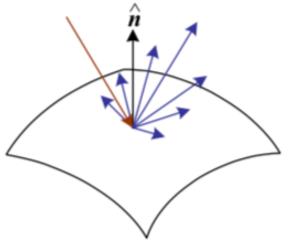


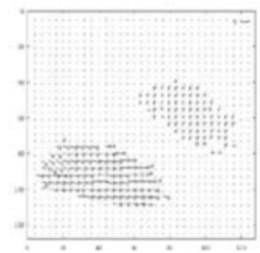
2. Image Formation

3. Image Processing

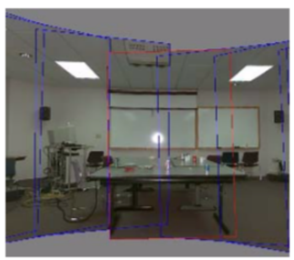
4. Features

5. Segmentation

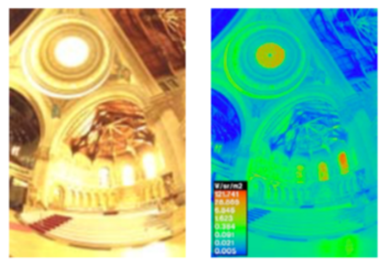
6-7. Structure from Motion



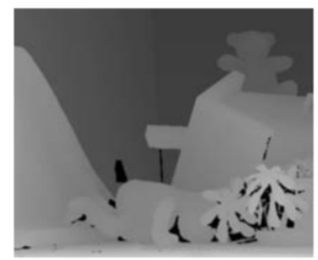
8. Motion



9. Stitching

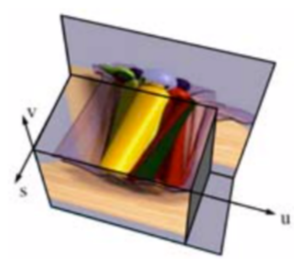


10. Computational Photography



11. Stereo

12. 3D Shape



13. Image-based Rendering

14. Recognition

Credits

- Images and formulas from Szeliski
- Second half from CVPR talk by Zhaoyang Lv

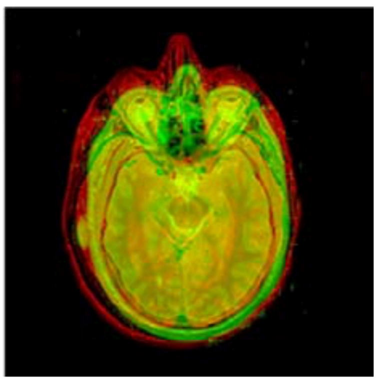
  Taking a Deeper Look at the Inverse Compositional Algorithm, Zhaoyang Lv , Frank Dellaert, James M. Rehg, Andreas Geiger, CVPR 2019

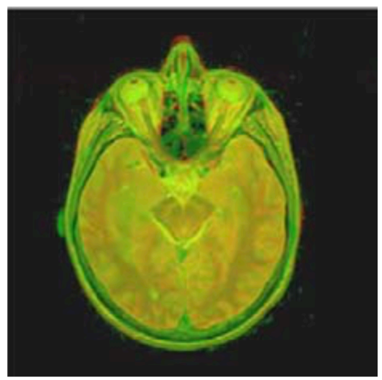
Motivating problem: Video Stabilization

Original

Stabilized

Dense Motion Estimation





- Widely used!
  - Aligning images
  - Motion in video
  - Video Stabilization
- We need:
  - Error metric
  - Search technique
    - Full search
    - Hierarchical
    - Incremental

Image credit: Kybic and Unser © 2003 IEEE

Outline

- Error metric/full search
- Hierarchical search
- Incremental refinement
  - Parametric Motion
- Deep Learning Approach (CVPR19)

Translational Alignment

- Shift image I_1 with respect to template I_0
- Before, feature-based error:

$$E_{\mathrm{LS}} = \sum_i \|\boldsymbol{r}_i\|^2 = \sum_i \|\boldsymbol{f}(\boldsymbol{x}_i;\boldsymbol{p}) - \boldsymbol{x}'_i\|^2,$$

Translational Alignment

- Shift image I_1 with respect to template I_0
- Before, feature-based error:

$$E_{\mathrm{LS}} = \sum_i \|\boldsymbol{r}_i\|^2 = \sum_i \|\boldsymbol{f}(\boldsymbol{x}_i;\boldsymbol{p}) - \boldsymbol{x}'_i\|^2,$$

- Now, image-based error:

$$E_{\mathrm{SSD}}(\boldsymbol{u}) = \sum_i [I_1(\boldsymbol{x}_i + \boldsymbol{u}) - I_0(\boldsymbol{x}_i)]^2 = \sum_i e_i^2,$$

SSD

$$E_{\mathrm{SSD}}(\boldsymbol{u}) = \sum_i [I_1(\boldsymbol{x}_i + \boldsymbol{u}) - I_0(\boldsymbol{x}_i)]^2 = \sum_i e_i^2,$$

- Sum of Squared Differences
- Assumes: brightness constancy
- If u fractional: interpolation needed
  - Bilinear (fast, good)
  - Bicubic (slower, slightly better)

Robust Error Metrics

$$E_{\mathrm{SAD}}(\boldsymbol{u}) = \sum_i |I_1(\boldsymbol{x}_i + \boldsymbol{u}) - I_0(\boldsymbol{x}_i)| = \sum_i |e_i|.$$

- Quadratic error is unforgiving!
- Absolute error (SAD): allows for outliers
- Differentiable robust error metrics exist

Dealing with Boundary Conditions

- Should not count pixels outside
- Add two "window" functions
- Windowed SSD metric:

$$E_{\mathrm{WSSD}}(\boldsymbol{u}) = \sum_i w_0(\boldsymbol{x}_i) w_1(\boldsymbol{x}_i + \boldsymbol{u}) [I_1(\boldsymbol{x}_i + \boldsymbol{u}) - I_0(\boldsymbol{x}_i)]^2,$$

- Invariant to overlap: Root mean square:

$$A = \sum_i w_0(\boldsymbol{x}_i) w_1(\boldsymbol{x}_i + \boldsymbol{u}) \qquad RMS = \sqrt{E_{\mathrm{WSSD}}/A}$$

Violations of Brightness Constancy

- Estimate Bias and Gain

$$I_1(\boldsymbol{x}+\boldsymbol{u}) = (1+\alpha)I_0(\boldsymbol{x}) + \beta,$$

$$E_{\mathrm{BG}}(\boldsymbol{u}) = \sum_i [I_1(\boldsymbol{x}_i+\boldsymbol{u}) - (1+\alpha)I_0(\boldsymbol{x}_i) - \beta]^2$$

