|_{W_m(x,y)}}$$ Normalized pixel

# Images as Vectors

Left Right

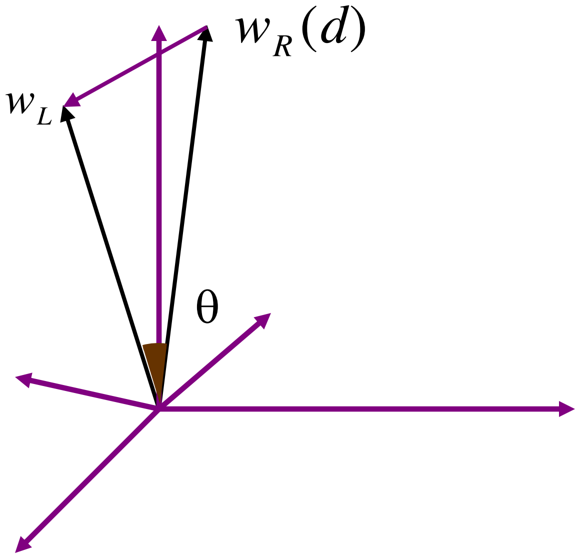


"Unwrap" image to form vector, using raster scan order




Each window is a vector in an $m^2$ dimensional vector space. Normalization makes them unit length.

# Image Metrics




(Normalized) Sum of Squared Differences

$$C_{\mathrm{SSD}}(d) = \sum_{(u,v)\in W_m(x,y)} [\hat{I}_L(u,v) - \hat{I}_R(u-d,v)]^2$$

$$= \left\| w_L - w_R(d) \right\|^2$$

Normalized Correlation

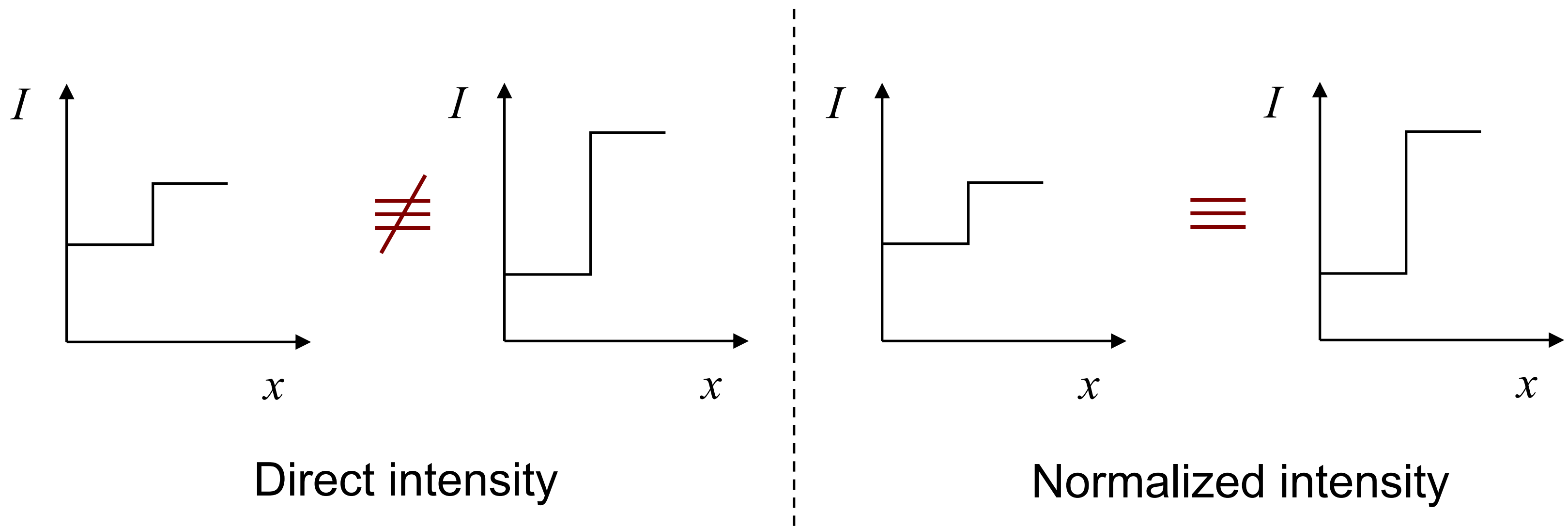
$$C_{\mathrm{NC}}(d) = \sum_{(u,v)\in W_m(x,y)} \hat{I}_L(u,v)\hat{I}_R(u-d,v)$$

$$= w_L \cdot w_R(d) = \cos\theta$$

$$d^* = \arg\min_d \left\| w_L - w_R(d) \right\|^2 = \arg\max_d w_L \cdot w_R(d)$$

