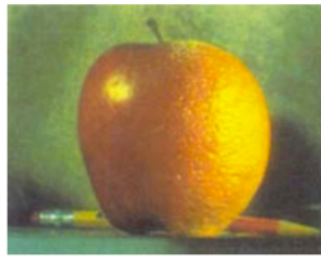


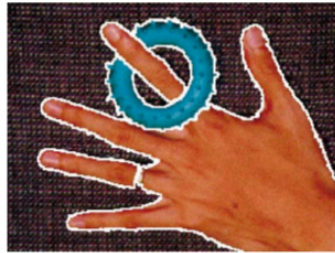
2. Image Formation



3. Image Processing



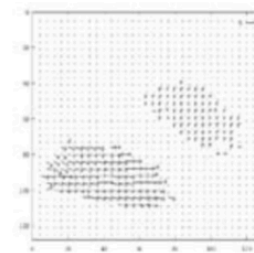
4. Features



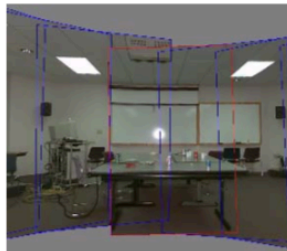
5. Segmentation



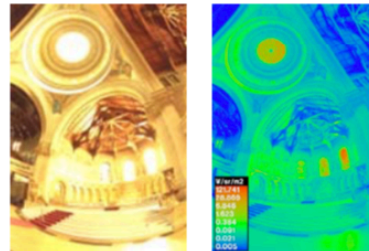
6-7. Structure from Motion



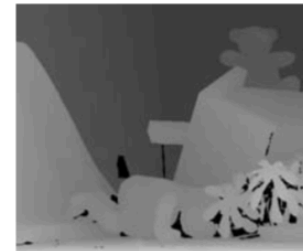
8. Motion



9. Stitching



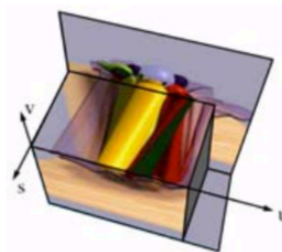
10. Computational Photography



11. Stereo



12. 3D Shape

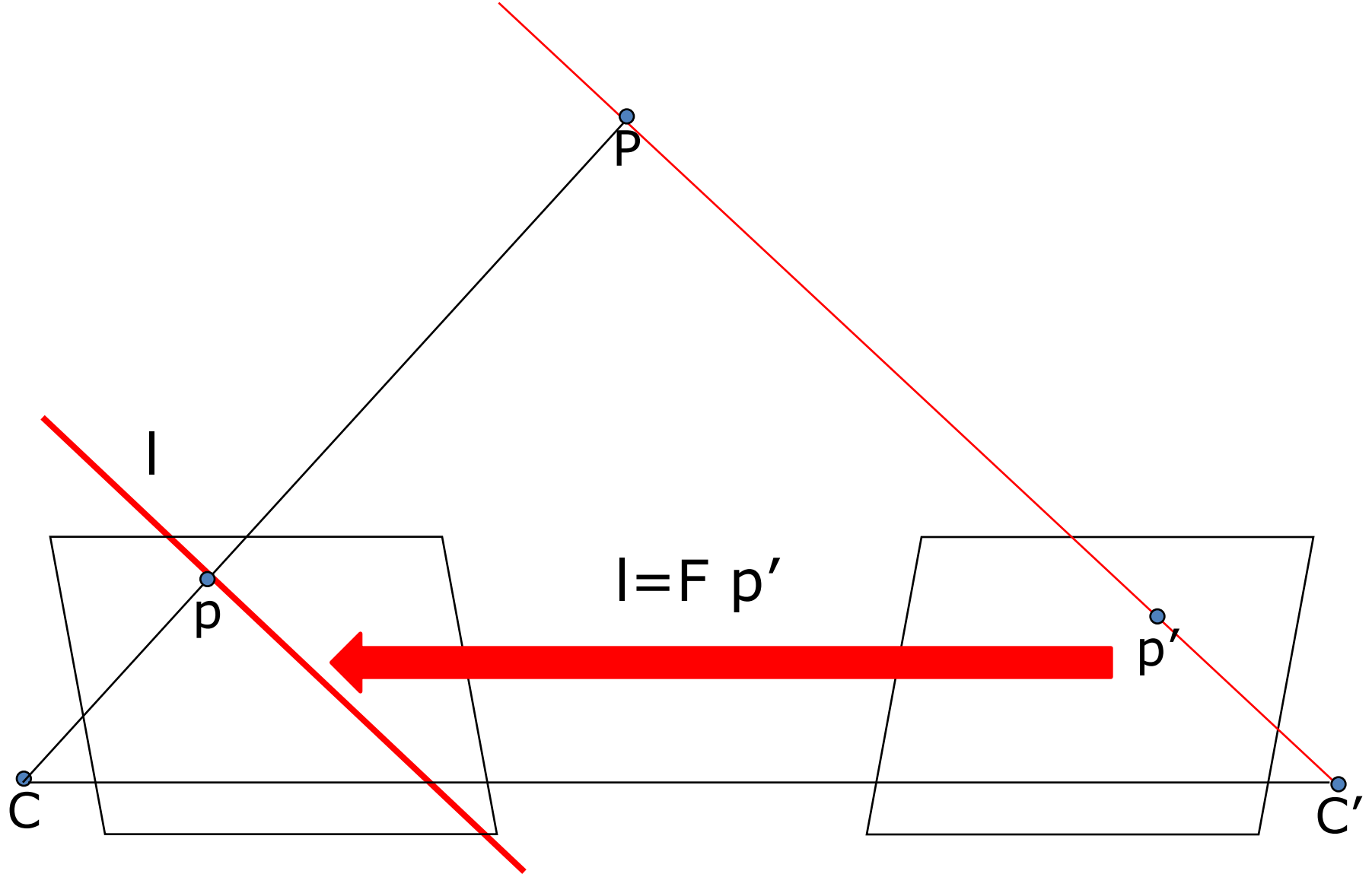


13. Image-based Rendering



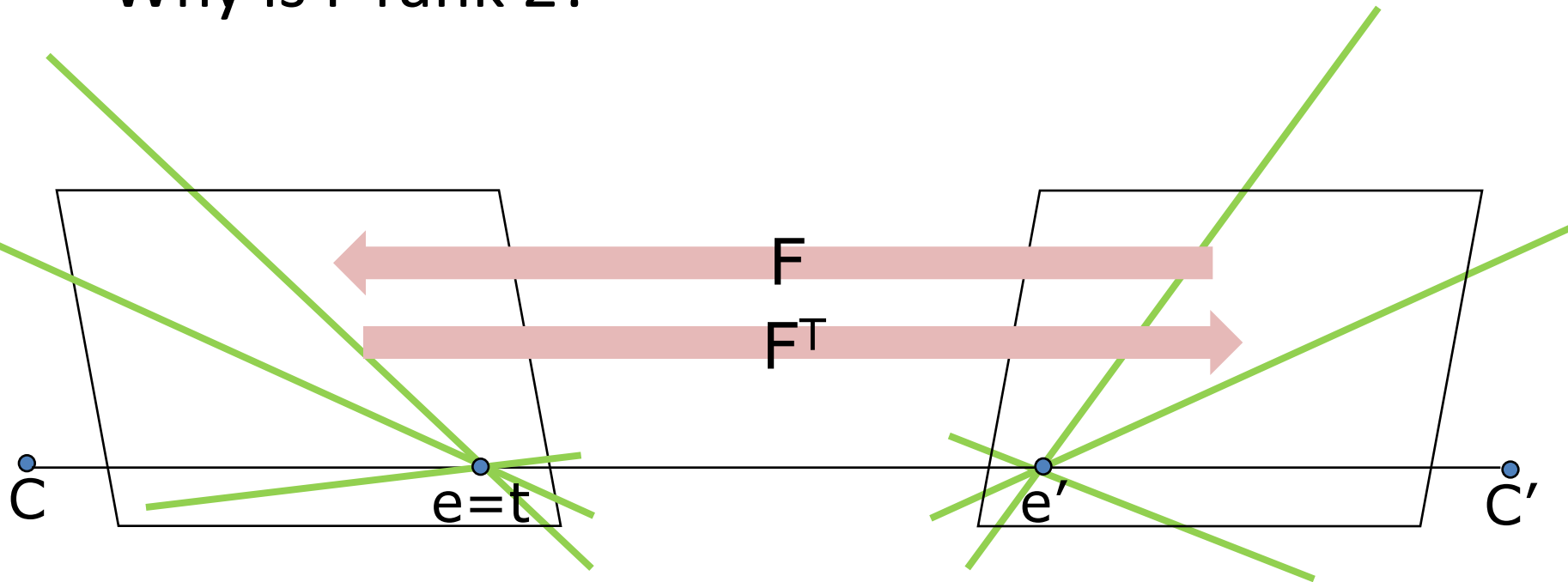
14. Recognition

Recap: Two views and Fundamental Matrix F



Rank 2 Constraint

- Why is F rank 2?



- Not invertible! Collection of points is mapped to a pencil of lines. Epipoles map to zero.
- What would it mean to be rank 1?

The Eight-Point Algorithm (Longuet-Higgins, 1981)

$$(u, v, 1) \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \quad \Rightarrow \quad (uu', uv', u, vu', vv', v, u', v', 1) \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$



$$\begin{pmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 \\ u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 \\ u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 \\ u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 \\ u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 \\ u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 \\ u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Minimize:

$$\sum_{i=1}^n (\mathbf{p}_i^T \mathcal{F} \mathbf{p}'_i)^2$$

under the constraint

$$|\mathcal{F}|^2 = 1.$$

The Normalized Eight-Point Algorithm (Hartley, 1995)

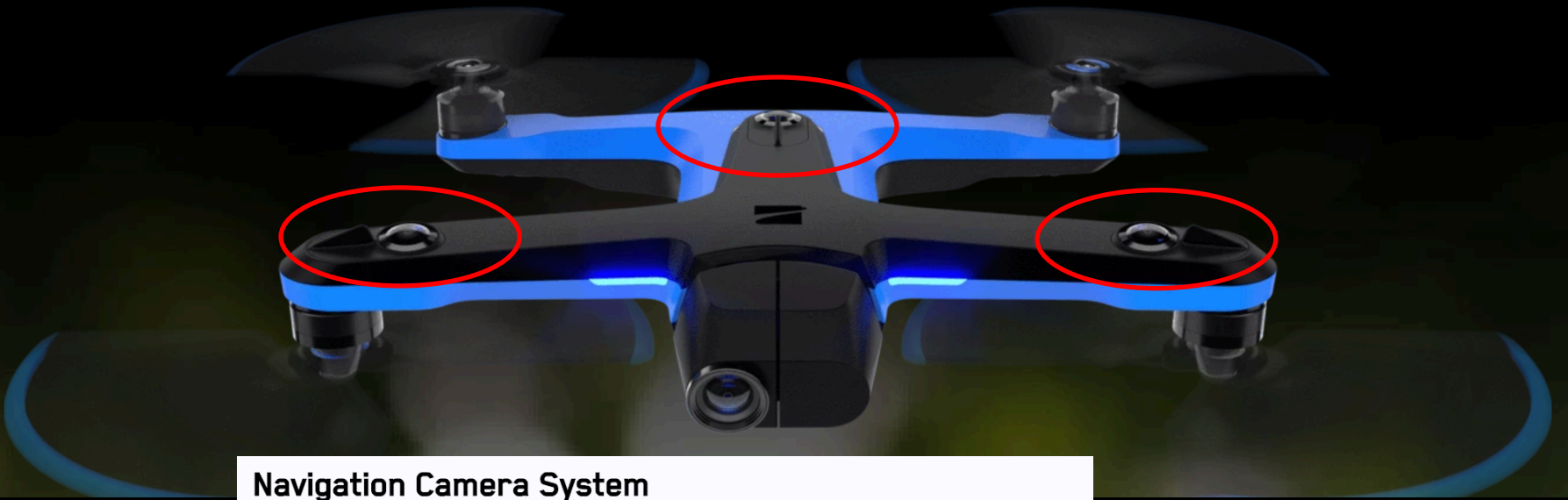
- Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels:

$$q_i = T p_i \quad q_i' = T' p_i'.$$

- Use the eight-point algorithm to compute \mathcal{F} from the points q_i and q_i' .
- Enforce the rank-2 constraint.
- Output $T^{-1} \mathcal{F} T'$.