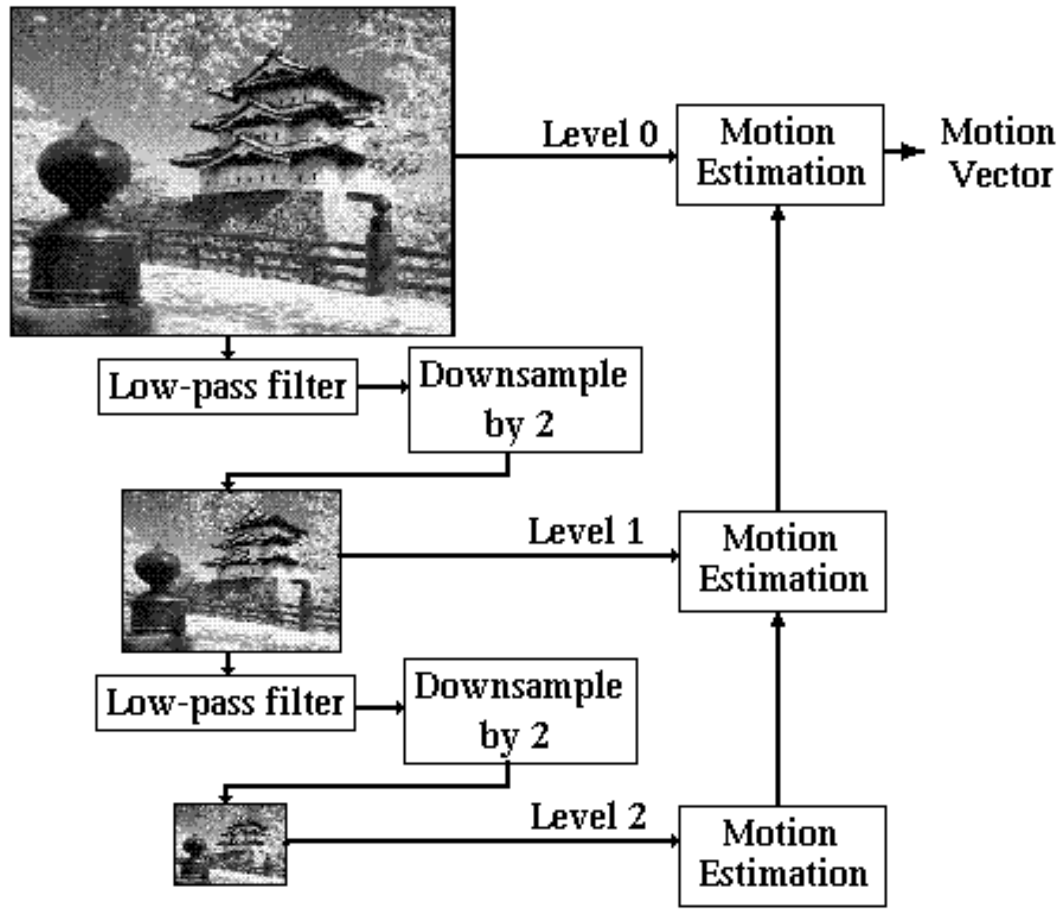
- Normalized Cross-Correlation

$$E_{\mathrm{CC}}(\boldsymbol{u}) = \sum_i I_0(\boldsymbol{x}_i) I_1(\boldsymbol{x}_i+\boldsymbol{u}).$$

$$E_{\mathrm{NCC}}(\boldsymbol{u}) = \frac{\sum_i [I_0(\boldsymbol{x}_i) - \overline{I_0}]\ [I_1(\boldsymbol{x}_i+\boldsymbol{u}) - \overline{I_1}]}{\sqrt{\sum_i [I_0(\boldsymbol{x}_i) - \overline{I_0}]^2}\sqrt{\sum_i [I_1(\boldsymbol{x}_i+\boldsymbol{u}) - \overline{I_1}]^2}}$$

Hierarchical Motion Estimation

- Build an image pyramid:
  - Low-pass
  - Decimate
- Recursively estimate motion:
  - Estimate motion at highest level
  - Use result as initial estimate at lower level



Sub-pixel Refinement

- Taylor expansion of SSD in sub-pixel update Δu:

$$
\begin{aligned}
E_{\mathrm{LK-SSD}}(\boldsymbol{u}+\Delta\boldsymbol{u}) &= \sum_i [I_1(\boldsymbol{x}_i+\boldsymbol{u}+\Delta\boldsymbol{u}) - I_0(\boldsymbol{x}_i)]^2 && (8.33)\\
&\approx \sum_i [I_1(\boldsymbol{x}_i+\boldsymbol{u}) + \boldsymbol{J}_1(\boldsymbol{x}_i+\boldsymbol{u})\Delta\boldsymbol{u} - I_0(\boldsymbol{x}_i)]^2 && (8.34)\\
&= \sum_i [\boldsymbol{J}_1(\boldsymbol{x}_i+\boldsymbol{u})\Delta\boldsymbol{u} + e_i]^2, && (8.35)
\end{aligned}
$$

where J is the Jacobian, i.e., gradients at x_i+u:

$$
\boldsymbol{J}_1(\boldsymbol{x}_i+\boldsymbol{u}) = \nabla I_1(\boldsymbol{x}_i+\boldsymbol{u}) = (\frac{\partial I_1}{\partial x}, \frac{\partial I_1}{\partial y})(\boldsymbol{x}_i+\boldsymbol{u}) \qquad (8.36)
$$

∇T_x

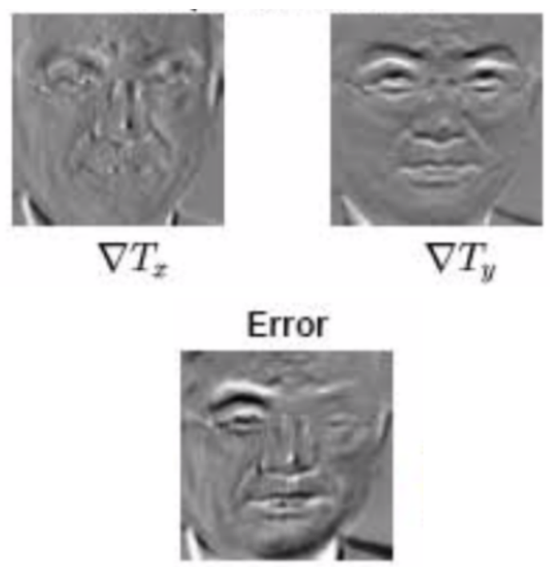
∇T_y

Solve using Normal Equations

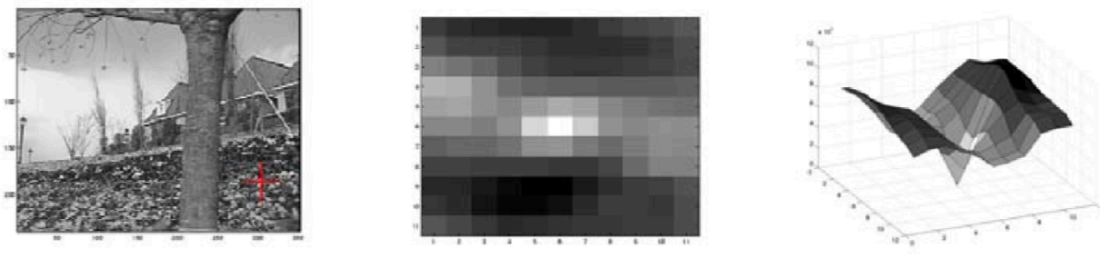
$$\boldsymbol{A}\Delta\boldsymbol{u} = \boldsymbol{b}$$

$$\boldsymbol{A} = \sum_i \boldsymbol{J}_1^T(\boldsymbol{x}_i + \boldsymbol{u})\boldsymbol{J}_1(\boldsymbol{x}_i + \boldsymbol{u}) \qquad \boldsymbol{b} = -\sum_i e_i \boldsymbol{J}_1^T(\boldsymbol{x}_i + \boldsymbol{u})$$

