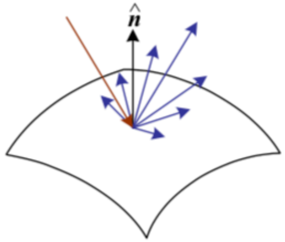
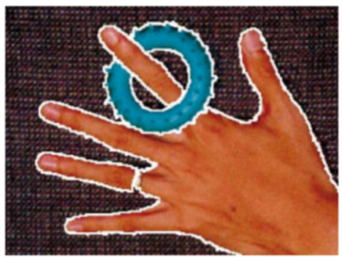


2. Image Formation

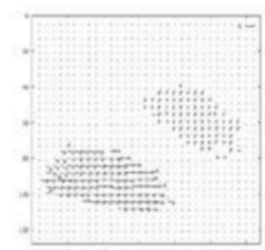
3. Image Processing

4. Features

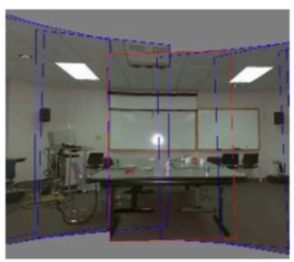


5. Segmentation

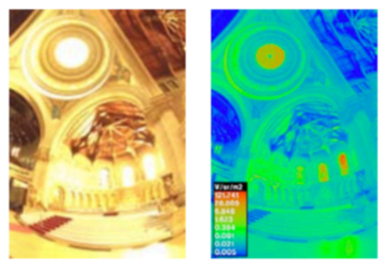
6-7. Structure from Motion



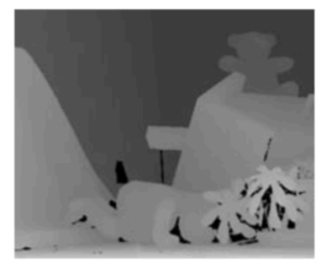
8. Motion



9. Stitching