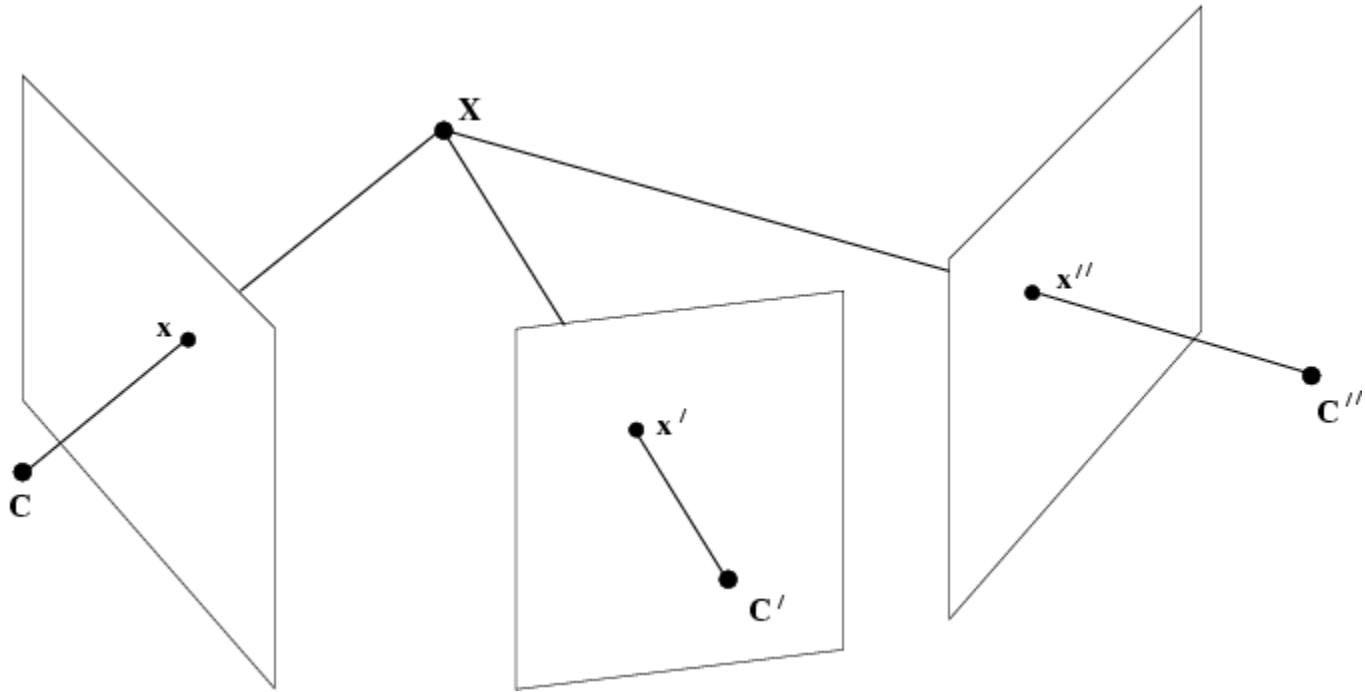
Trinocular Camera rigs

Skydio 2



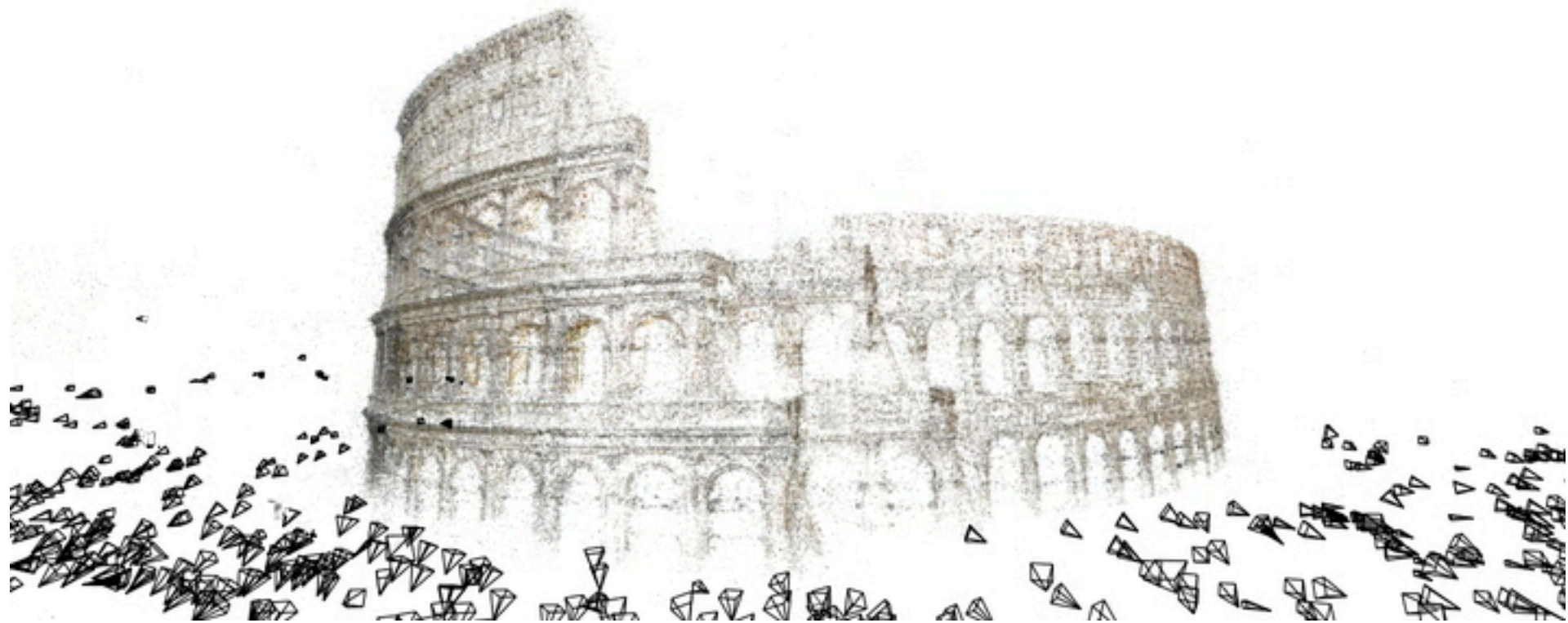
Navigation Camera System	
CONFIGURATION	6x cameras in trinocular configuration top and bottom
SENSOR TYPE	Sony 1/3" 4K color CMOS
LENS APERTURE	f/1.8
FIELD-OF-VIEW	200°
ENVIRONMENT COVERAGE	True 360°

Trifocal Geometry



$$[\mathbf{x}']_{\times} \left(\sum_i x^i \mathbf{T}_i \right) [\mathbf{x}'']_{\times} = \mathbf{0}_{3 \times 3}$$

Structure from Motion



Building Rome in a Day
Agarwal et al

Motivation

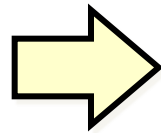
- Photo Tourism
- Photosynth
- Multi-view stereo
- Building Rome in a Day
- Rome on a Cloudless Day

Photo Tourism

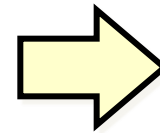
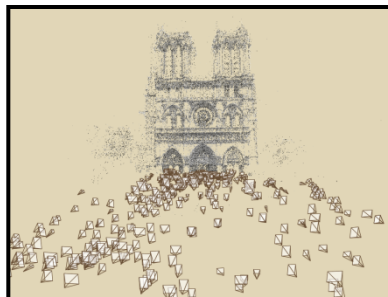
Noah Snavely, Steven M. Seitz, Richard Szeliski, [Photo tourism: Exploring photo collections in 3D,](#) ACM Transactions on Graphics (SIGGRAPH Proceedings), 25(3), 2006, 835-846.