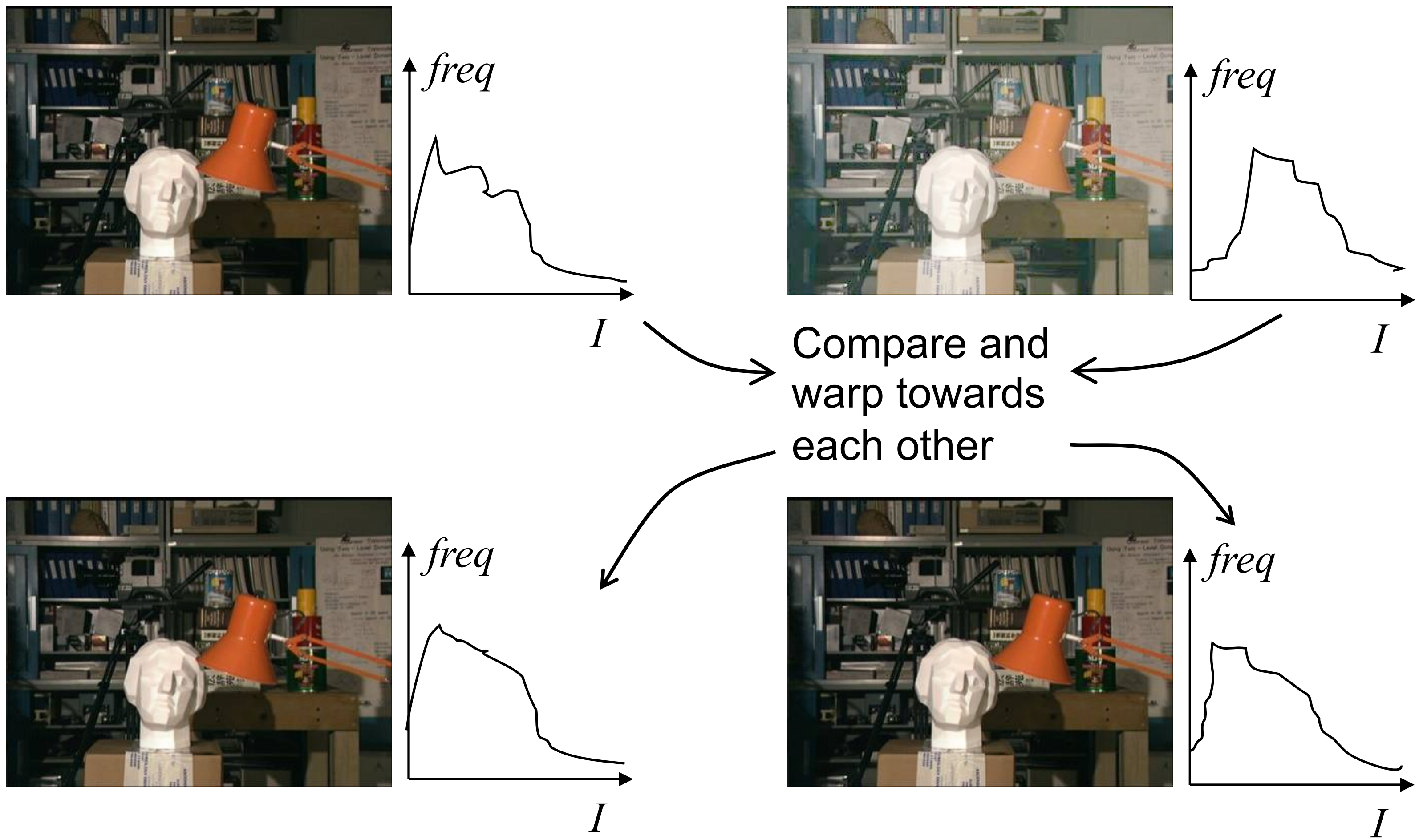
# Caveat

- Image normalization should be used *only* when deemed necessary
- The equivalence classes of things that look "similar" are substantially larger, leading to more matching ambiguities



# Alternative: Histogram Warping

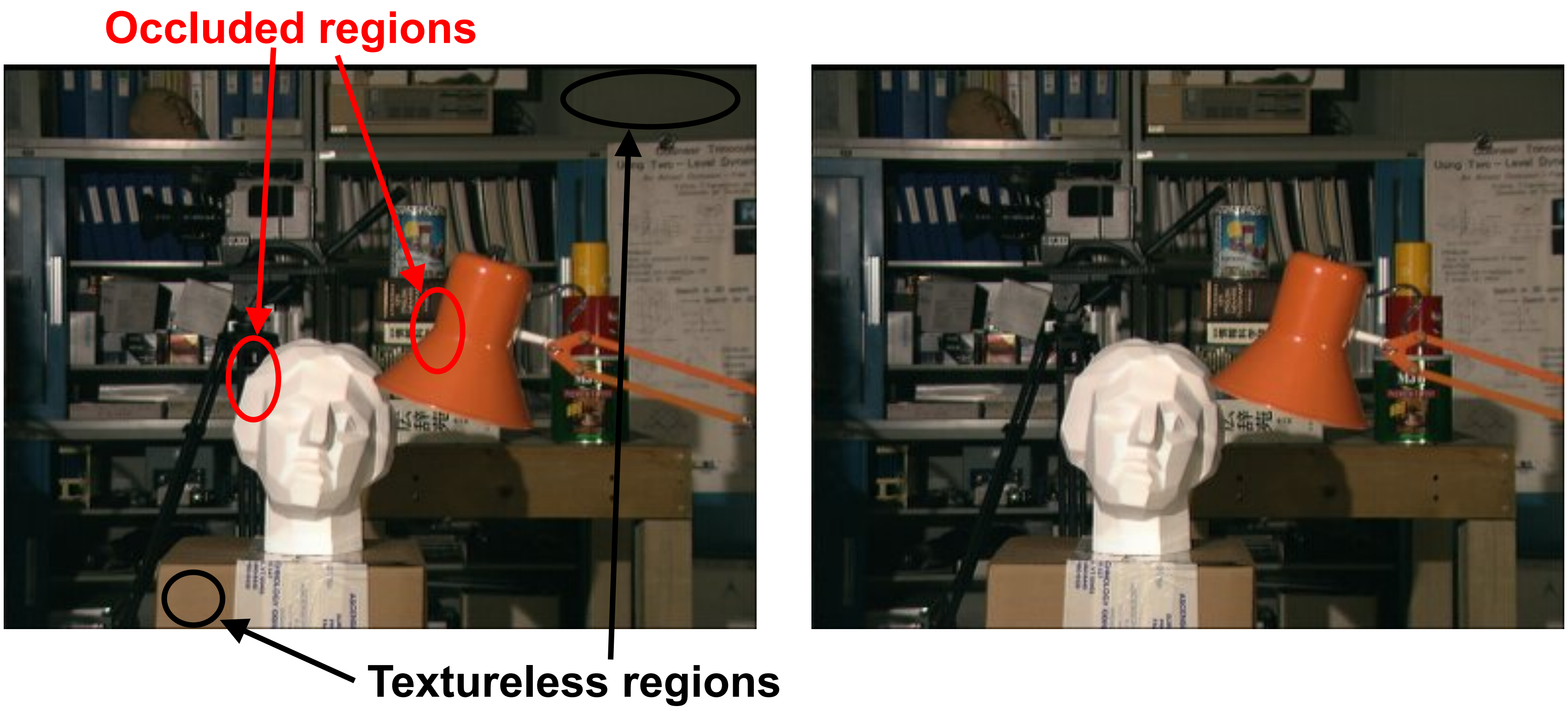
(Assumes significant visual overlap between images)