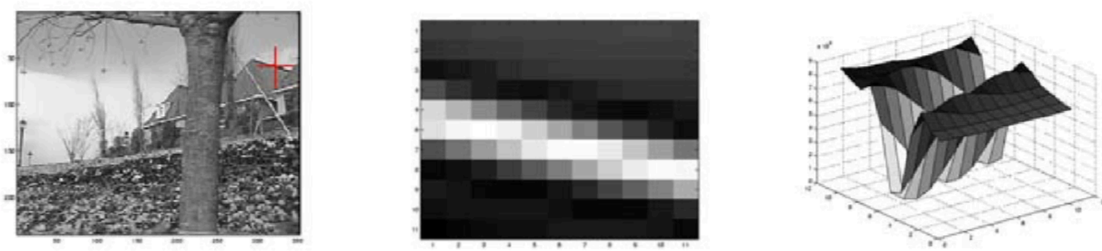
- *A* is Hessian or "information matrix", same as Harris uses!
- RHS *b* is just dot product of gradient images with error ->
- Remember: feature-based translation: just mean of flow vectors !



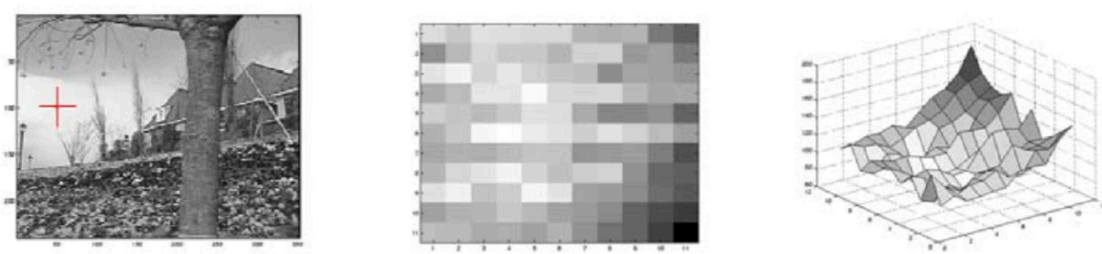
Aperture Problems and Harris



(a)



(b)



(c)

Revisiting Video Stabilization

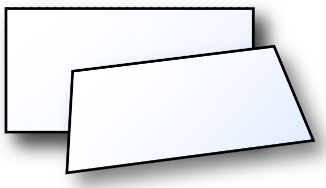
Motion Models: Translation

* Translation in x and y
* 2 DOF
* Still very shaky

Motion Models: Similarity

- Translation in x and y
- Uniform scale and rotation
- 4 DOF
- Not shaky, but wobbly

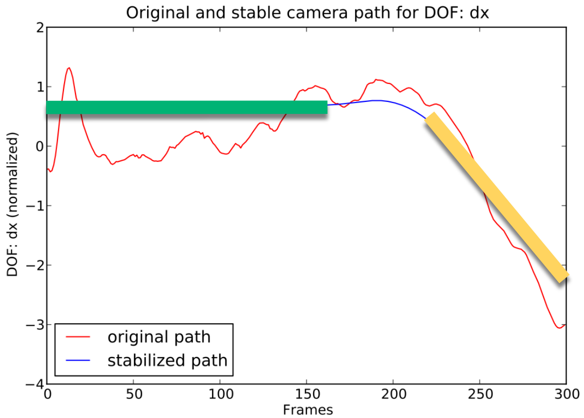
Motion Models: Homography

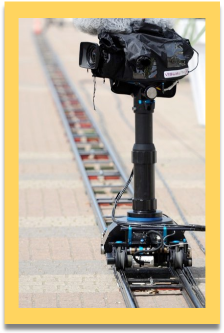


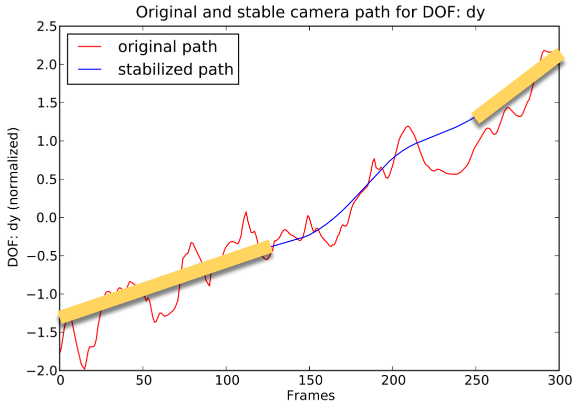
* Translation in x and y, scale and rotation
* Skew and perspective
* 8 DOF
* Stable

Path Smoothing

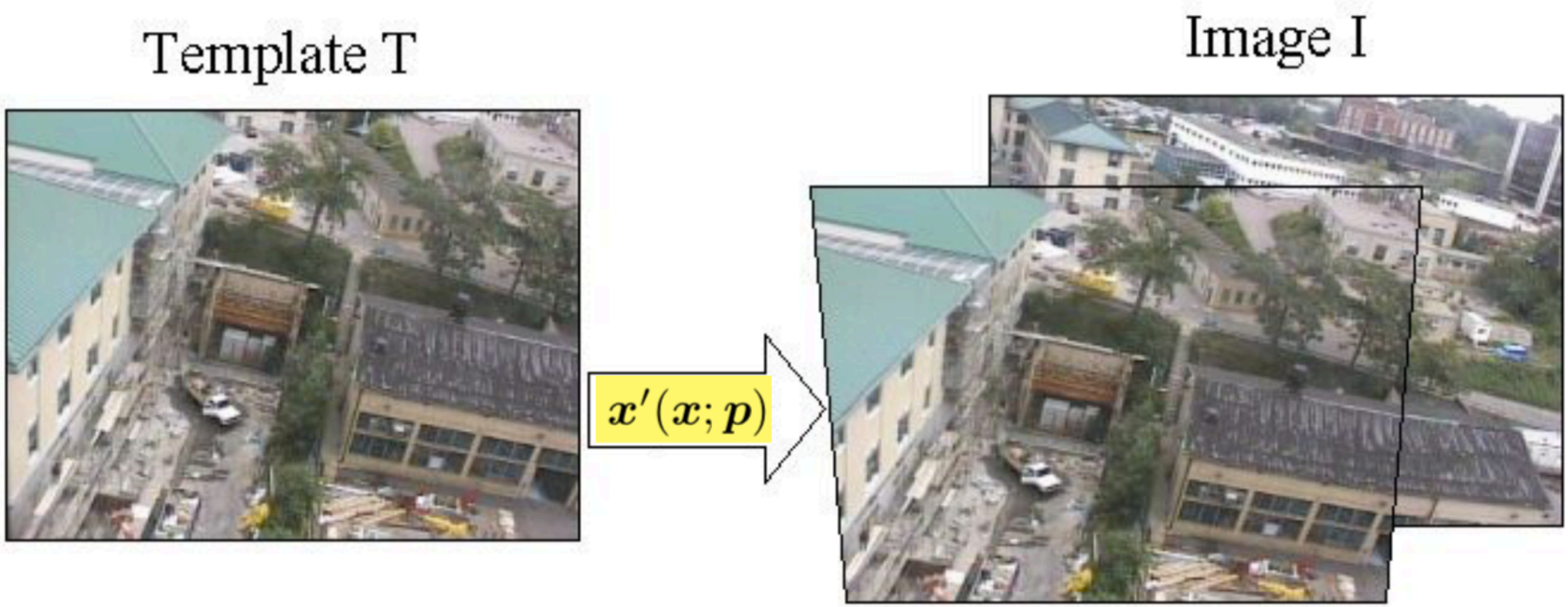
- Goal: Approximate original path with stable one
- Cinematography inspired: Properties of a stable path?
  - Tripod → Constant segment
  - Dolly or pan → Linear segment
  - Ease in and out transitions → Parabolic segment