Input photographs



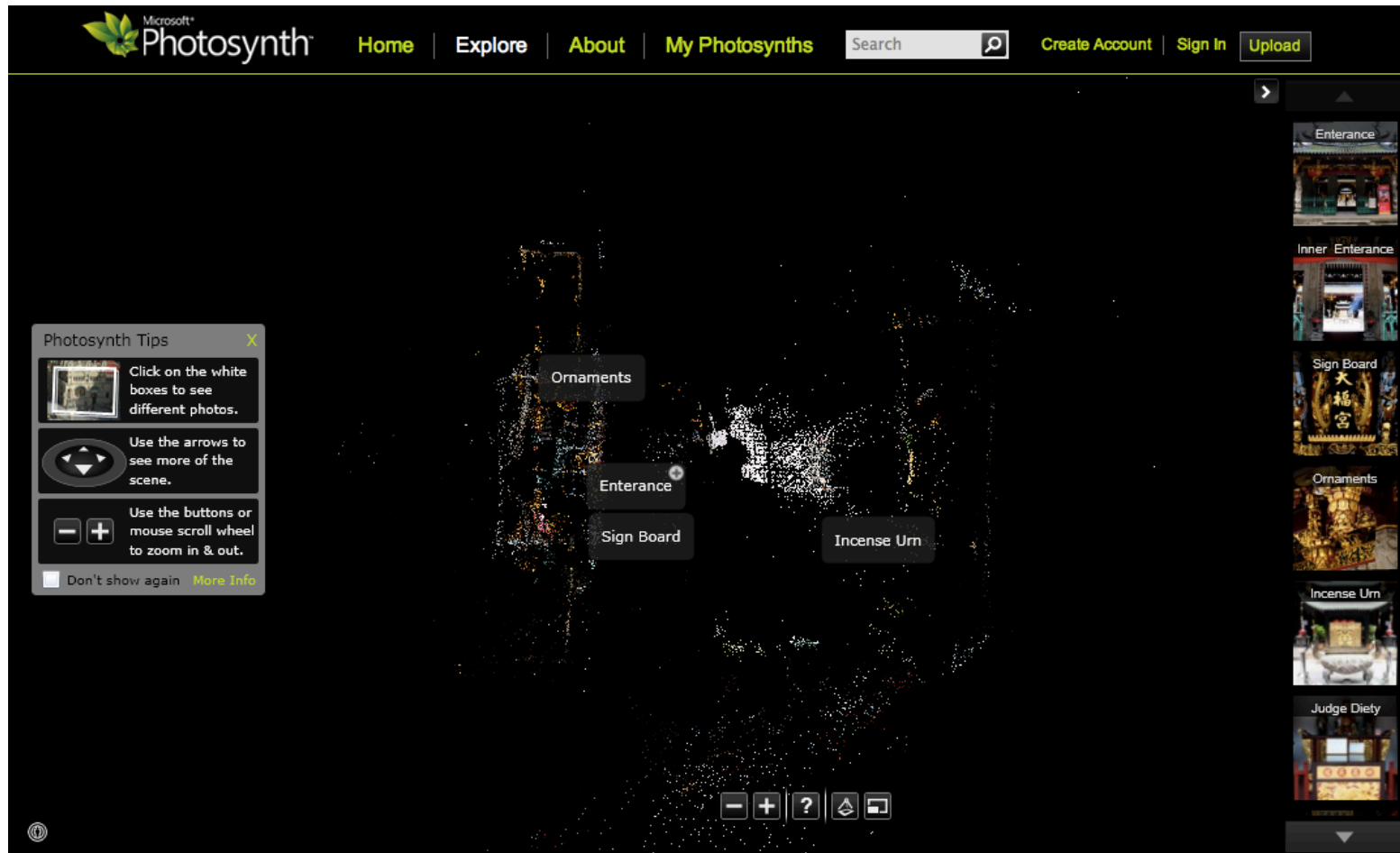
Scene
reconstruction



<http://phototour.cs.washington.edu/>

Photosynth

photosynth.net

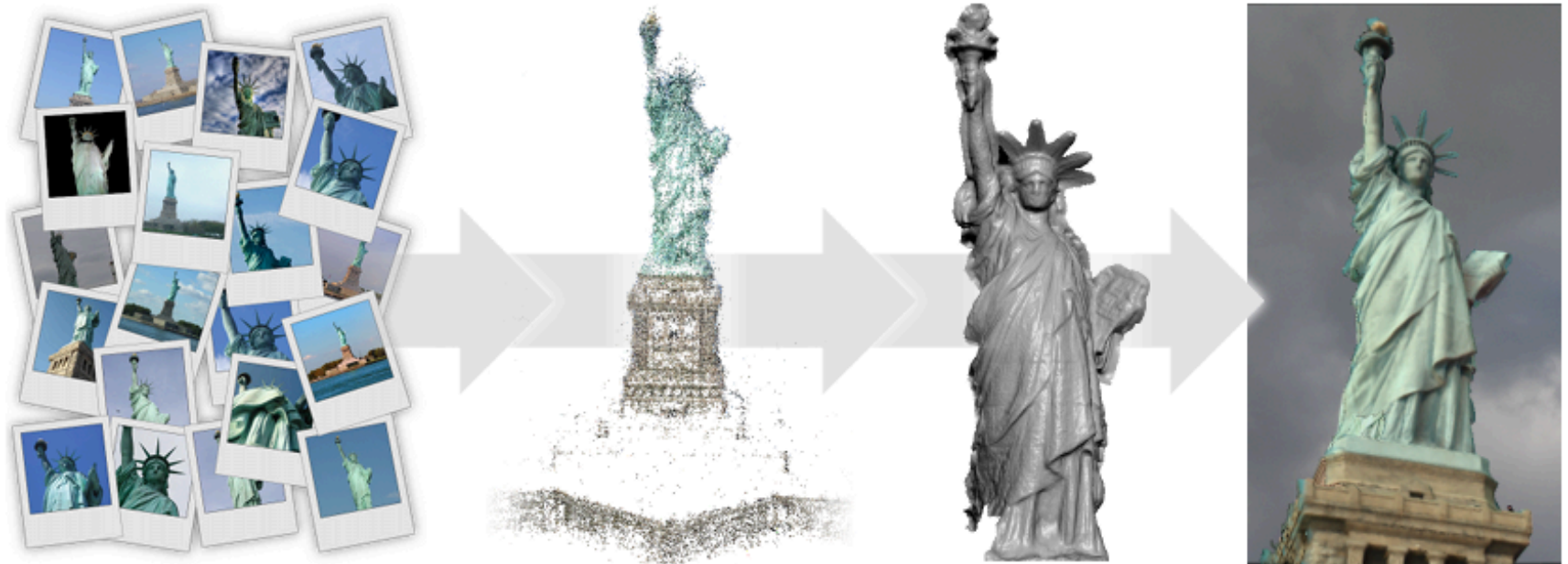


- <http://photosynth.net/view.aspx?cid=29aa8616-a43a-43e4-9d6e-b8ad9b50483e>

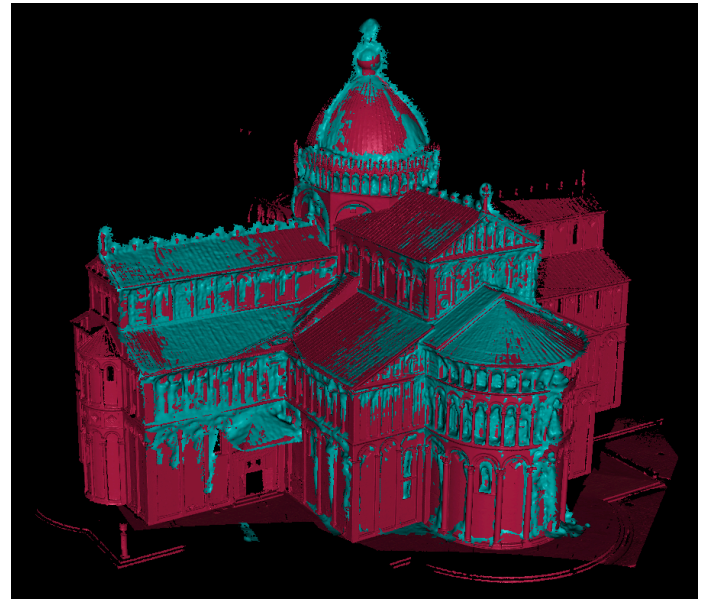
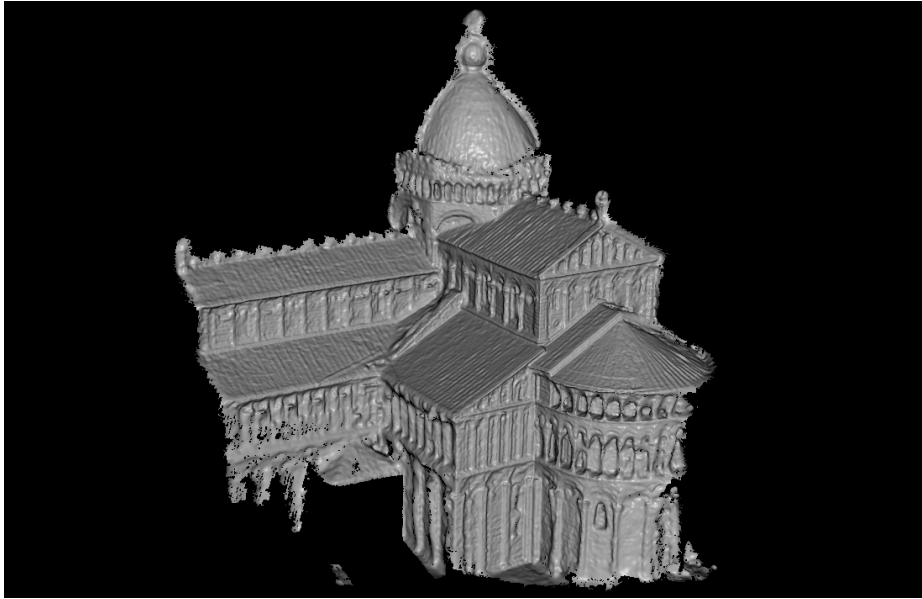
Multi-view Stereo

Multi-View Stereo for Community Photo Collections

Michael Goesele, Noah Snavely, Brian Curless, Hugues Hoppe, and Steven M. Seitz
ICCV 2007



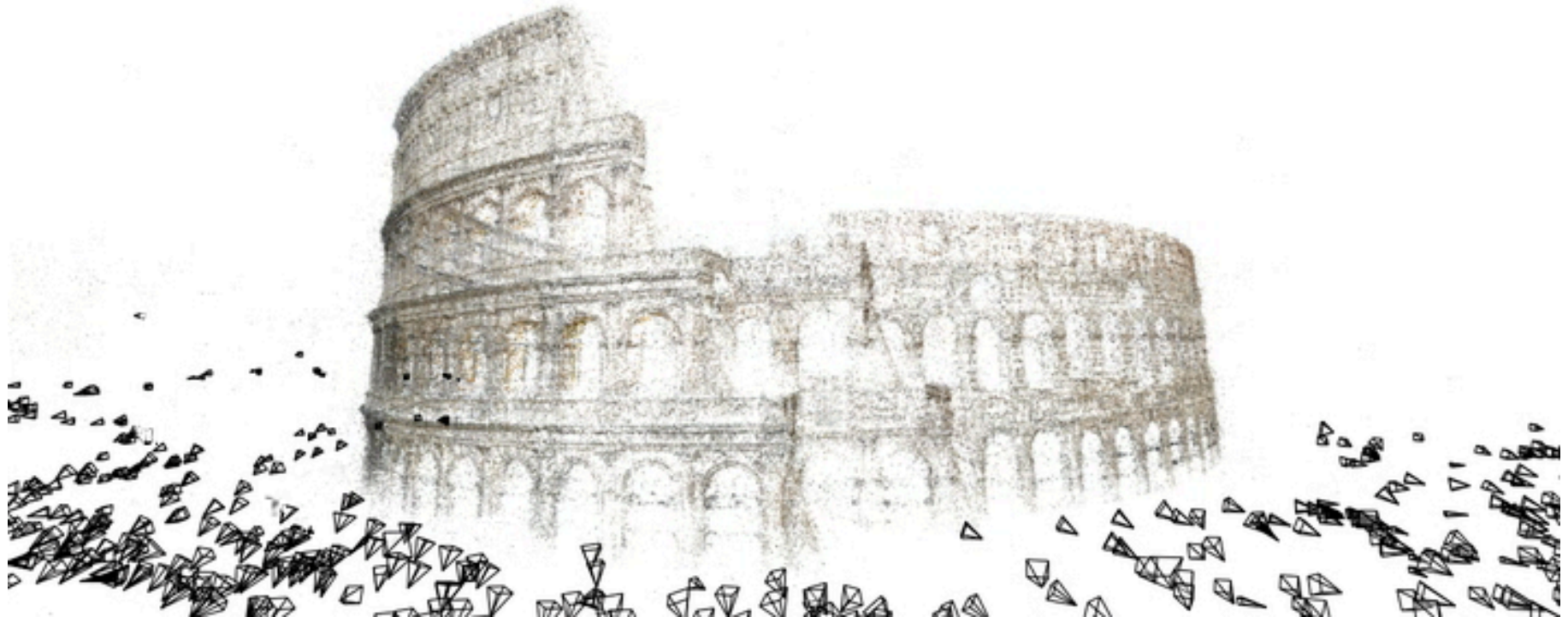
Multi-view Stereo



Compared with Laser-Scanner

Building Rome in a Day

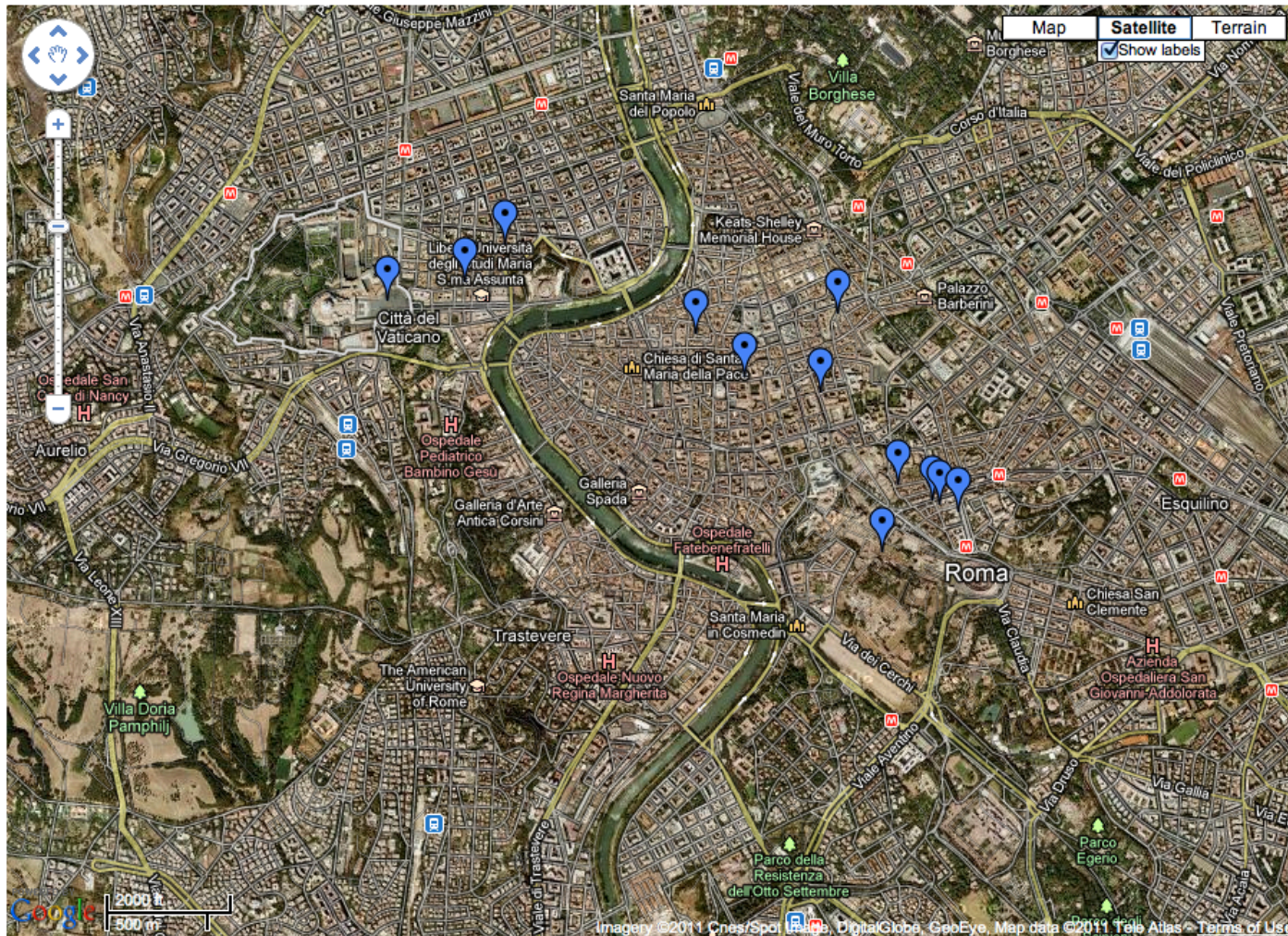
[Building Rome in a Day Sameer Agarwal, Noah Snavely, Ian Simon, Steven M. Seitz and Richard Szeliski International Conference on Computer Vision, 2009, Kyoto, Japan.](#)