Cox, Roy, & Hingorani' 95: "Dynamic Histogram Warping"

# Two major roadblocks

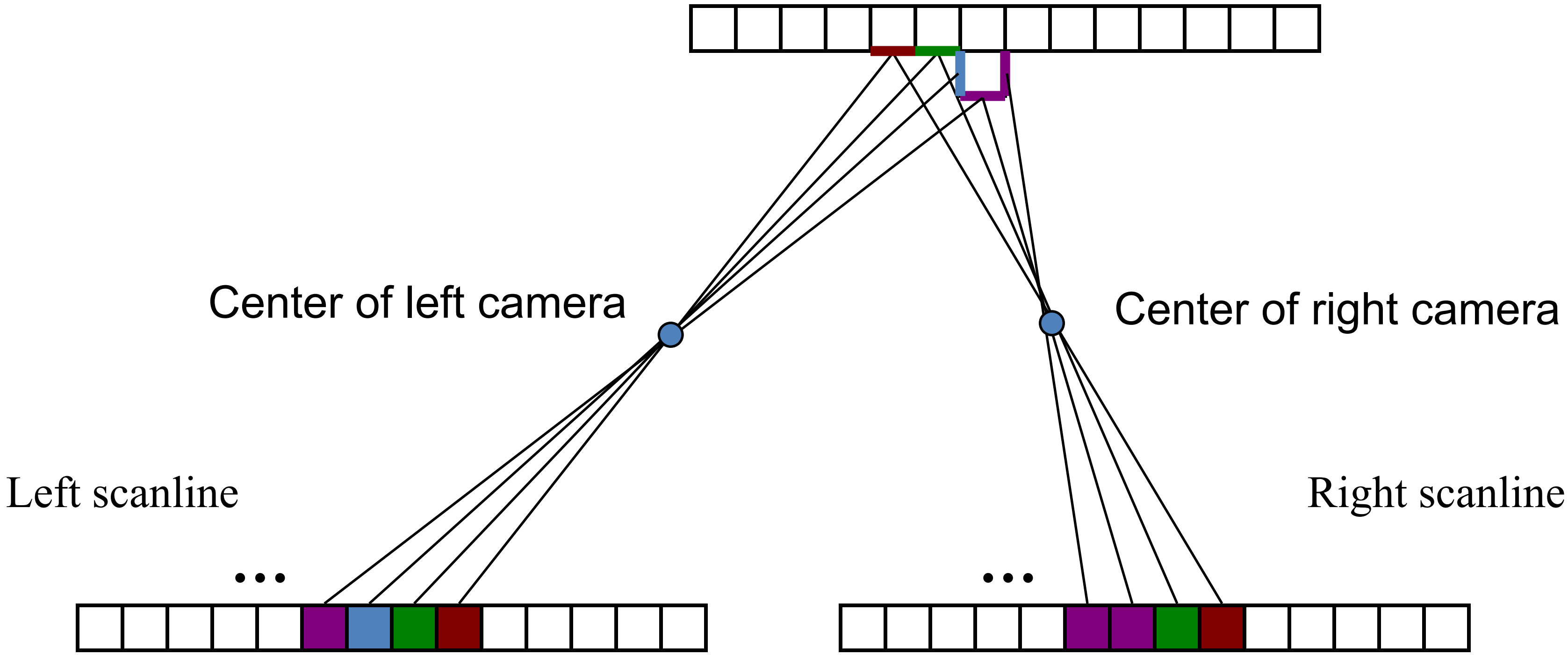
- Textureless regions create ambiguities
- Occlusions result in missing data



# Dealing with ambiguities and occlusion

- Ordering constraint:
  - Impose same matching order along scanlines
- Uniqueness constraint:
  - Each pixel in one image maps to unique pixel in other
- Can encode these constraints easily in dynamic programming