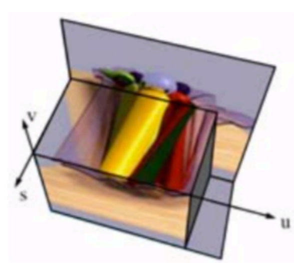


10. Computational Photography



11. Stereo

12. 3D Shape



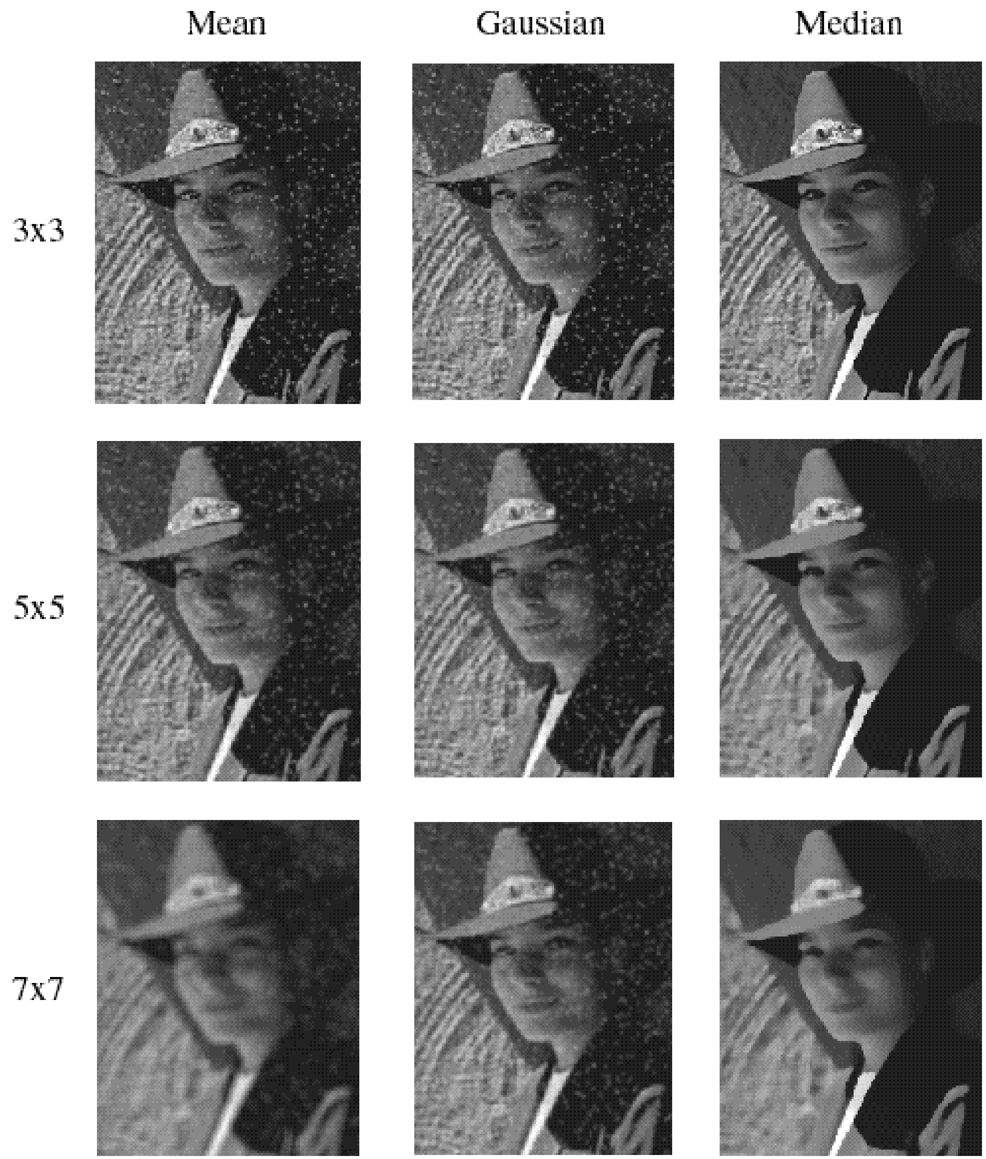
13. Image-based Rendering

14. Recognition

Median filters

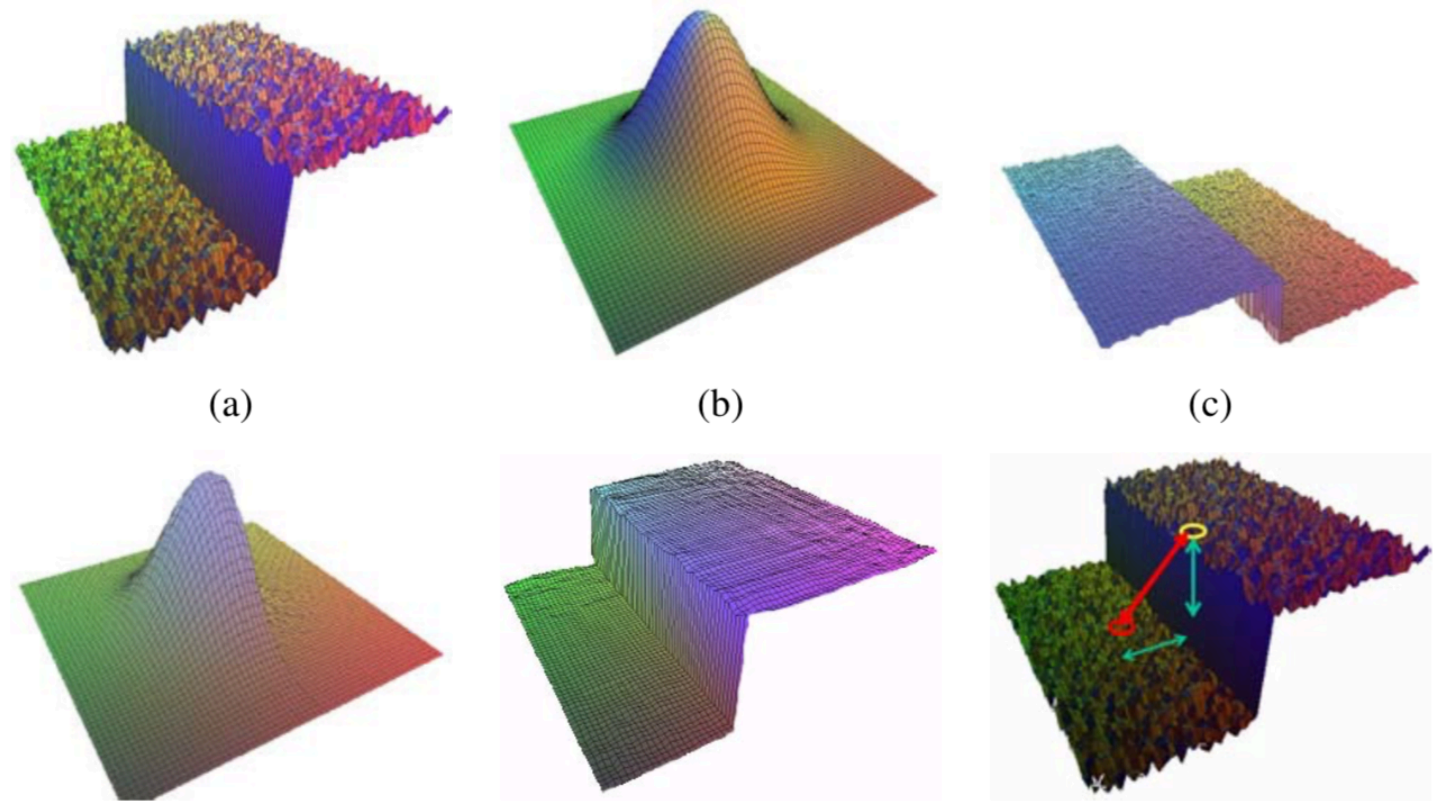
- A **Median Filter** operates over a window by selecting the median intensity in the window.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