<http://grail.cs.washington.edu/rome/>

Rome on a Cloudless Day

Jan-Michael Frahm, Pierre Georgel, David Gallup, Tim Johnson, Rahul Raguram, Changchang Wu, Yi-Hung Jen, Enrique Dunn, Brian Clipp, Svetlana Lazebnik, Marc Pollefeys, *ECCV 2010*



http://www.cs.unc.edu/~jmf/rome_on_a_cloudless_day/

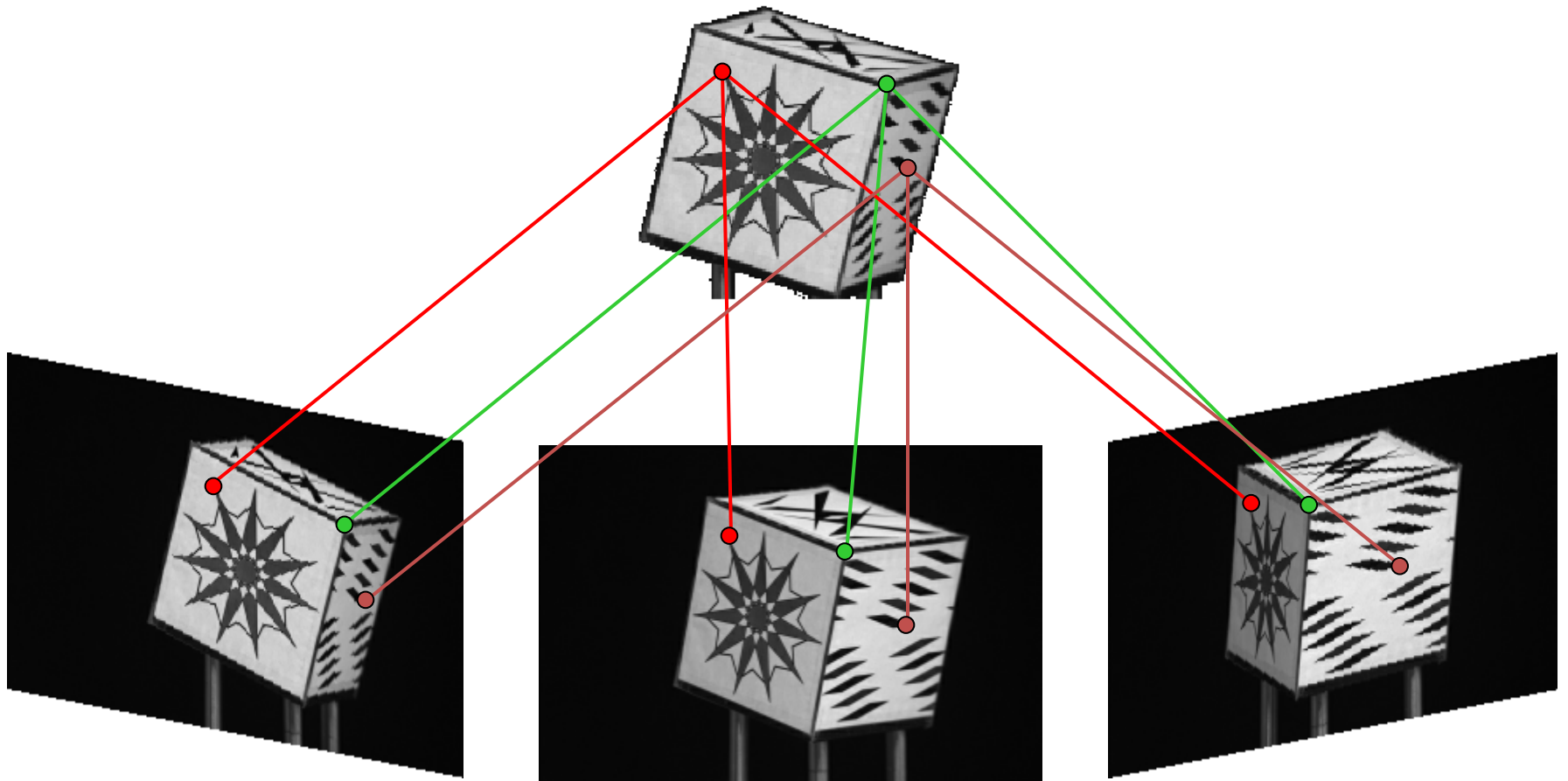
Frank Dellaert Fall 2019

2 Problems !

Correspondence

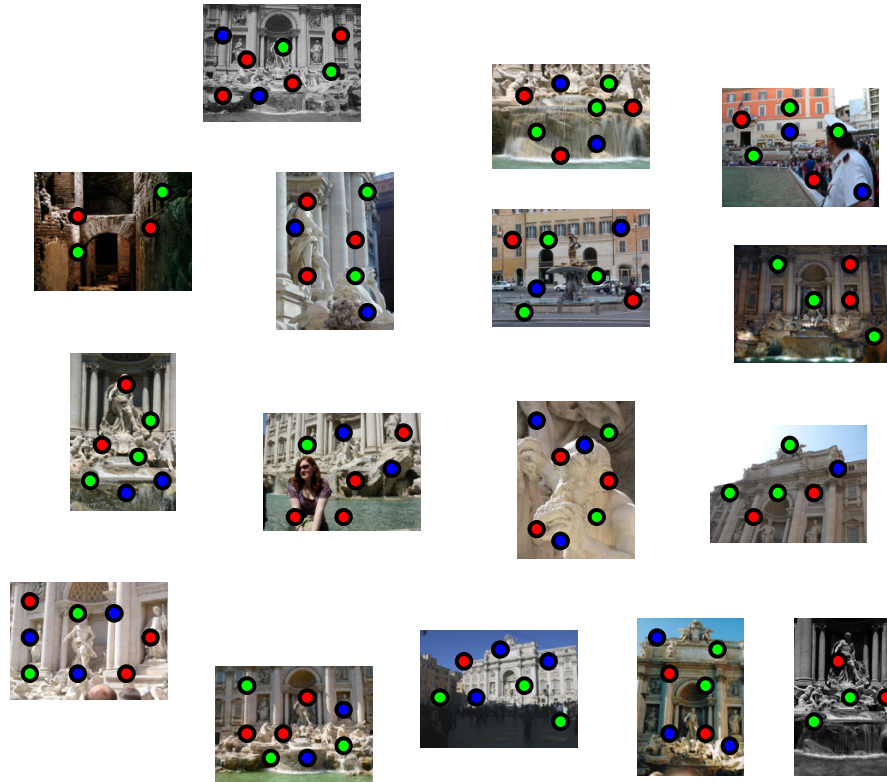
Optimization

A Correspondence Problem



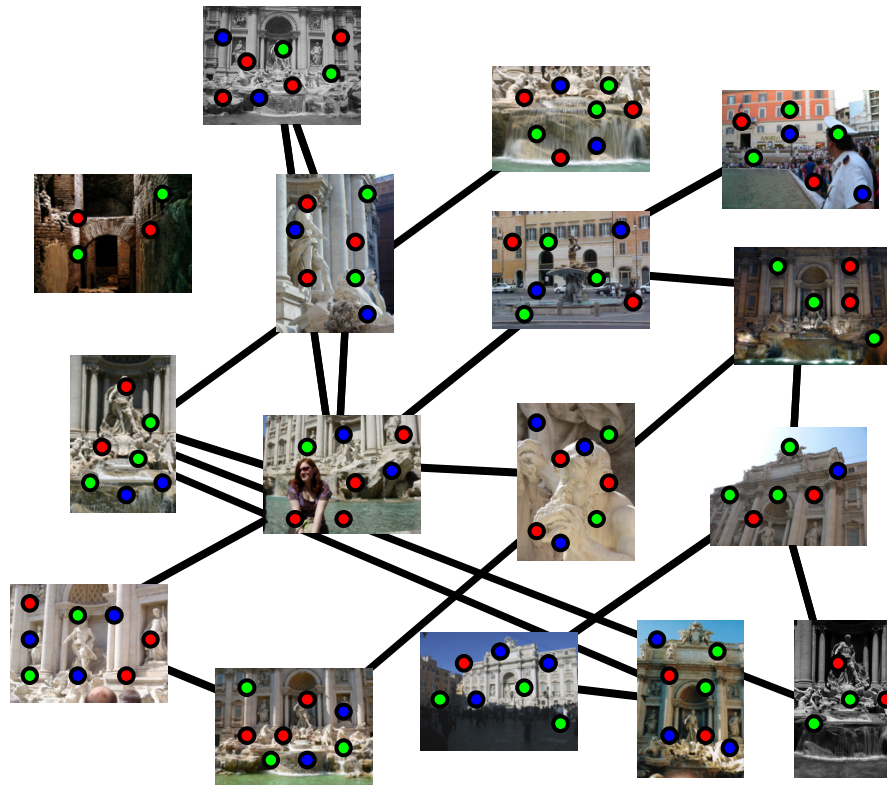
Feature detection

- Detect features using SIFT [Lowe, IJCV 2004]



Feature matching

Refine matching using RANSAC [Fischler & Bolles 1987] to estimate fundamental matrices between pairs



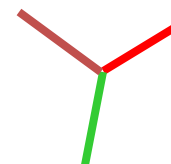
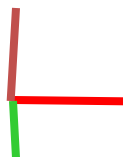
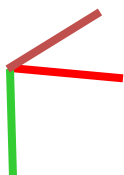
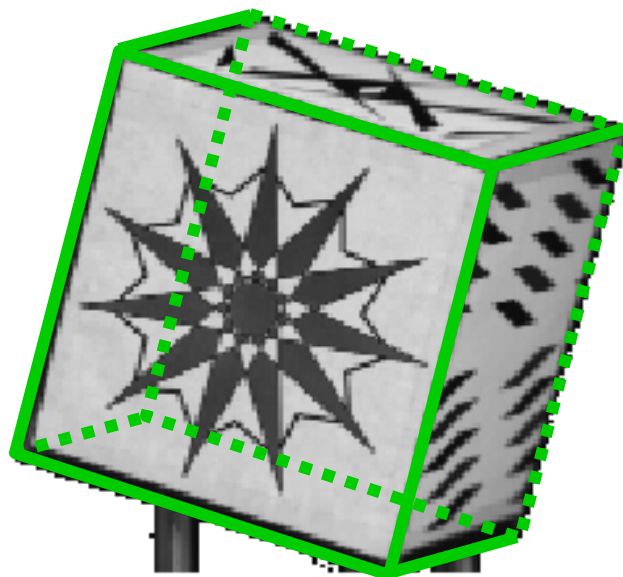
2 Problems !