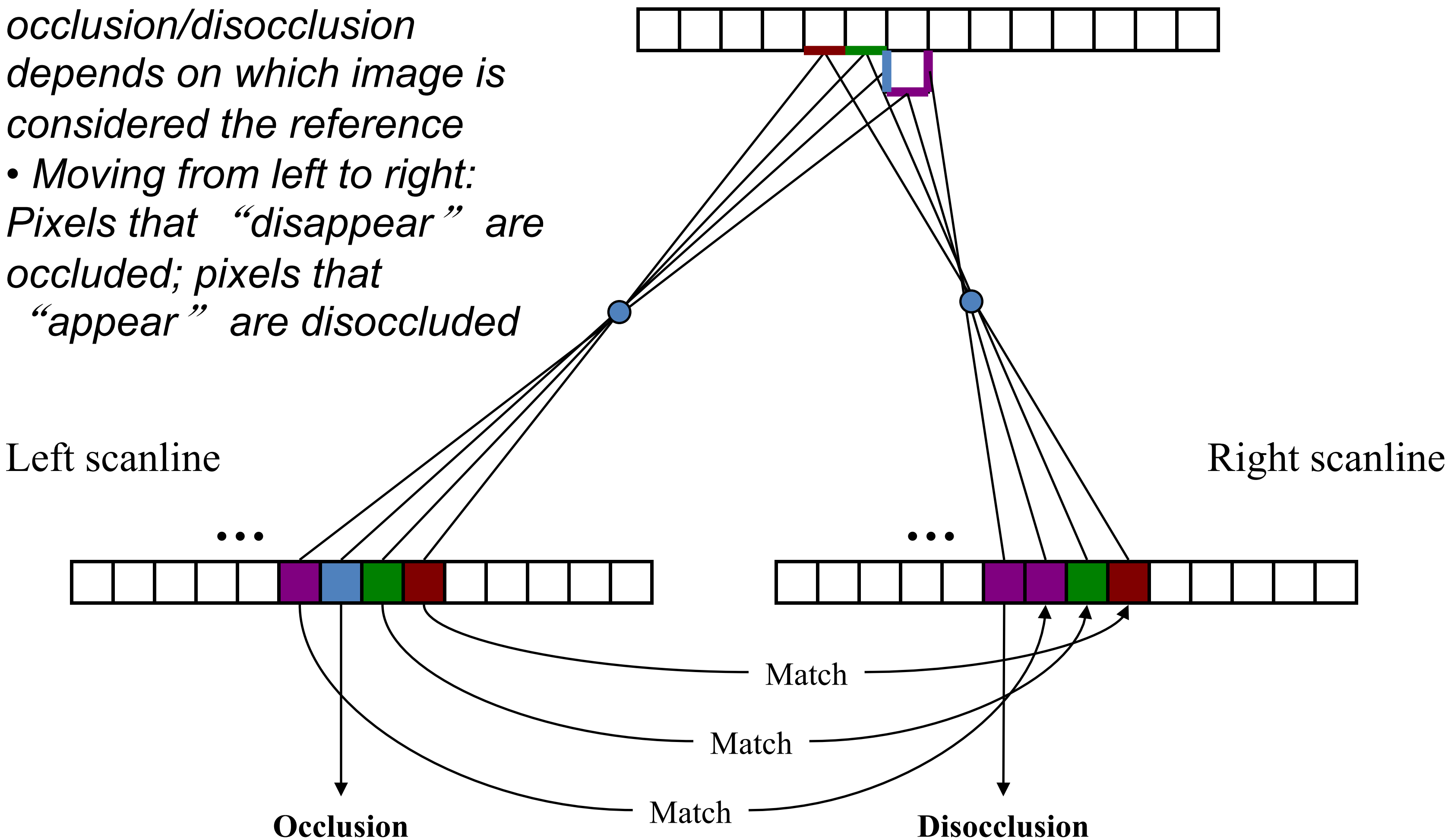
# Pixel-based Stereo



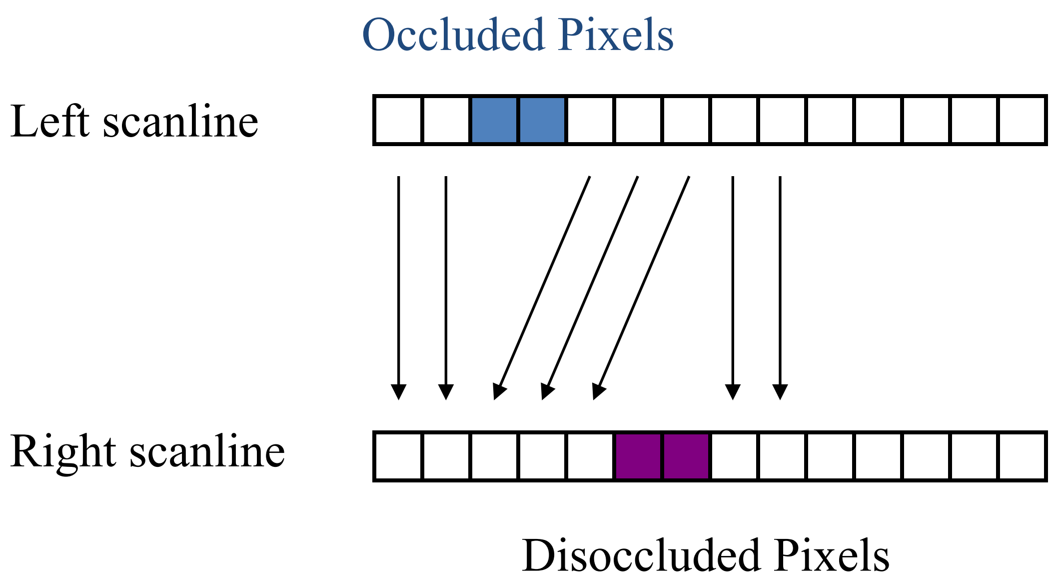

(NOTE: I’m using the actual, not virtual, image here.)