Parametric Motion

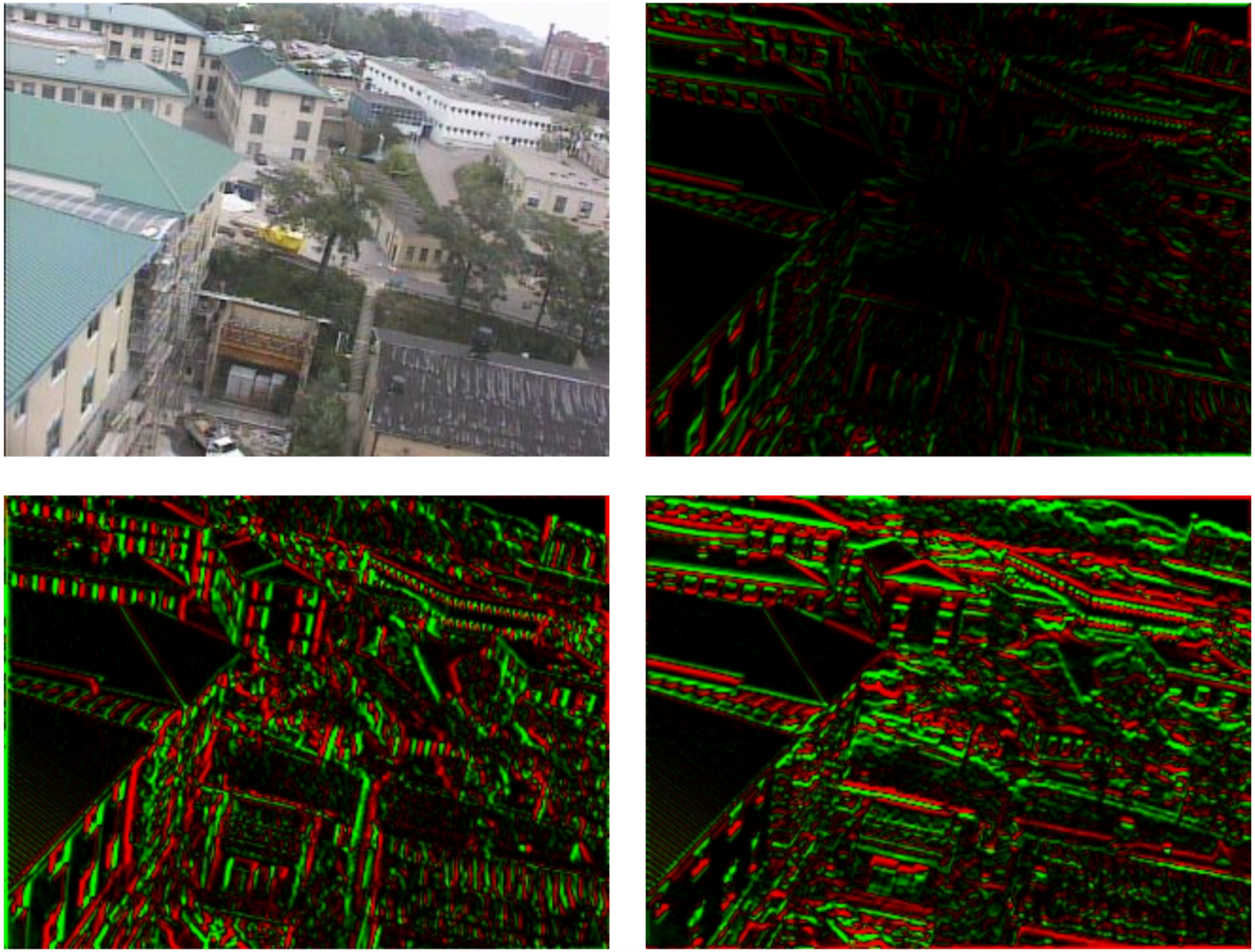


- E.g., image-based homography estimation

$$\begin{aligned} E_{\mathrm{LK-PM}}(\boldsymbol{p}+\Delta\boldsymbol{p}) &= \sum_i [I_1(\boldsymbol{x}'(\boldsymbol{x}_i;\boldsymbol{p}+\Delta\boldsymbol{p})) - I_0(\boldsymbol{x}_i)]^2 \\ &\approx \sum_i [I_1(\boldsymbol{x}'_i) + \boldsymbol{J}_1(\boldsymbol{x}'_i)\Delta\boldsymbol{p} - I_0(\boldsymbol{x}_i)]^2 \end{aligned}$$

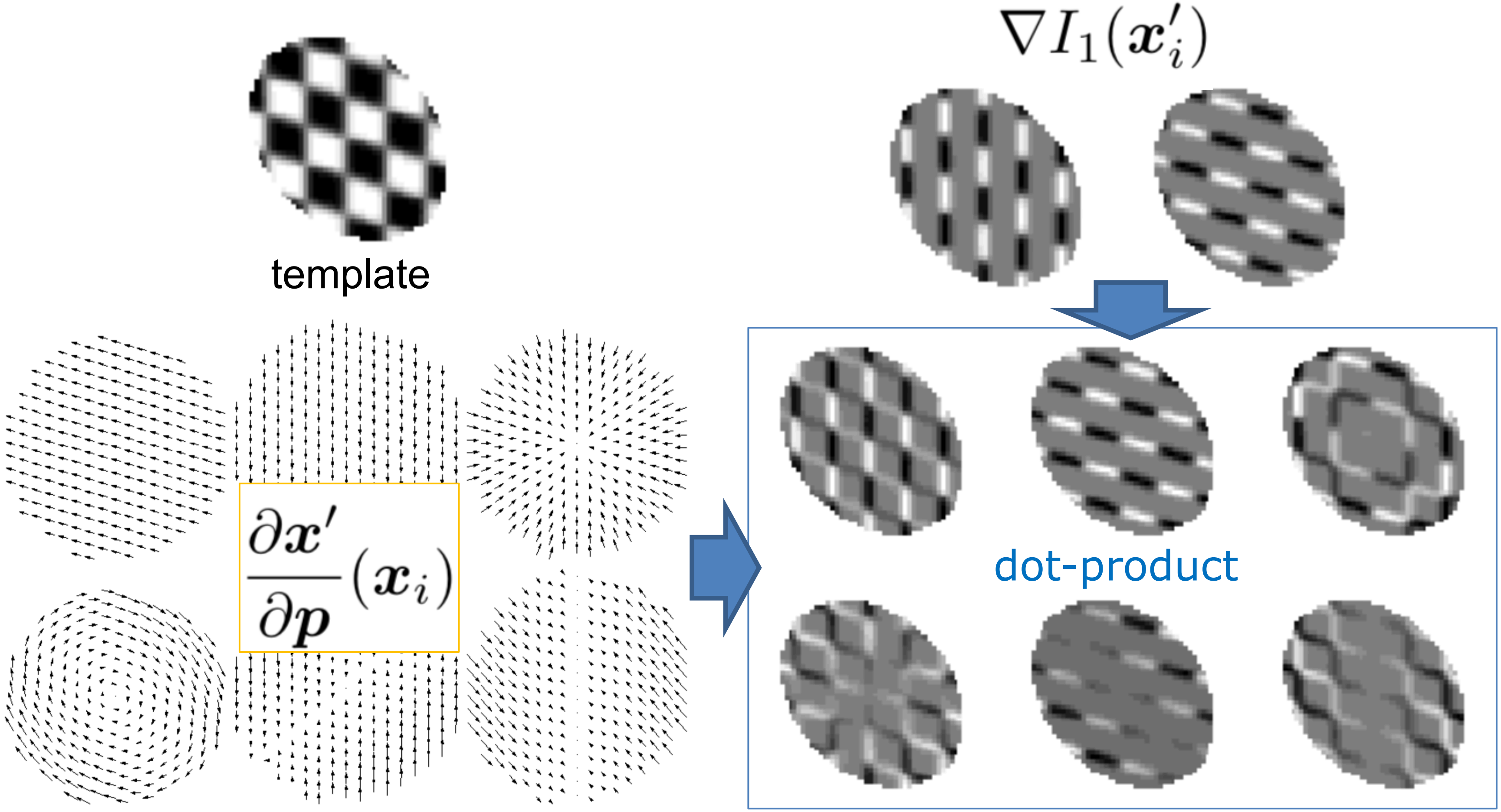
Dellaert & Collins, 1999, Fast Image-Based Tracking by Selective Pixel Integration