Correspondence

Optimization

An Optimization Problem

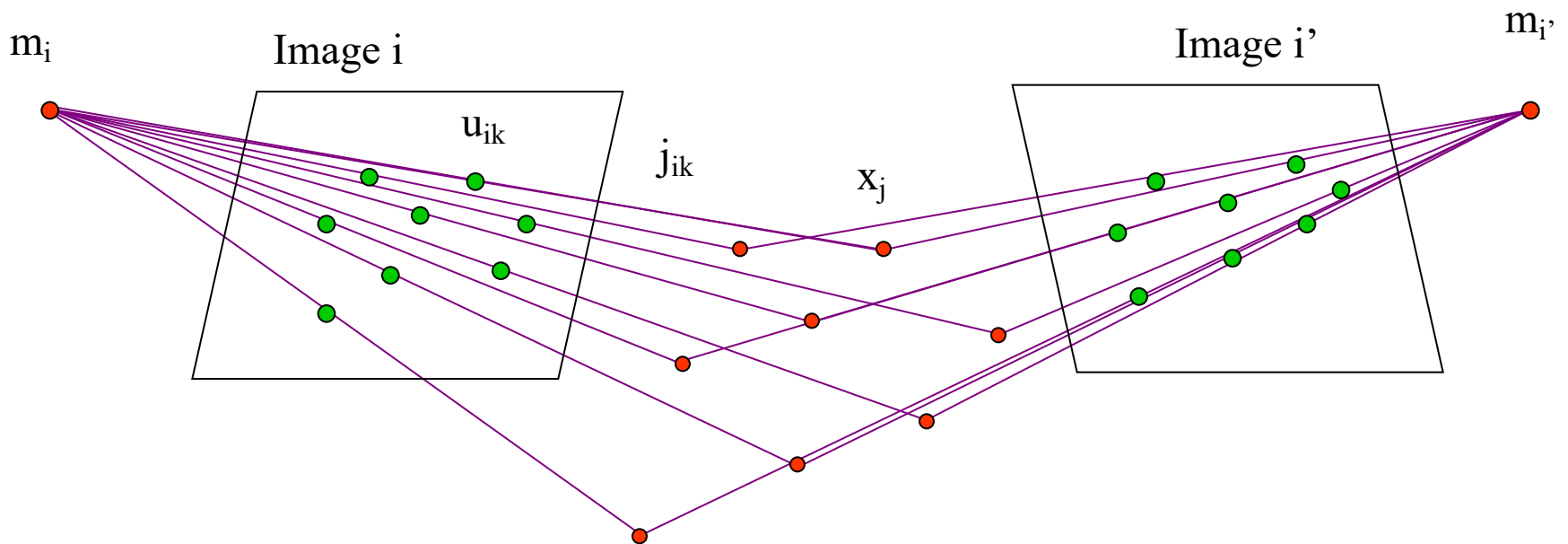
- Find the **most likely** structure and motion Θ



Optimization

=Non-linear Least-Squares !

$$\sum_{i=1}^m \sum_{k=1}^{K_i} \|\mathbf{u}_{ik} - \mathbf{h}(\mathbf{m}_i, \mathbf{x}_{j_{ik}})\|^2$$



Recall: Nonlinear Least Squares

$$E_{NLS} = \sum_i \|f(x_i; p) - x'_i\|^2$$

Linearize around a current guess p :

$$f(x; p + \Delta p) = f(x; p) + J(x; p)\Delta p$$

$$r = x' - f(x; p) = J(x; p)\Delta p$$

$$E_{NLS} = \sum_i \|f(x; p) + J(x; p)\Delta p - x'_i\|^2 = \sum_i \|J(x; p)\Delta p - r_i\|^2$$

Differentiate and set to 0:

$$2 \sum_i J^T(x_i; p) (J(x_i; p)\Delta p - r_i) = 0$$

Normal equations — $\left[\sum_i J^T(x_i; p) J(x_i; p) \right] \Delta p = \sum_i J^T(x_i; p) r_i$

$$A\Delta p = b$$

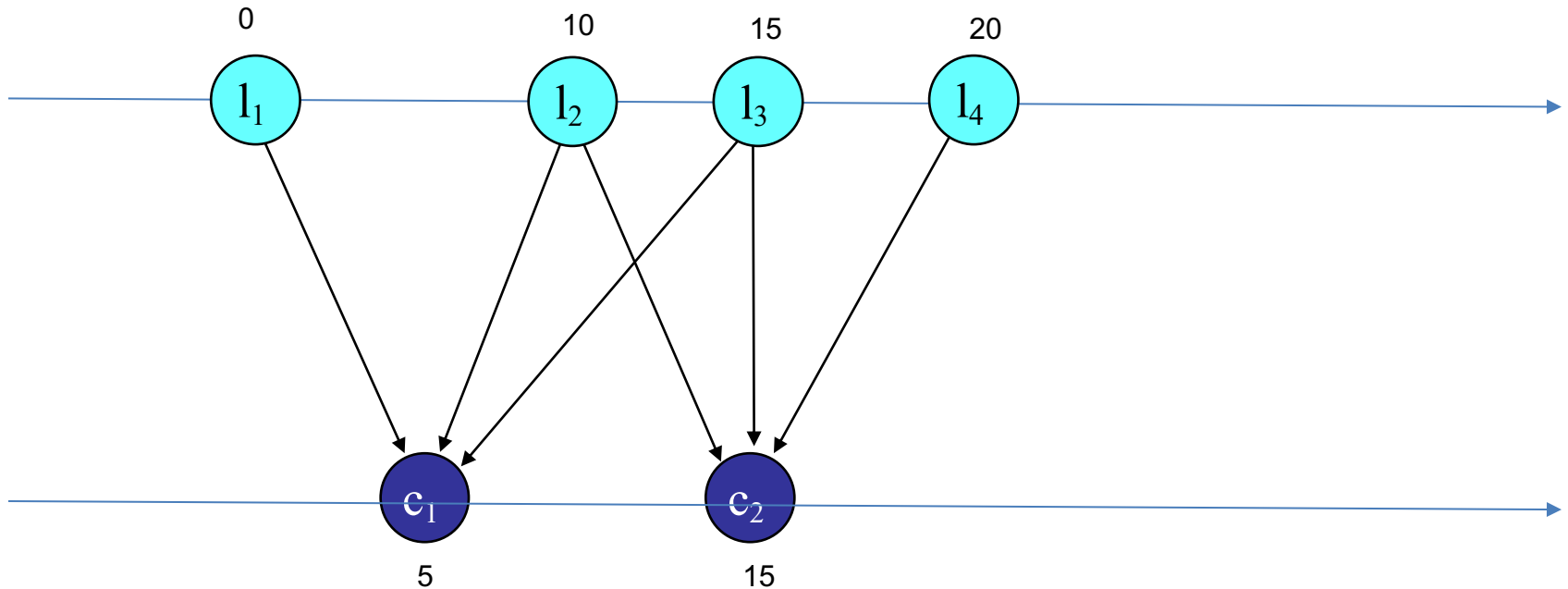
$$\Delta p^* = A^{-1}b$$

Hessian

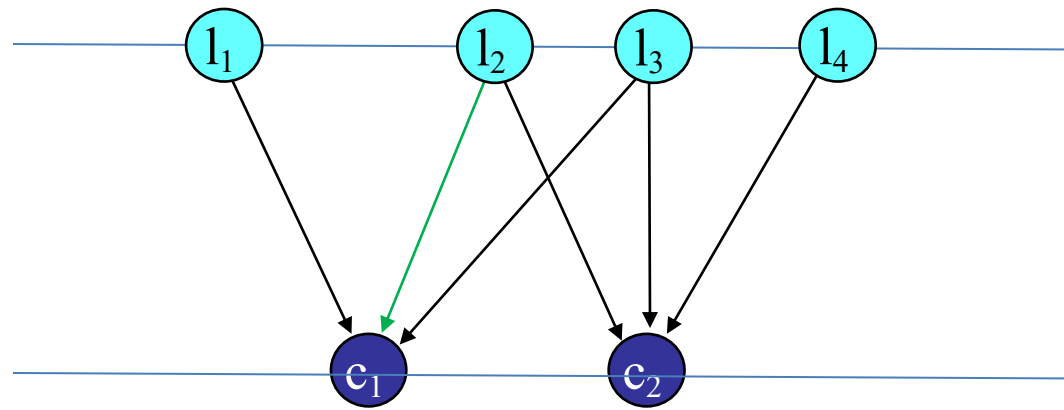
Jacobian

Sparse nonlinear least squares

- Simple 1-Dimensional Example
- $p = 2$ cameras and 4 points $\{c_1, c_2, l_1, l_2, l_3, l_4\}$
- $f(u_{ik}; p) = \text{difference in } x \text{ position} = l_{j(i,k)} - c_i$



Sparse Jacobian and Hessian



$A =$

	c1	c2	l1	l2	l3	l4
1	1	0	0	0	0	0
-1	-1	0	1	0	0	0
-1	-1	0	0	1	0	0
-1	0	0	0	0	1	0
0	0	-1	1	0	0	0
0	0	-1	0	1	0	0
0	0	-1	0	0	1	0
0	0	-1	0	0	0	1

$b =$

5
-5
5
10
-15
-5
0
5



$A' * A$

	c1	c2	l1	l2	l3	l4
4	4	0	-1	-1	-1	0
0	0	4	-1	-1	-1	-1
-1	-1	-1	2	0	0	0
-1	-1	-1	0	2	0	0
-1	-1	-1	0	0	2	0
0	0	-1	0	0	0	1