# Stereo Correspondences

- *Right image is reference*
- *Definition of occlusion/disocclusion depends on which image is considered the reference*
- *Moving from left to right: Pixels that "disappear" are occluded; pixels that "appear" are disoccluded*



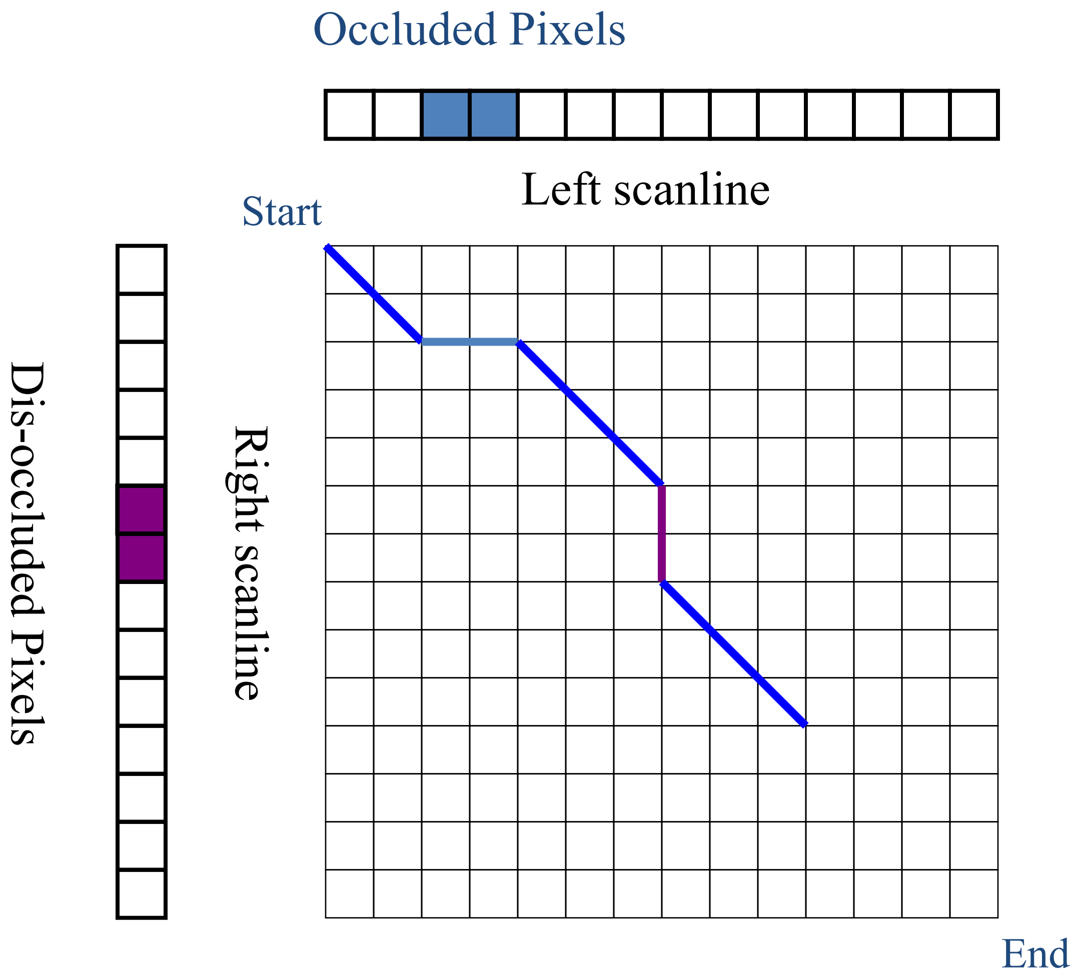
# Search Over Correspondences




Three cases:

- –Sequential – cost of match
- –Occluded – cost of no match
- –Disoccluded – cost of no match

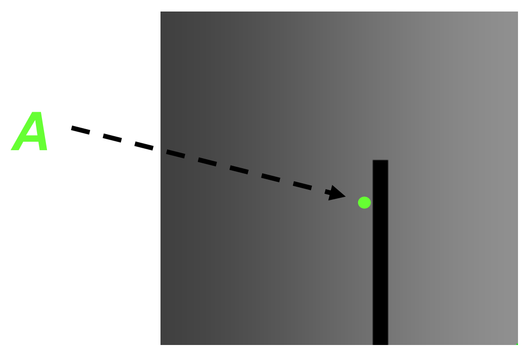
# Stereo Matching with Dynamic Programming




Dynamic programming yields the optimal path through grid. This is the best set of matches that satisfy the ordering constraint