”Jacobian Images”



Dellaert & Collins, 1999, Fast Image-Based Tracking by Selective Pixel Integration

Computing Jacobian Images



$$\boldsymbol{J}_1(\boldsymbol{x}'_i) = \frac{\partial I_1}{\partial \boldsymbol{p}} = \boxed{\nabla I_1(\boldsymbol{x}'_i)\frac{\partial \boldsymbol{x}'}{\partial \boldsymbol{p}}(\boldsymbol{x}_i),} \qquad (8.52)$$

Dellaert & Collins, 1999, Fast Image-Based Tracking by Selective Pixel Integration

Compositional and Inverse Compositional

- Compare three variants:
  - Original:
  - Compositional:
  - Inverse Comp:

$$\sum_i [I_1(\boldsymbol{x}'(\boldsymbol{x}_i; \boldsymbol{p} + \Delta\boldsymbol{p})) - I_0(\boldsymbol{x}_i)]^2 \qquad (8.49)$$

$$\sum_i [\tilde{I}_1(\tilde{\boldsymbol{x}}(\boldsymbol{x}_i; \Delta\boldsymbol{p})) - I_0(\boldsymbol{x}_i)]^2 \qquad (8.60)$$

$$\sum_i [\tilde{I}_1(\boldsymbol{x}_i) - I_0(\tilde{\boldsymbol{x}}(\boldsymbol{x}_i; \Delta\boldsymbol{p}))]^2 \qquad (8.64)$$

- In compositional approach we *warp* the image I_1 and solve for an incremental update.
- Inverse compositional: search for incremental update to template instead
  - Jacobians and Hessian can now be *precomputed*

The Inverse Compositional Algorithm [S. Baker and I. Matthews, 04]

$$\mathbf{r}_k(\mathbf{0}) = \mathbf{I}(\boldsymbol{\xi}_k) - \mathbf{T}(\mathbf{0})$$

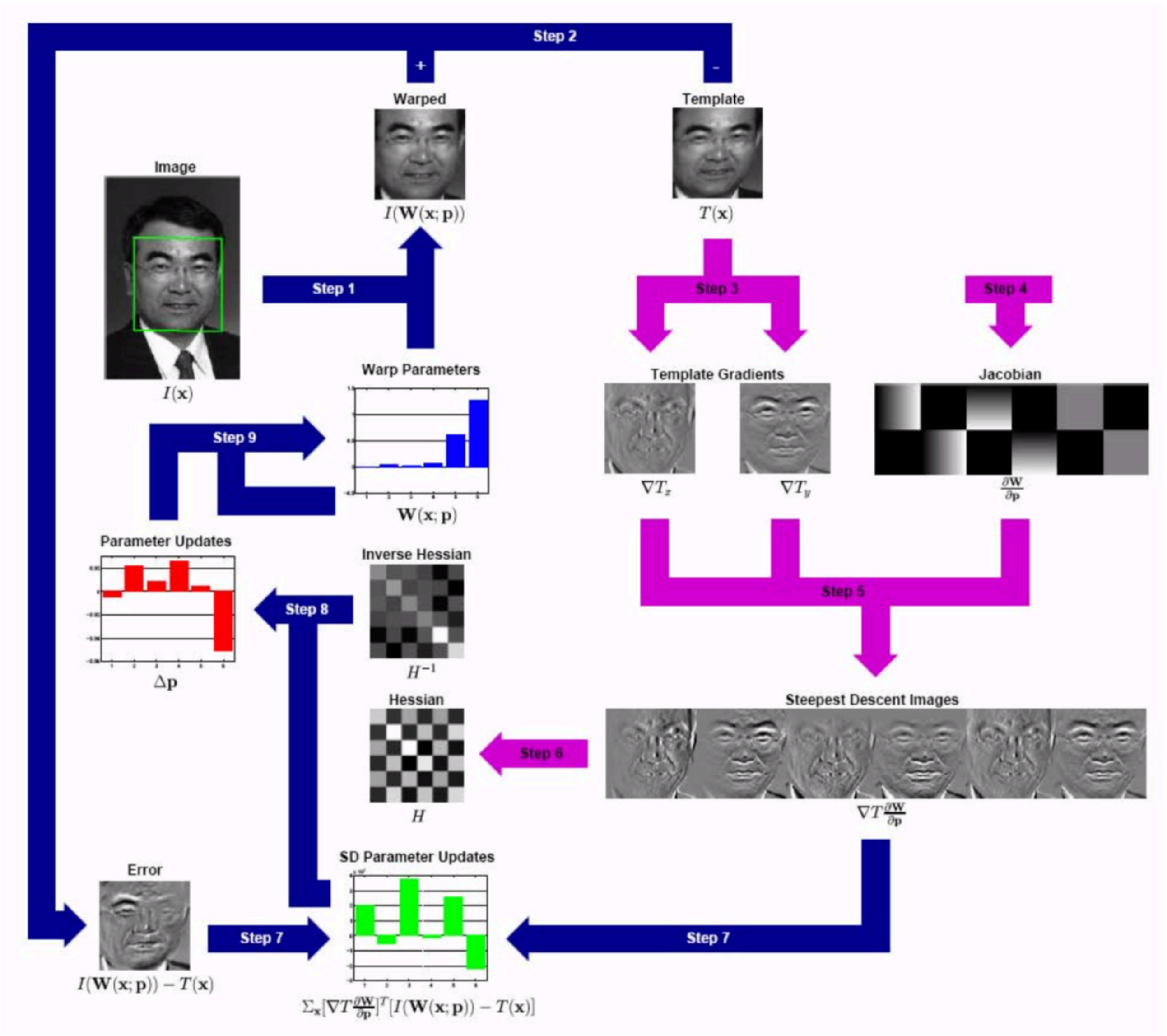
$$\Delta\boldsymbol{\xi} = \left(\mathbf{J}^T\mathbf{W}\mathbf{J} + \lambda\,\mathrm{diag}(\mathbf{J}^T\mathbf{W}\mathbf{J})\right)^{-1}\mathbf{J}^T\mathbf{W}\,\mathbf{r}_k(\mathbf{0})$$

$$\boldsymbol{\xi}_{k+1} = \boldsymbol{\xi}_k \circ (\Delta\boldsymbol{\xi})^{-1}$$