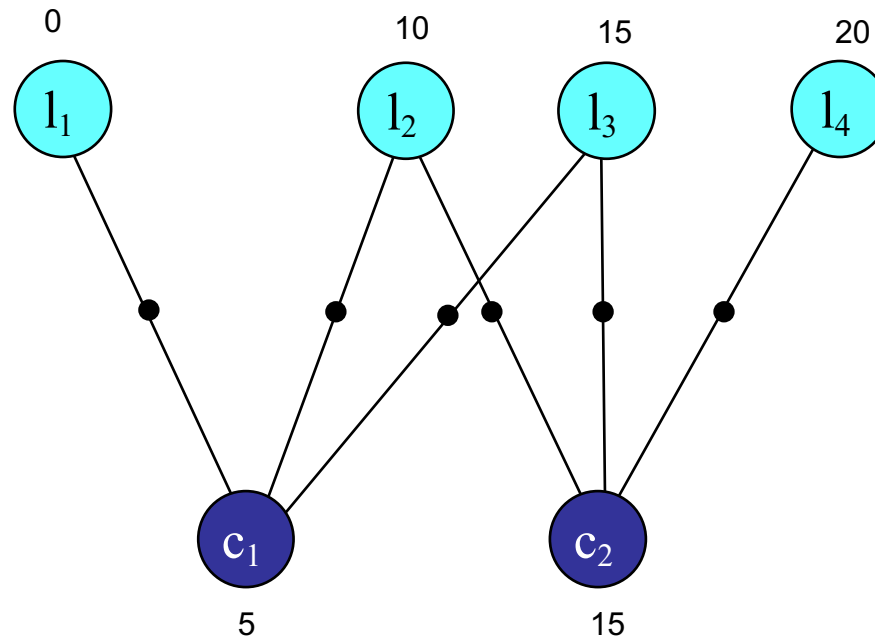
$(A' * A) \setminus A' * b =$

5.0000
15.0000
0.0000
10.0000
15.0000
20.0000



A general formalism: Factor Graphs

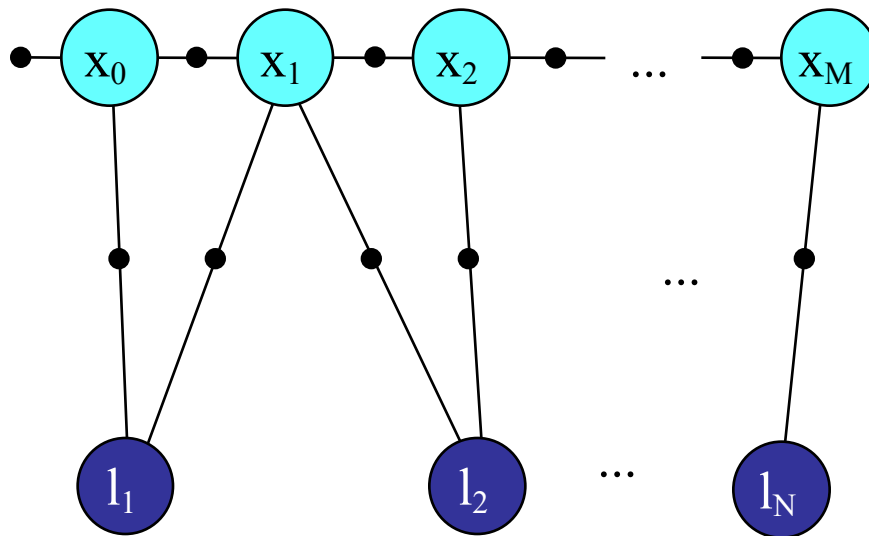
- Bipartite graph
- Two types of nodes:
 - Unknowns
 - Factors: correspond to squared errors
- Connectivity = sparsity! Factor is function of small set.



SLAM: Simultaneous Localization and Mapping



SLAM Factor Graph



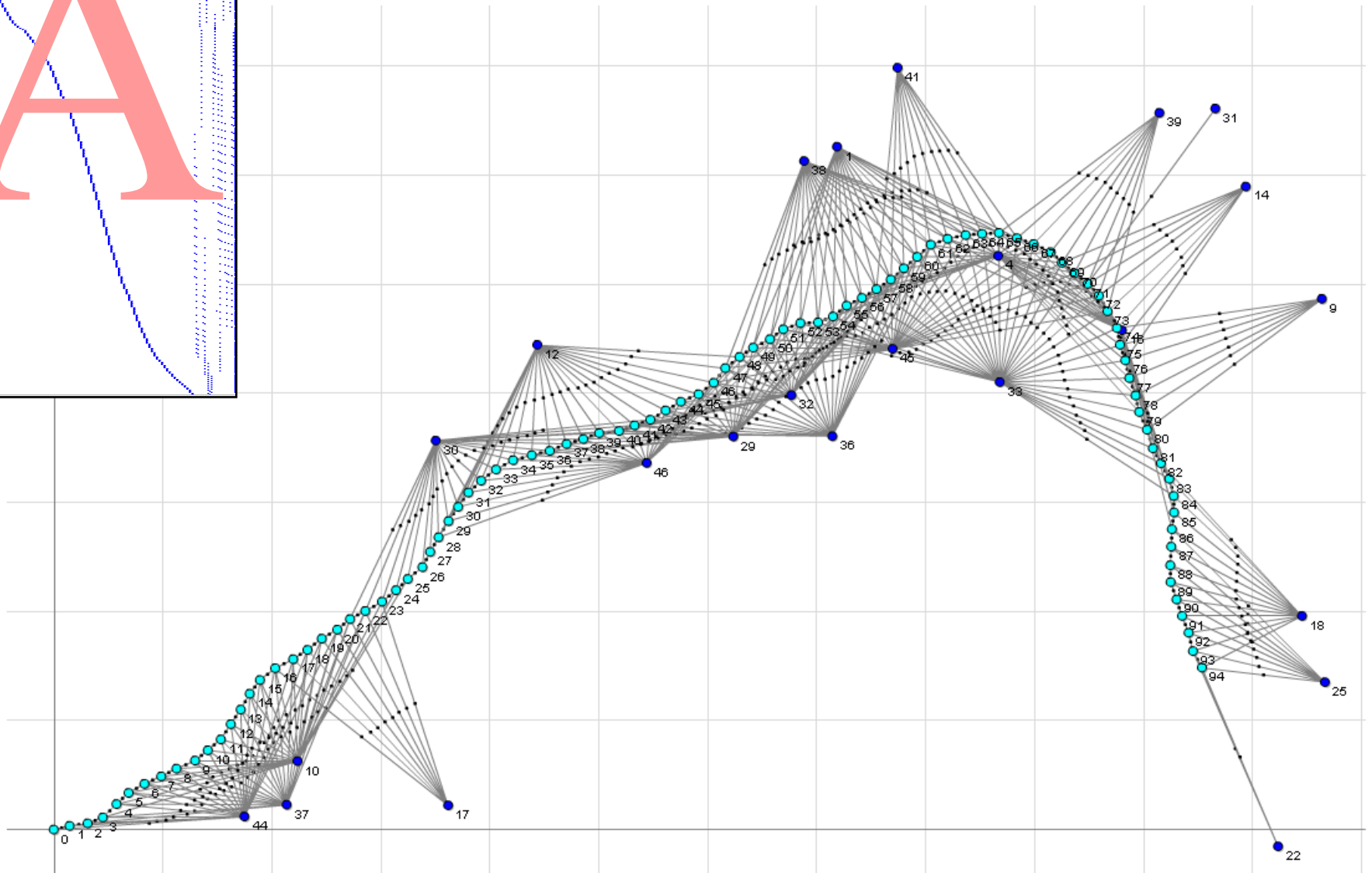
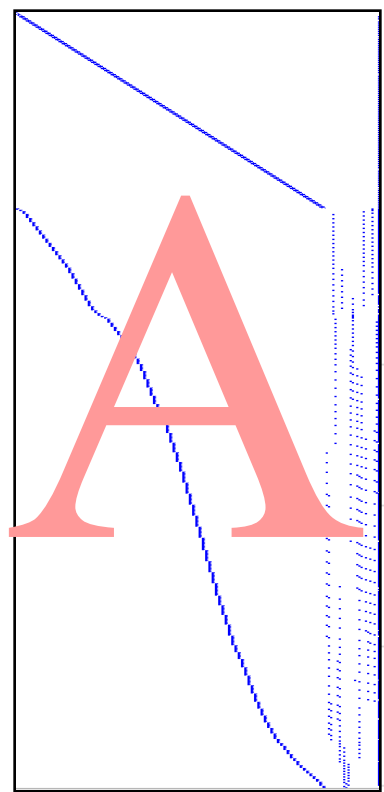
- Trajectory of Robot

- Landmark Measurements

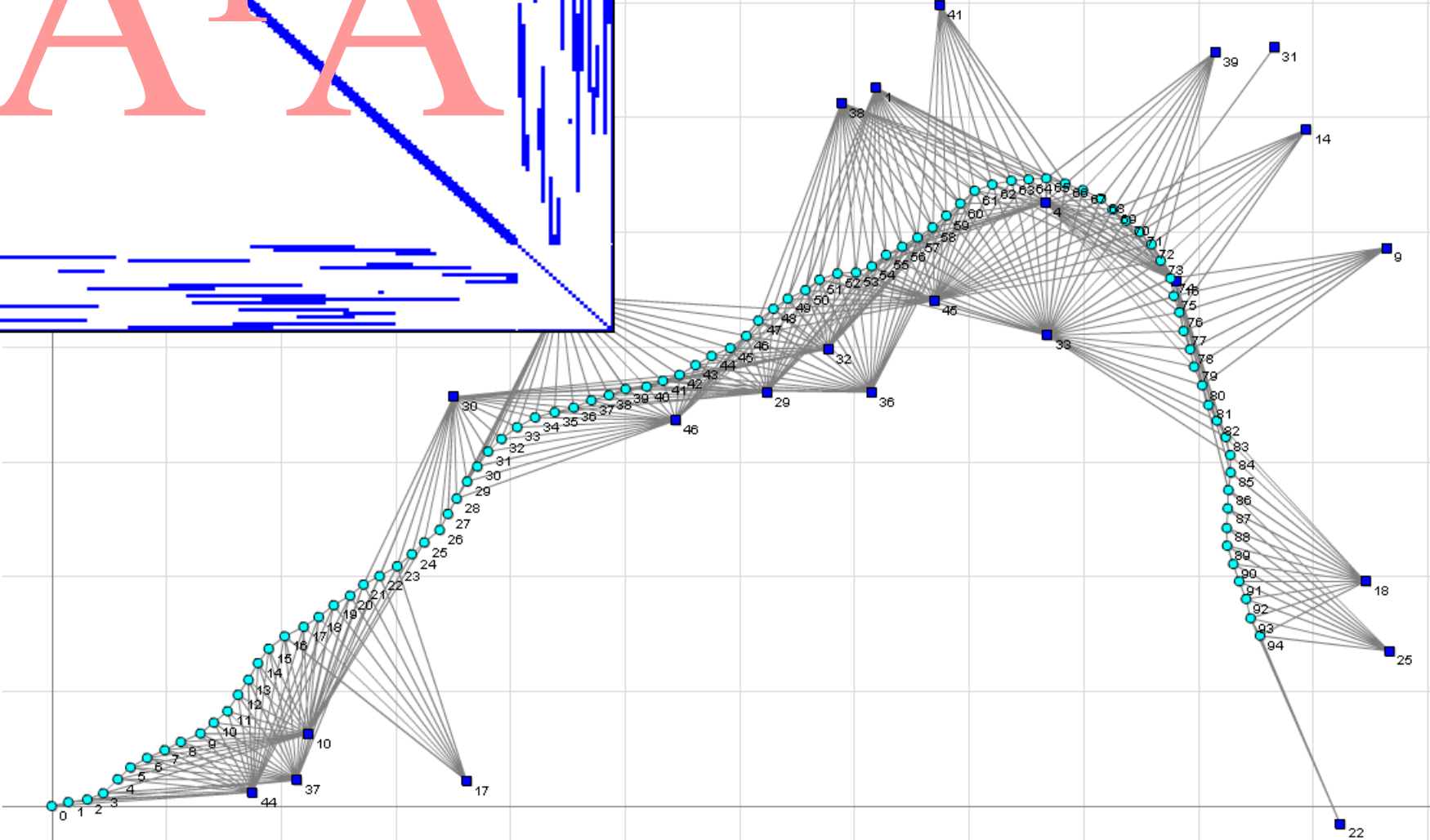
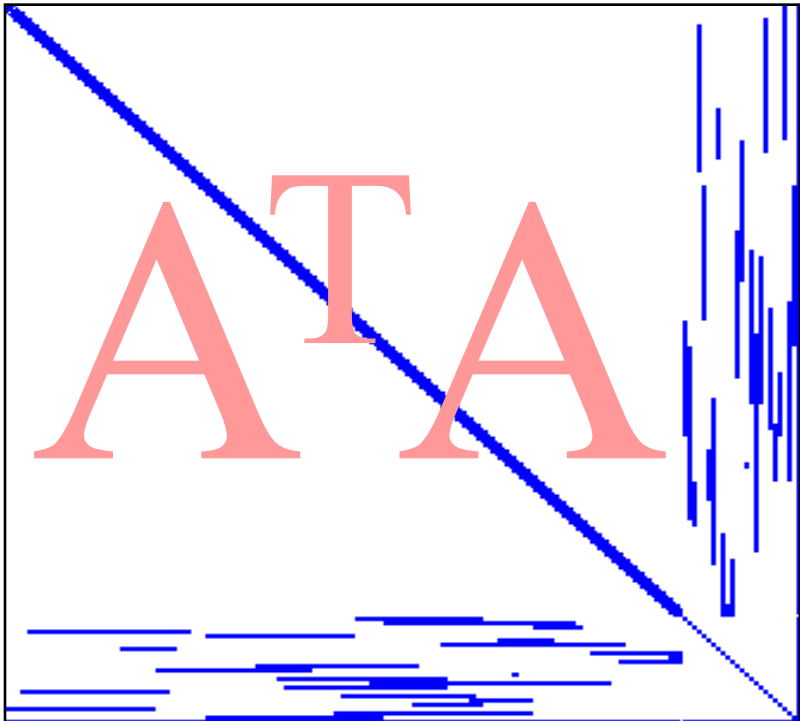
- “Landmarks”

$$P(X, M) = k^* P(x_0) \prod_{i=1}^M P(x_i | x_{i-1}, u_i) \times \prod_{k=1}^K P(z_k | x_{i_k}, l_{j_k})$$