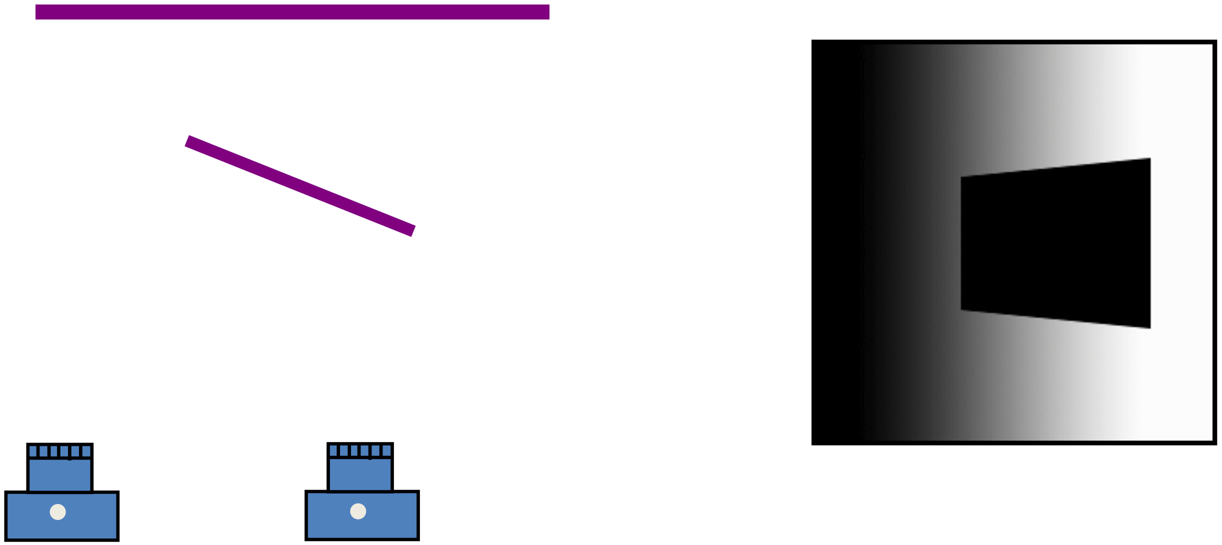
# Ordering Constraint is not Generally Correct

- Preserves matching order along scanlines, but cannot handle "double nail illusion"



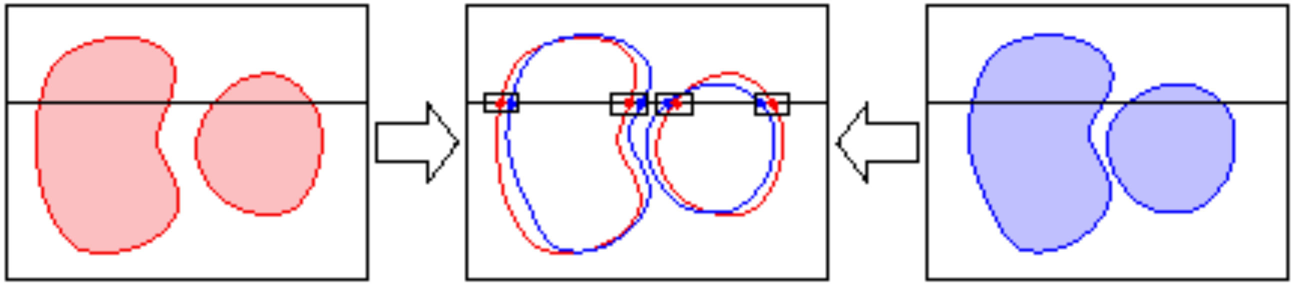
# Uniqueness Constraint is not Generally Correct