Comparison: salt and pepper noise



Slide by Steve Seitz

Bilateral filtering



(a) (b) (c)

(d) (e) (f)

Figure 3.20 Bilateral filtering (Durand and Dorsey 2002) © 2002 ACM: (a) noisy step edge input; (b) domain filter (Gaussian); (c) range filter (similarity to center pixel value); (d) bilateral filter; (e) filtered step edge output; (f) 3D distance between pixels.

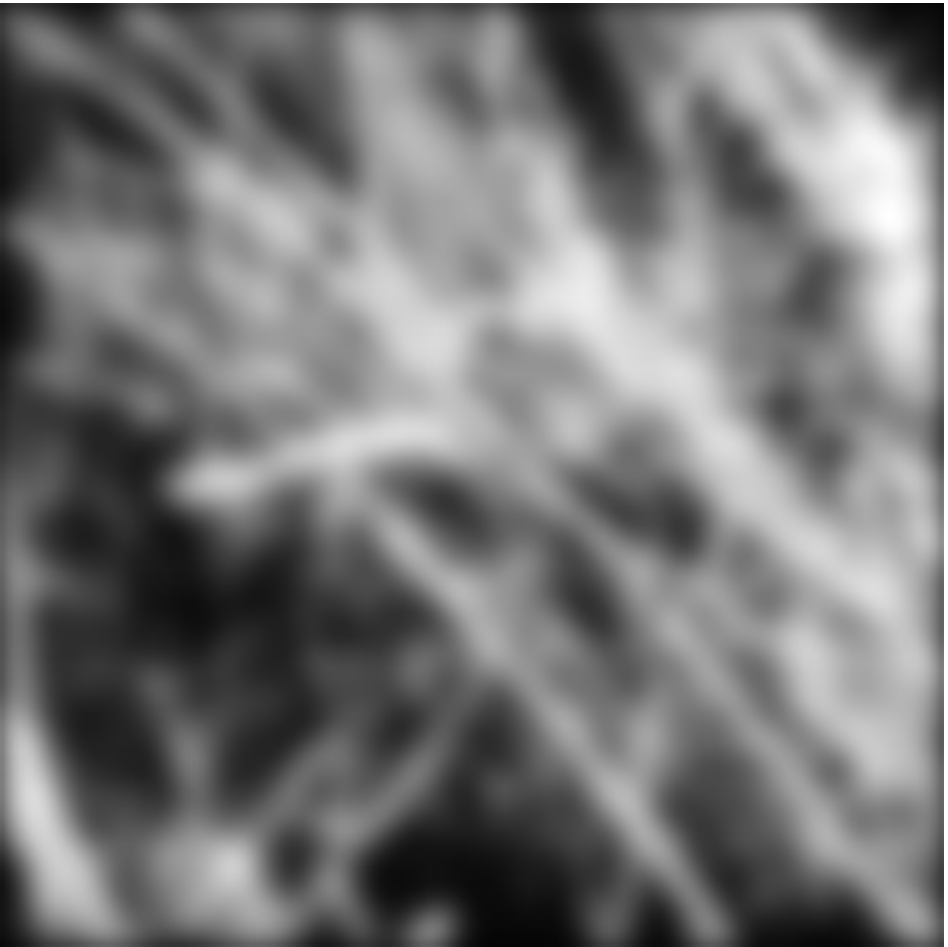