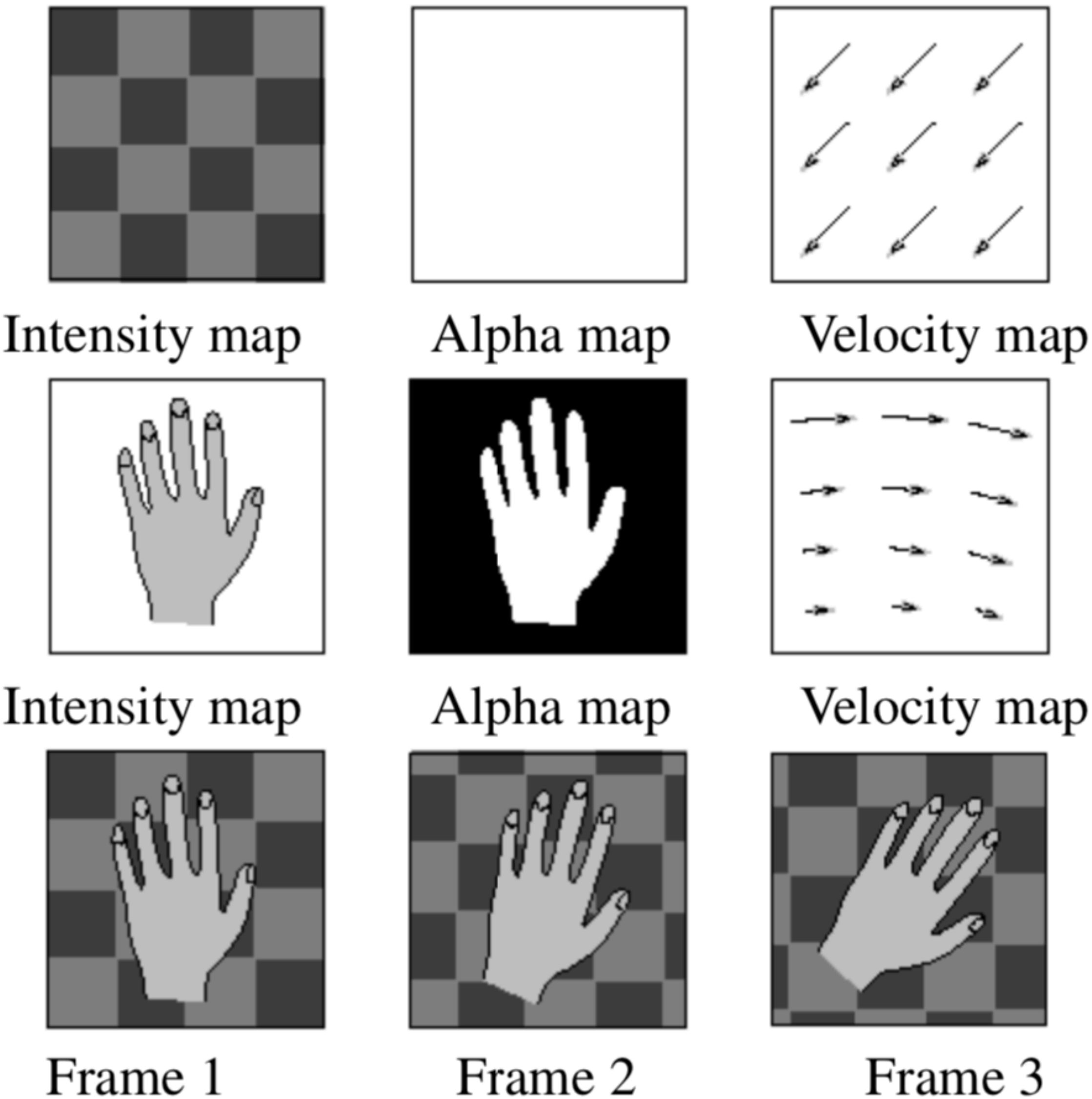
$\mathbf{W}$ Weight matrix

$\lambda\,\mathrm{diag}(\mathbf{J}^T\mathbf{W}\mathbf{J})$ Damping: very frequently used in non-linear optimization to make sure gradients are valid; “Levenberg-Marquardt”