- Slanted plane: Matching between M pixels and N pixels



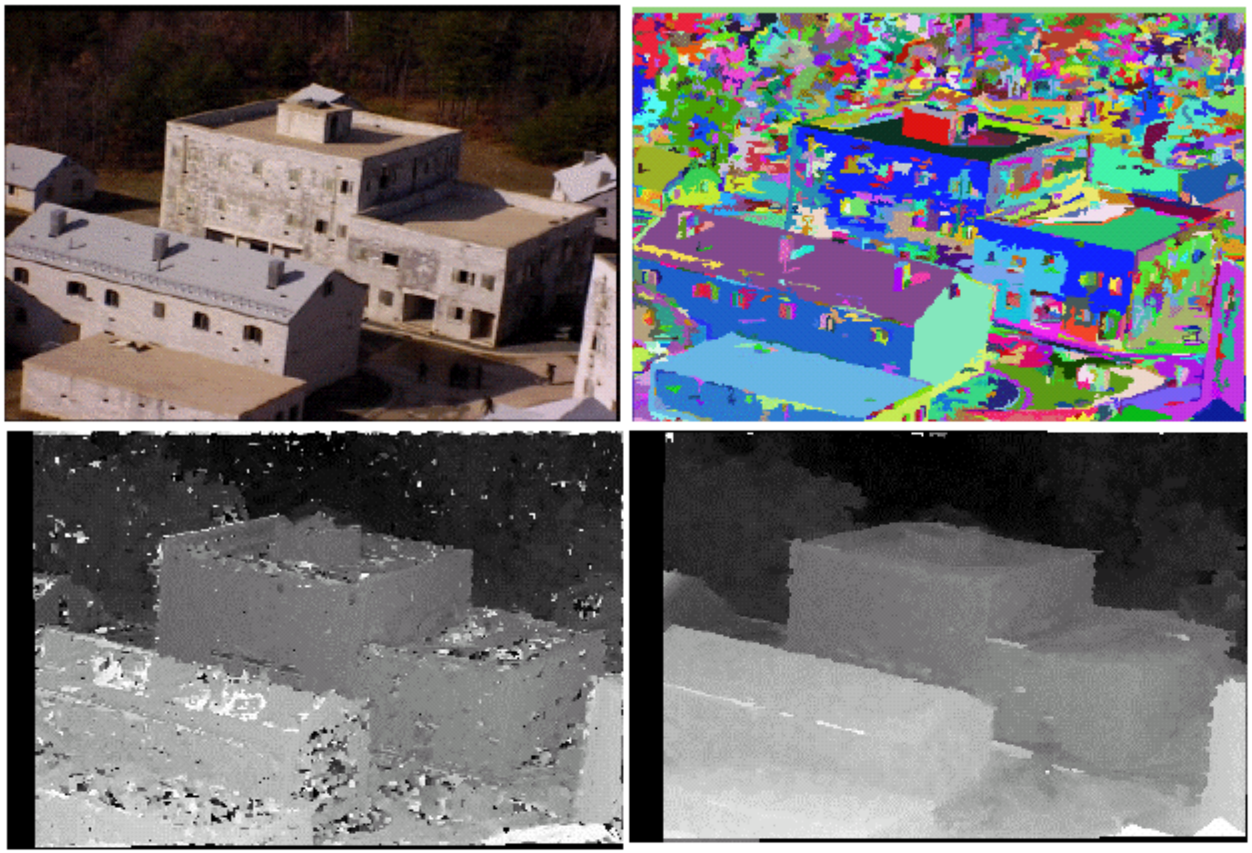
# Edge-based Stereo

- Another approach is to match *edges* rather than windows of pixels:



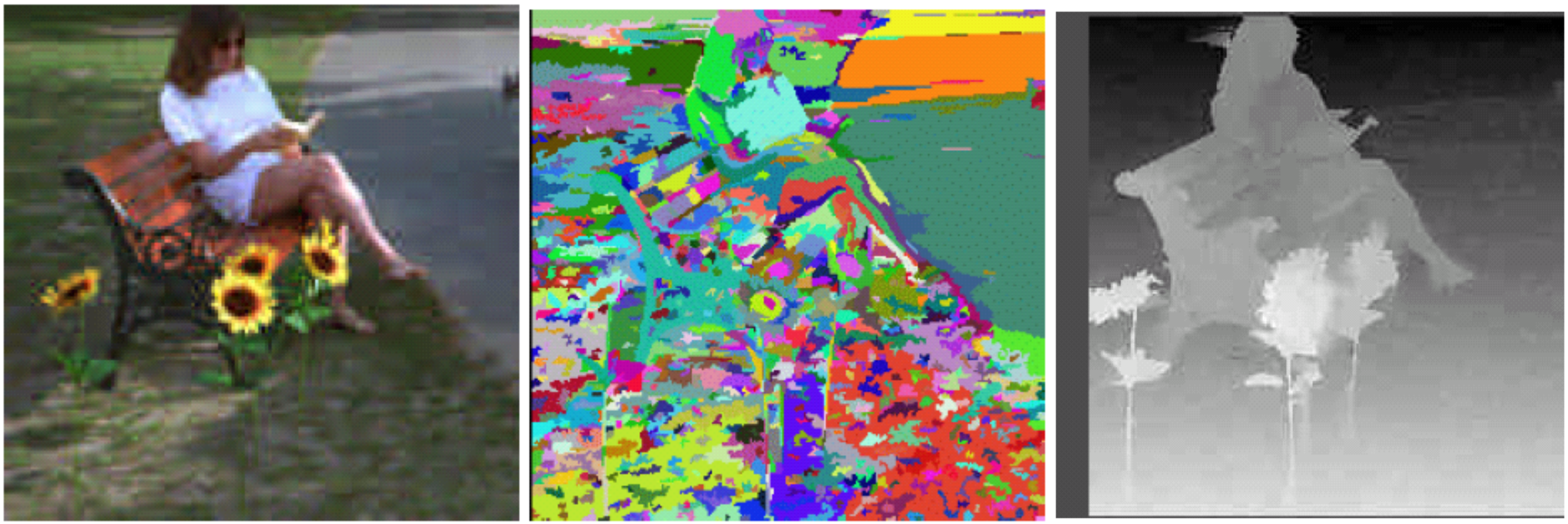
- Which method is better?
  – Edges tend to fail in dense texture (outdoors)
  – Correlation tends to fail in smooth featureless areas
  – Sparse correspondences

# Segmentation-based Stereo



**Hai Tao and Harpreet W. Sawhney**

# Another Example



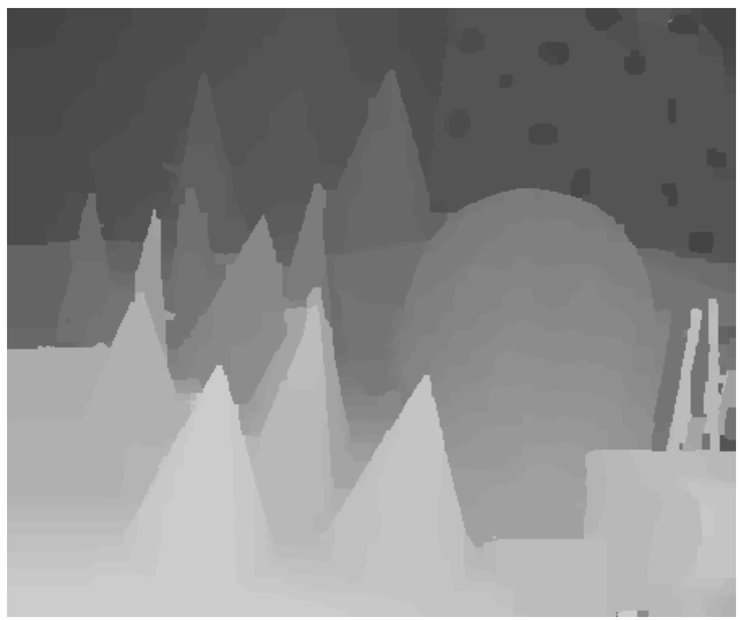
# Hallmarks of
# A Good Stereo Technique