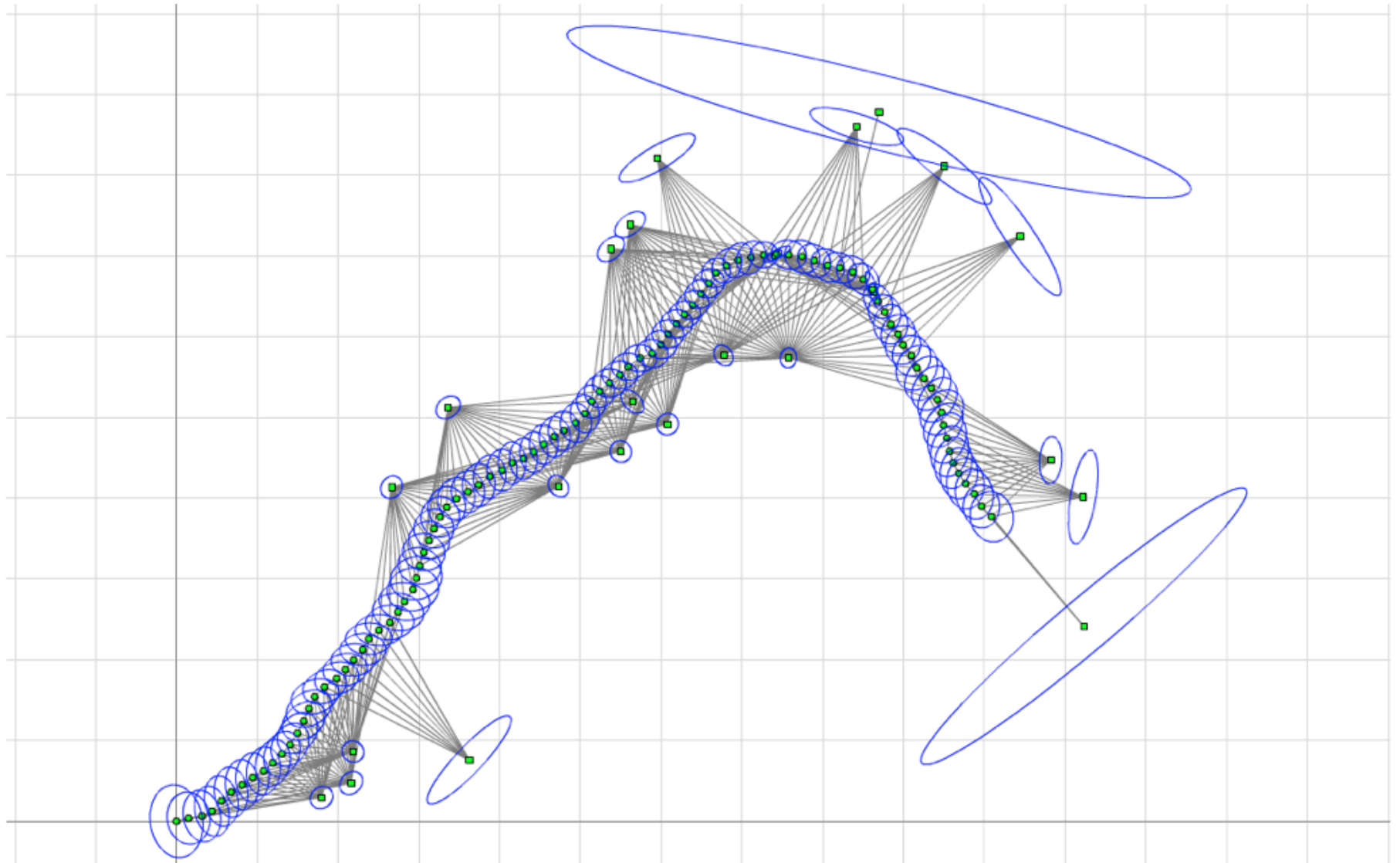
SLAM Factor Graph



Hessian



End result: Solution + Sigma

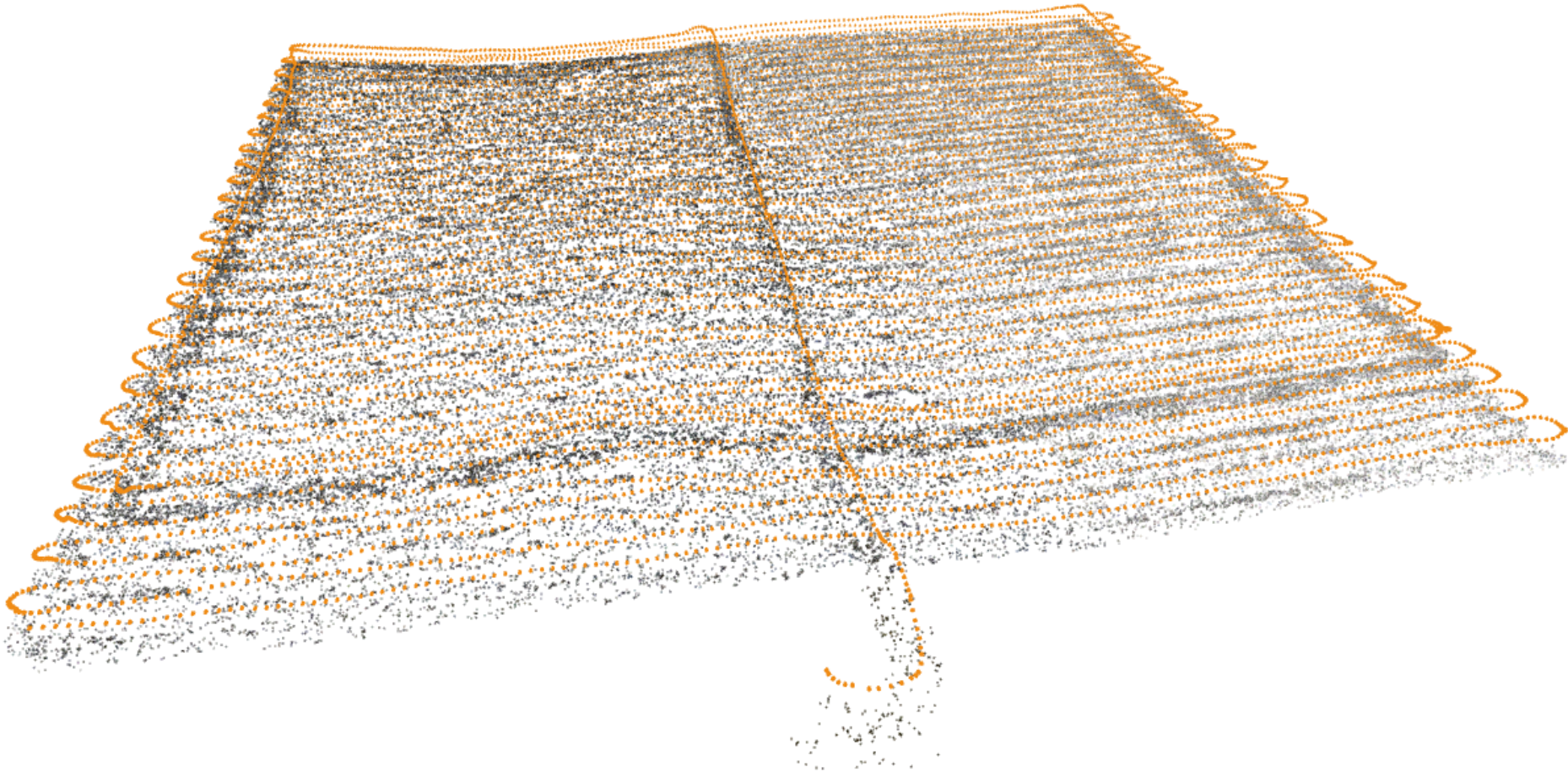


Example:
Victoria Park,
Sidney



Example: Underwater SLAM

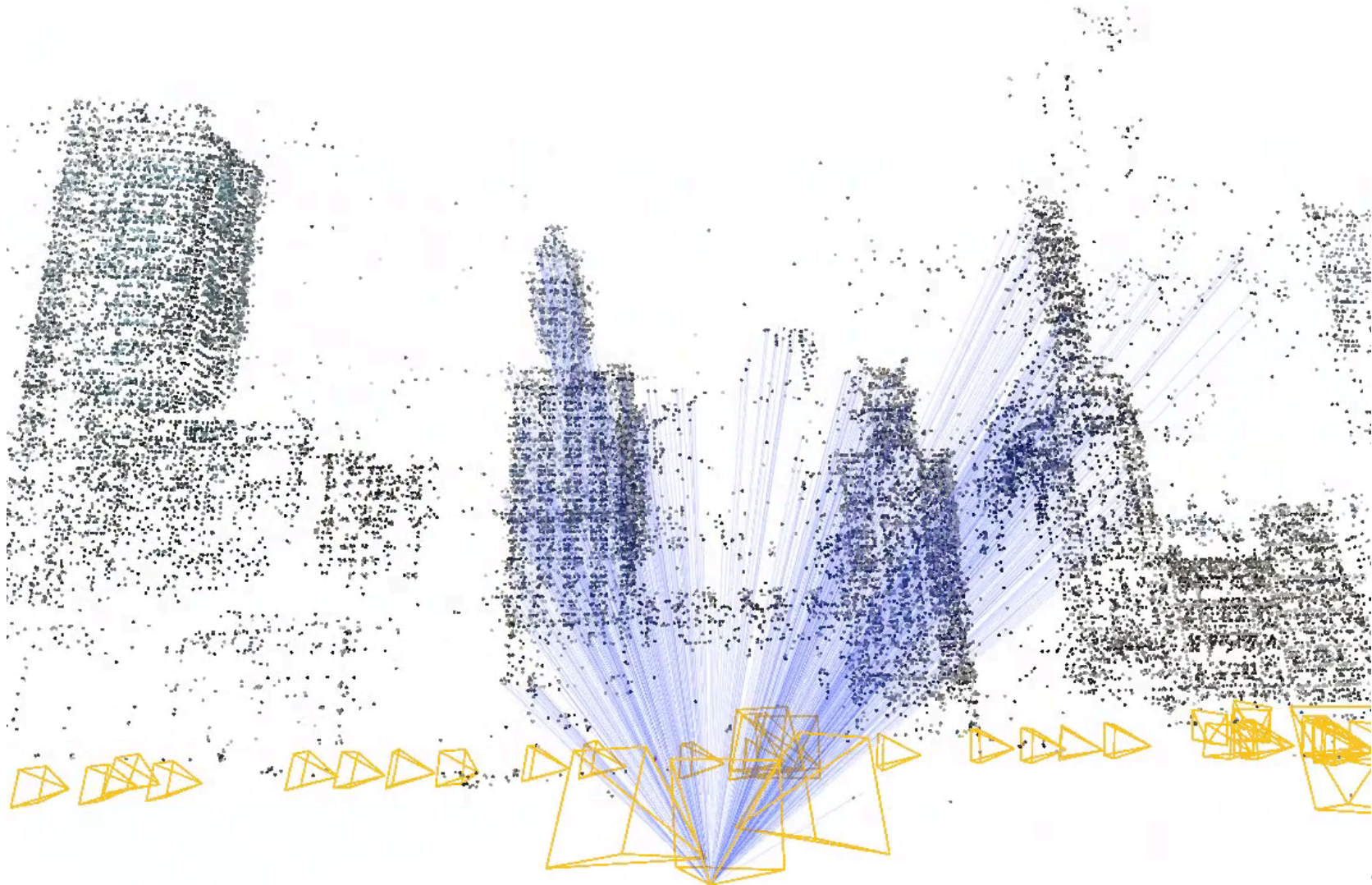
9831 camera poses, 185261 landmarks, and 350988 factors