Morphological Operators

(a) (b) (c) (d) (e) (f)

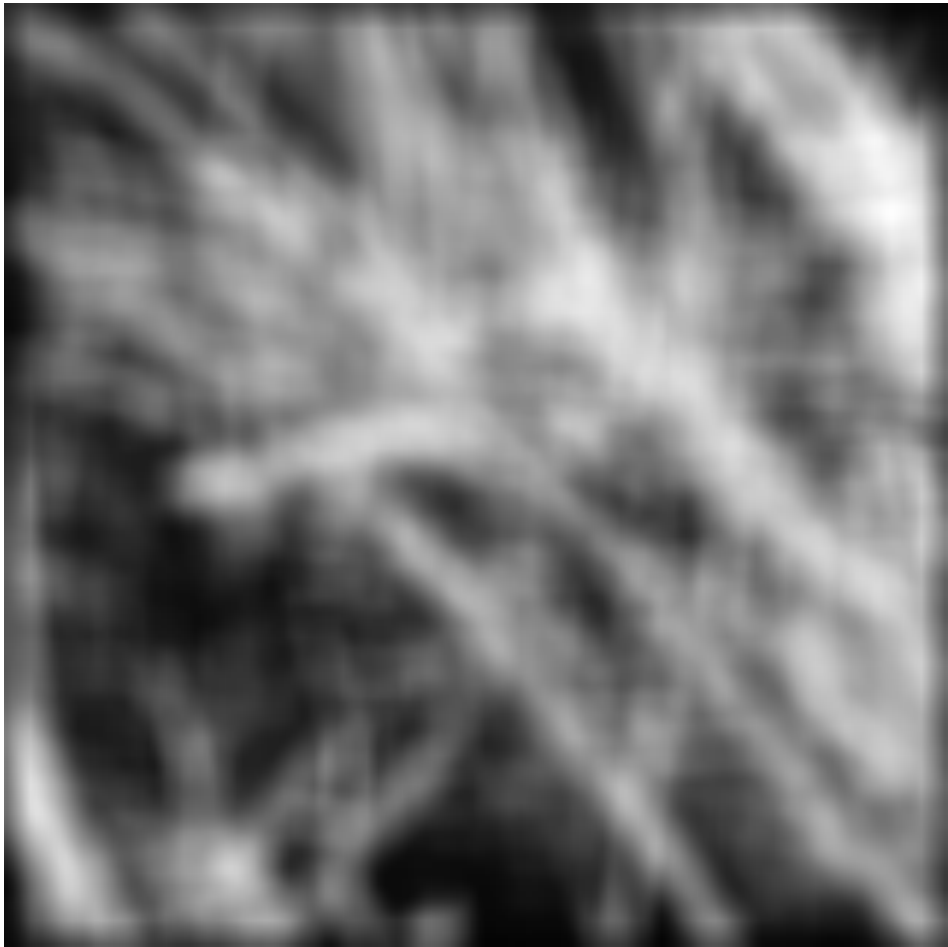
Figure 3.21 Binary image morphology: (a) original image; (b) dilation; (c) erosion; (d) majority; (e) opening; (f) closing. The structuring element for all examples is a 5×5 square. The effects of majority are a subtle rounding of sharp corners. Opening fails to eliminate the dot, since it is not wide enough.