- Should not rely on order and uniqueness constraints
- Should account for occlusions
- Should account for depth discontinuity
- Should have reasonable shape priors to handle textureless regions (e.g., planar or smooth surfaces)
- Should account for non-Lambertian surfaces
- There is a database with ground truth for testing: http://cat.middlebury.edu/stereo/data.html



Left

Right

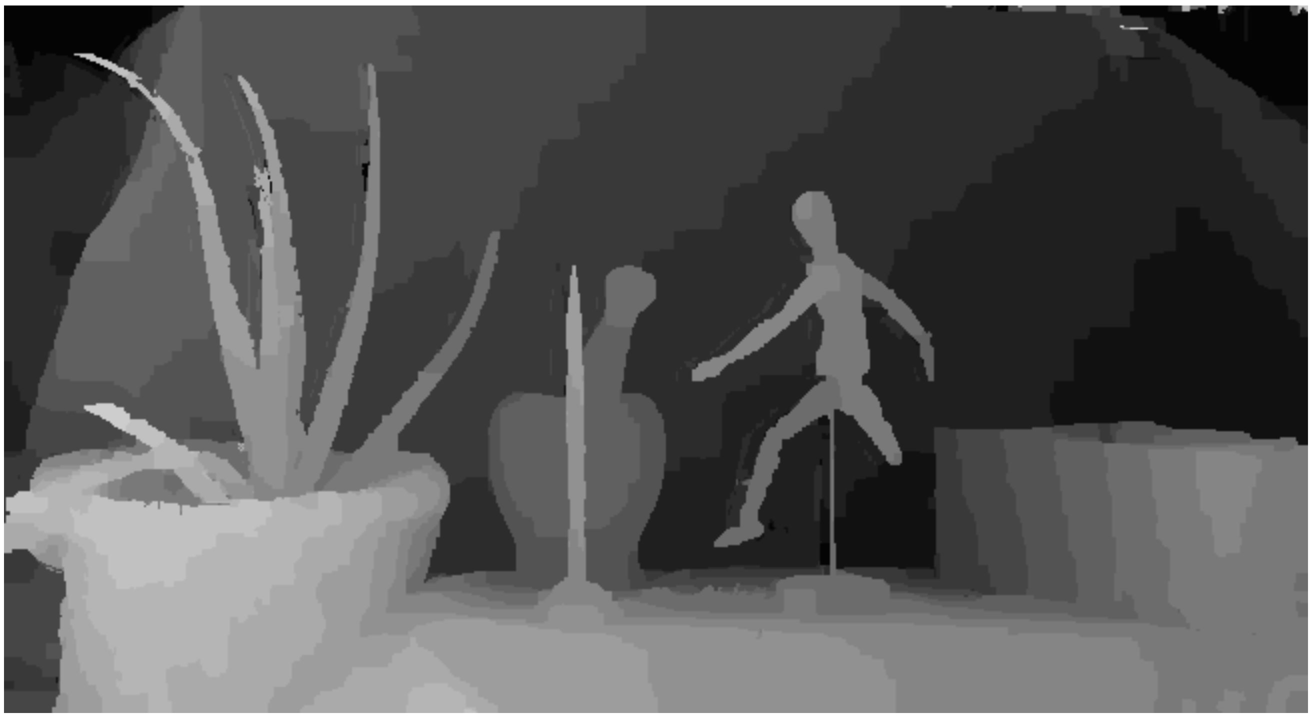


Disparity Map

Result of using a more sophisticated stereo algorithm

# View Interpolation

# Result using a good technique

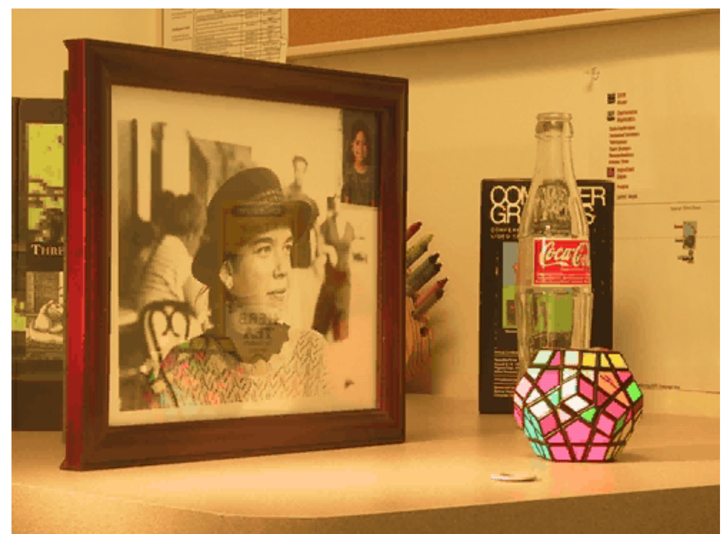


Right image
Left image
Disparity

# View Interpolation

# Bottom Line:
# Stereo is Still Difficult

- Depth discontinuities
- Lack of texture (depth ambiguity)
- Non-rigid effects (highlights, reflection, translucency)



# From 2 views to >2 views

- More pixels voting for the right depth
- Statistically more robust
- However, occlusion reasoning is more complicated, since we have to account for *partial occlusion*:
  – Which subset of cameras sees the same 3D point?