Inverse Compositional Approach

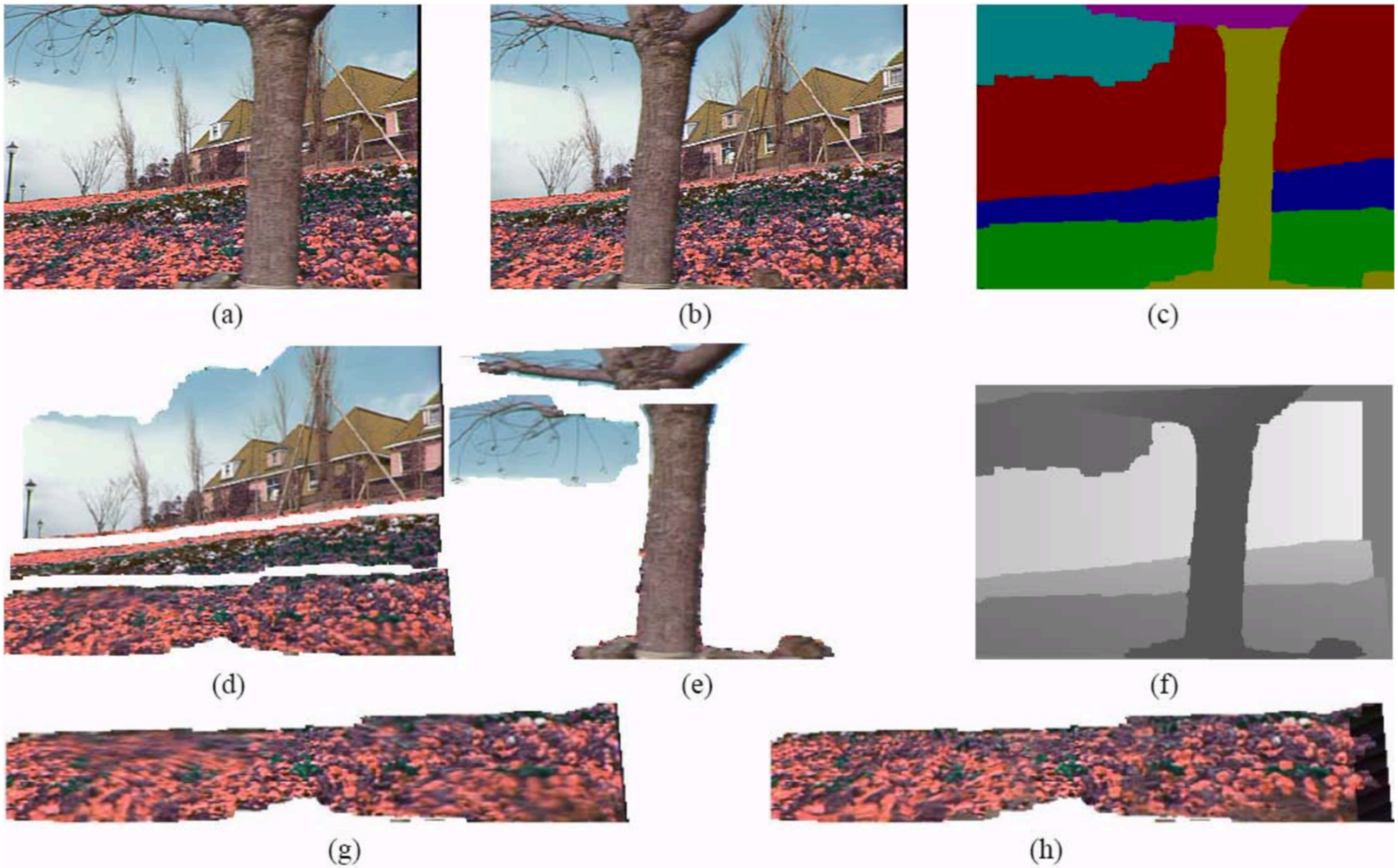


Layered Motion



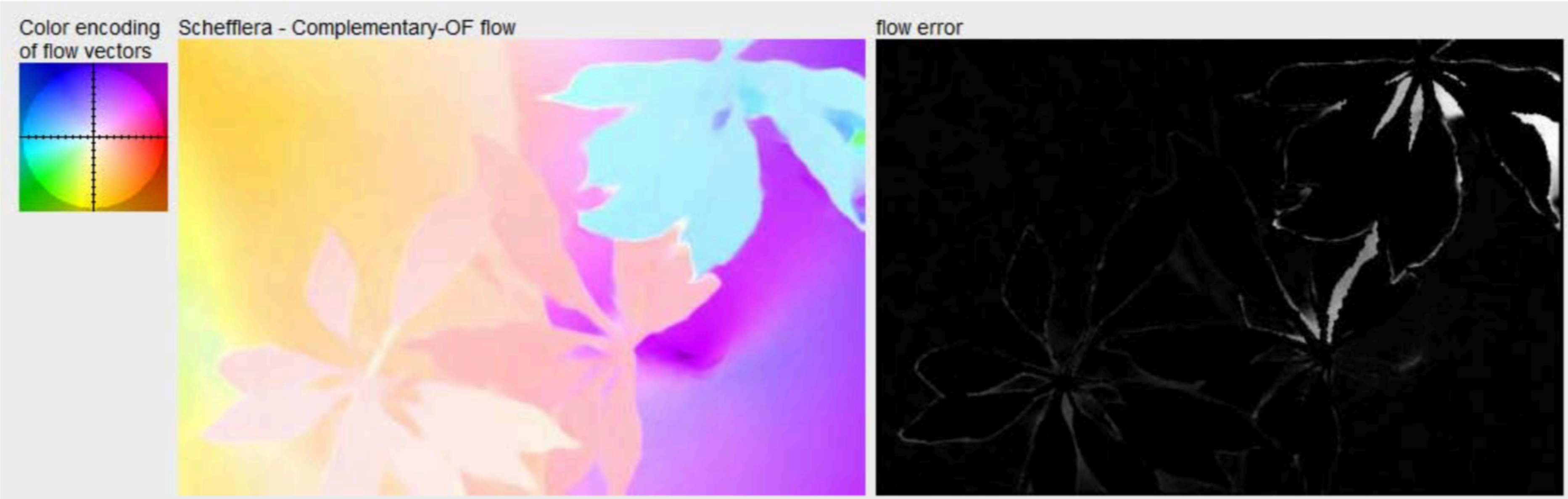
- One type of assumption to "regularize" optical flow
- Estimate FG and BG layers

Layered Motion Results



Baker, Szeliski, and Anandan 1998

Optical Flow: fully non-parametric



- Fully non-parametric model of motion
- N pixels -> N flow vectors -> 2N parameters
- Need some smoothness assumptions!
- Hard to deal with occlusion

Taking a Deeper Look at the Inverse Compositional Algorithm

Zhaoyang Lv[1] , Frank Dellaert[1], James M. Rehg[1], Andreas Geiger[2]

[1]Georgia Institute of Technology

[2]Autonomous Vision Group, MPI-IS and University of Tübingen

The Inverse Compositional Algorithm [S. Baker and I. Matthews, 04]

$$\mathbf{r}_k(\mathbf{0}) = \mathbf{I}(\boldsymbol{\xi}_k) - \mathbf{T}(\mathbf{0})$$

$$\Delta\boldsymbol{\xi} = \left(\mathbf{J}^T\mathbf{W}\mathbf{J} + \lambda\,\mathrm{diag}(\mathbf{J}^T\mathbf{W}\mathbf{J})\right)^{-1}\mathbf{J}^T\mathbf{W}\,\mathbf{r}_k(\mathbf{0})$$

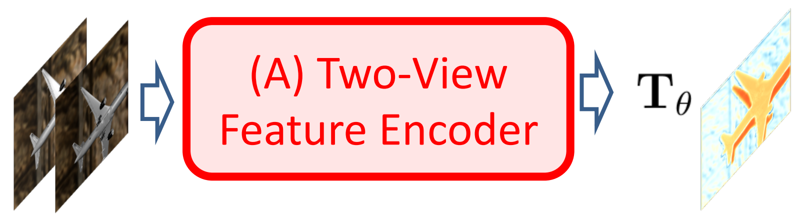
$$\boldsymbol{\xi}_{k+1} = \boldsymbol{\xi}_k \circ (\Delta\boldsymbol{\xi})^{-1}$$

We propose to take a **deeper** look at the Inverse Compositional algorithm **from a learning perspective.**

Take a Deeper Look at the Inverse Compositional algorithm

Contribution (A): Two-view Feature Encoder

$$\mathbf{r}_k = \boxed{\mathbf{I}_\theta(\boldsymbol{\xi}_k)} - \boxed{\mathbf{T}_\theta(\mathbf{0})}$$

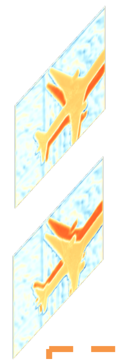


$$\Delta\boldsymbol{\xi} = (\mathbf{J}^T\mathbf{W}\mathbf{J} + \lambda\,\mathrm{diag}(\mathbf{J}^T\mathbf{W}\mathbf{J}))^{-1}\mathbf{J}^T\mathbf{W}\,\mathbf{r}_k(\mathbf{0})\ ;$$

$$\boldsymbol{\xi}_{k+1} = \boldsymbol{\xi}_k \circ (\Delta\boldsymbol{\xi})^{-1}$$