Structure from Motion (Chicago, movie by Yong Dian Jian)

180 cameras, 88723 points
458642 projections
active camera: 4

Original graph



3D Models from Community Databases

- E.g., Google image search on “Dubrovnik”

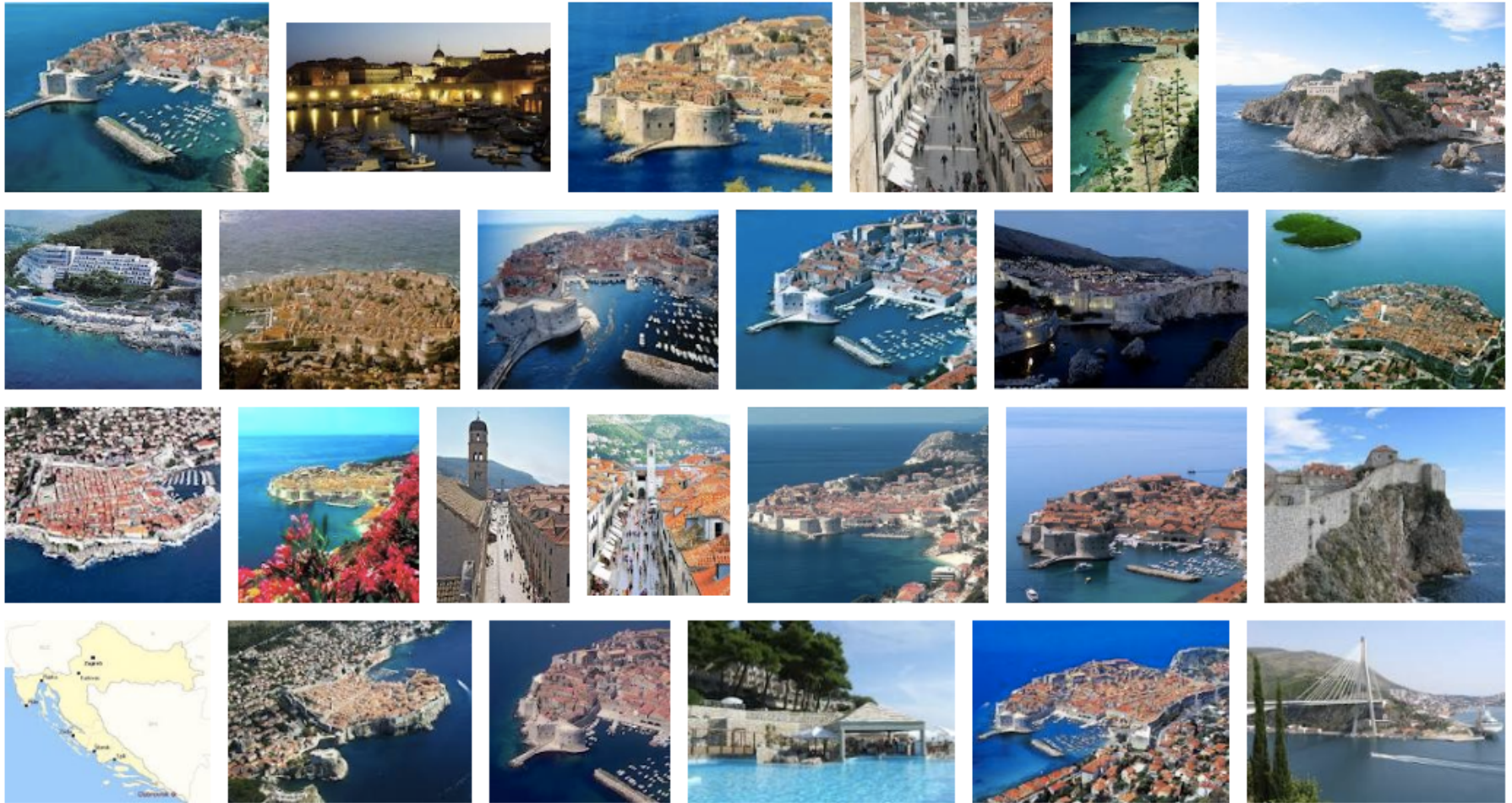


Figure by Aggarwal et al.

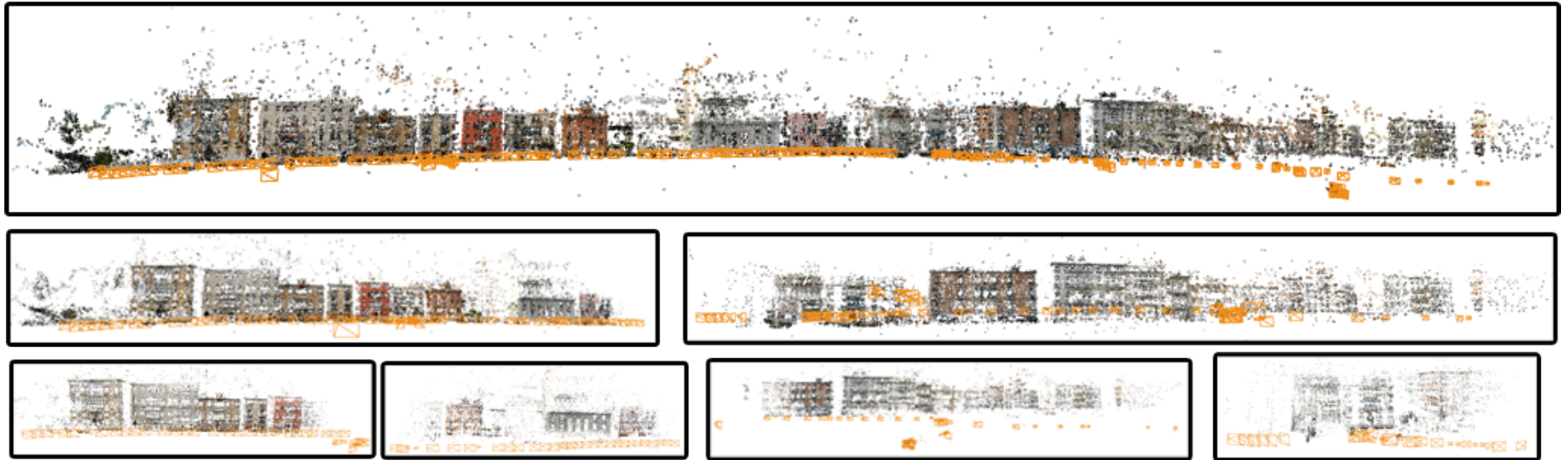
3D Models from Community Databases

Agarwal, Snavely, Seitz et al. at UW <http://grail.cs.washington.edu/rome/>



5K images, 3.5M points, >10M factors

Hyper-SfM: Efficient Multi-core



Kai now leads an autonomous driving startup in China

[Kai Ni](#), and [Frank Dellaert](#), [HyperSfm](#), [IEEE International Conference on 3D Imaging, Modeling, Processing, Visualization and Transmission \(3DIMPVT\)](#), 2012.

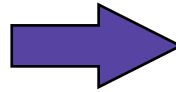
4D Reconstruction

Spatiotemporal Reconstruction

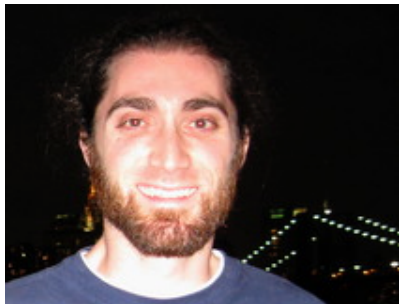
4D Cities: 3D + Time



Historical Image Collection



Supported by NSF CAREER, Microsoft
Recent revival: NSF NRI award on 4D
crops for precision agriculture...



Grant Schindler

4D Reconstruction of Lower Manhattan

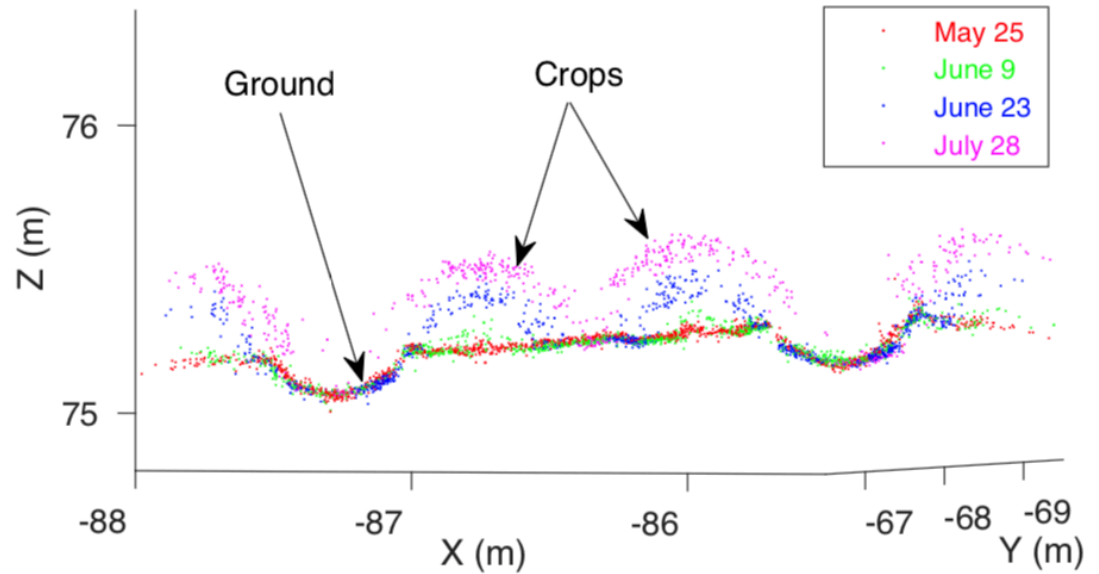
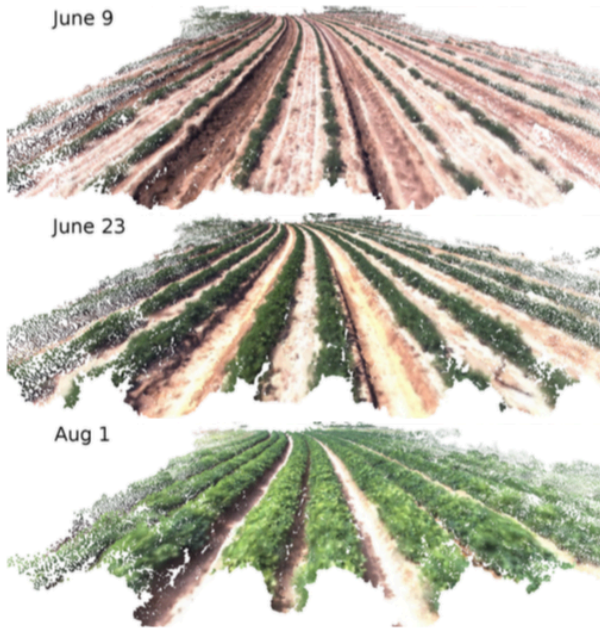


[Probabilistic Temporal Inference on Reconstructed 3D Scenes](#), G. Schindler and F. Dellaert, IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR), 2010.

4D Structure over Time



4D crop monitoring (Jing Dong)



Results: video (by Jing Dong)

May 25, 2016



May 25, 2016



4D reconstruction results (by PMVS)
and its cross section