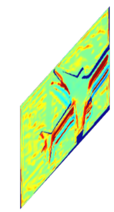
Take a Deeper Look at the Inverse Compositional algorithm

Contribution (B): Convolutional M-estimator



(B) Convolutional M-estimator

$\mathbf{W}_\theta$



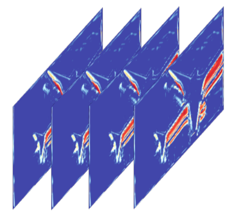
$$\mathbf{r}_k = \mathbf{I}_\theta(\boldsymbol{\xi}_k) - \mathbf{T}_\theta(\mathbf{0})$$

$$\Delta\boldsymbol{\xi} = (\mathbf{J}^T \boxed{\mathbf{W}_\theta} \mathbf{J} + \mathrm{diag}(\mathbf{J}^T \boxed{\mathbf{W}_\theta} \mathbf{J})^{-1} \mathbf{J}^T \boxed{\mathbf{W}_\theta} \mathbf{r}_k(\mathbf{0})$$

$$\boldsymbol{\xi}_{k+1} = \boldsymbol{\xi}_k \circ (\Delta\boldsymbol{\xi})^{-1}$$

Take a Deeper Look at the Inverse Compositional algorithm

Contribution (C): Trust Region Network



(C) Trust Region Network

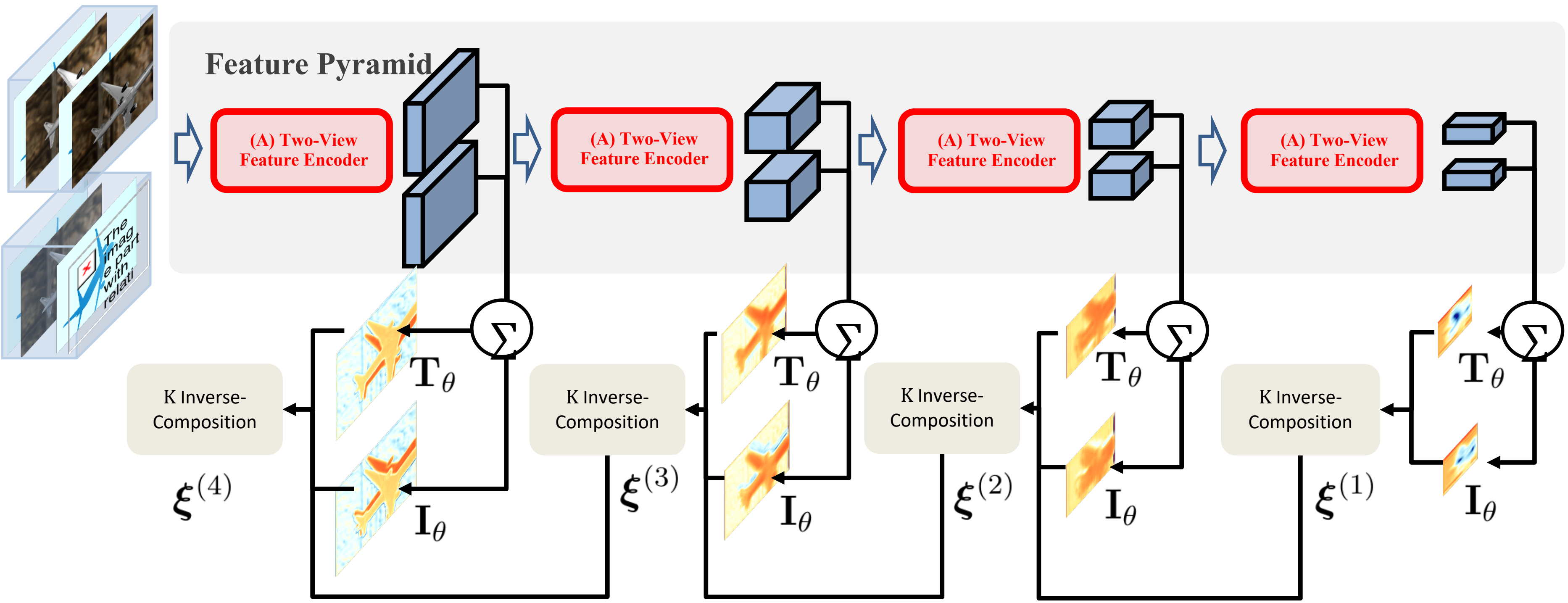
$\boldsymbol{\lambda}_\theta$

$$\mathbf{r}_k = \mathbf{I}_\theta(\boldsymbol{\xi}_k) - \mathbf{T}_\theta(\mathbf{0})$$

$$\Delta\boldsymbol{\xi} = (\mathbf{J}^T\mathbf{W}_\theta\mathbf{J} + \boxed{\mathrm{diag}(\boldsymbol{\lambda}_\theta)})^{-1}\mathbf{J}^T\mathbf{W}_\theta\,\mathbf{r}_k(\mathbf{0})$$

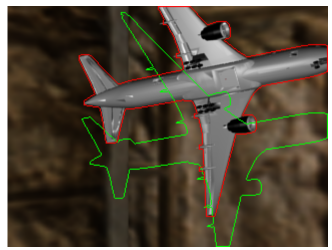
$$\boldsymbol{\xi}_{k+1} = \boldsymbol{\xi}_k \circ (\Delta\boldsymbol{\xi})^{-1}$$

Coarse-to-Fine Inverse Compositional Algorithm



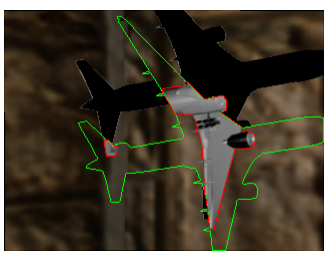
Visualization of Iterative 3D Rigid Motion Alignment

$\mathbf{T}$

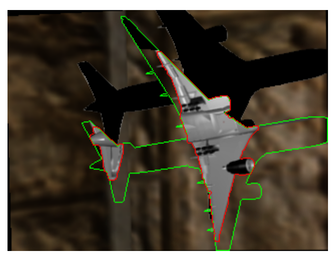


$\mathbf{I}$

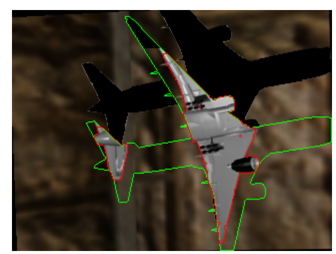
$\mathbf{I}(\boldsymbol{\xi}^{\mathrm{GT}})$



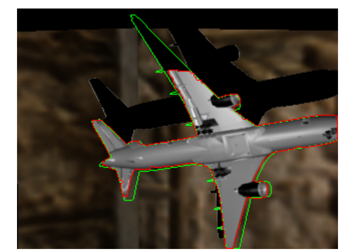
DeepLK
[Wang et al. ICRA, 2018]



Ours (A)



Ours (A)+(B)



Ours (A)+(B)+(C)

Conclusion

We have taken a deeper look at the inverse compositional algorithm by reformulating it with

(A) Two-view Feature Encoder

(B) Convolutional M-estimator

(C) Trust Region Network

The proposed solution is **learnable**, **accurate**, **small**, and **fast** in inference.