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

Gaussian 



Box filter 



Why does a lower resolution image still make sense to us? What do we lose?

Image: http://www.flickr.com/photos/igorms/136916757/

Thinking in Frequency

Fourier, Joseph (1768-1830)

French mathematician who discovered that any periodic motion can be written as a superposition of sinusoidal and cosinusoidal vibrations. He developed a mathematical theory of heat in *Théorie Analytique de la Chaleur (Analytic Theory of Heat)*, (1822), discussing it in terms of differential equations.

Fourier was a friend and advisor of Napoleon. Fourier believed that his health would be improved by wrapping himself up in blankets, and in this state he tripped down the stairs in his house and killed himself. The paper of Galois which he had taken home to read shortly before his death was never recovered.

SEE ALSO: Galois

Additional biographies: MacTutor (St. Andrews), Bonn

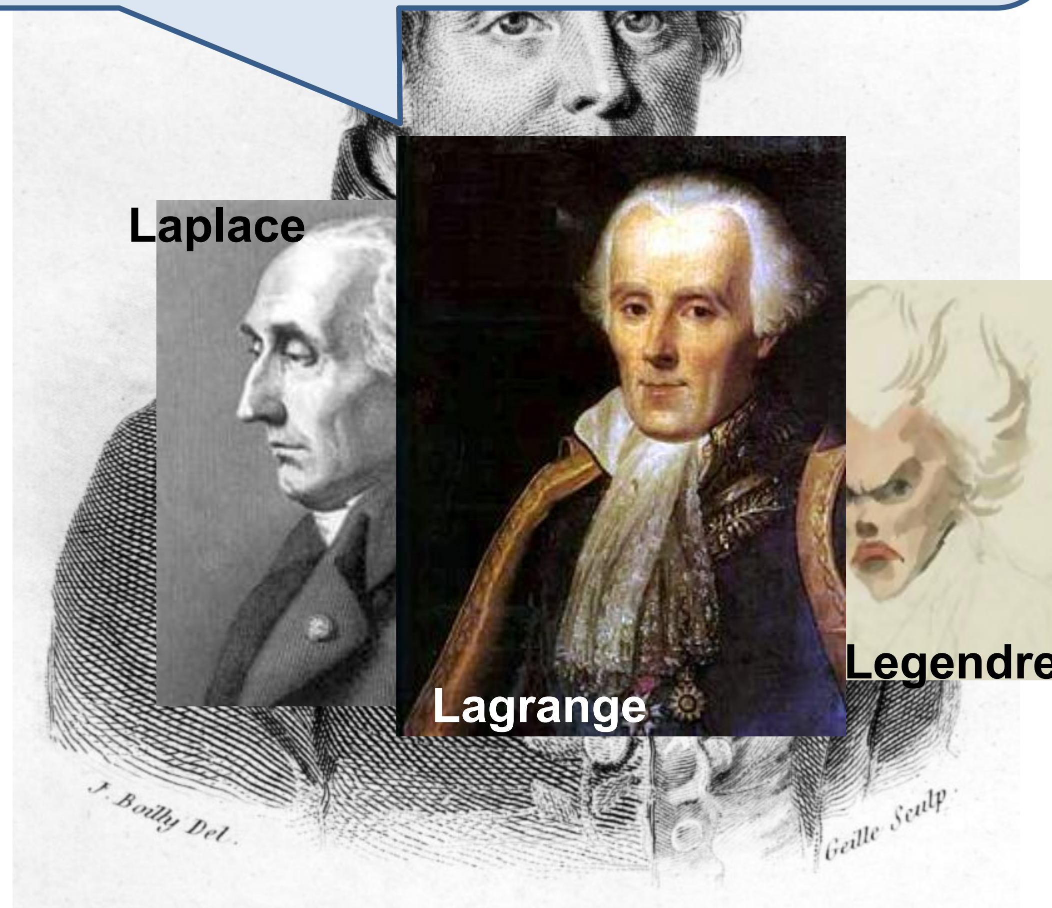
Jean Baptiste Joseph Fourier (1768-1830)

had crazy idea (1807):

> ***Any*** *univariate function can*
> *rewritten as a weighted sum*
> *sines and cosines of different*
> *frequencies.*

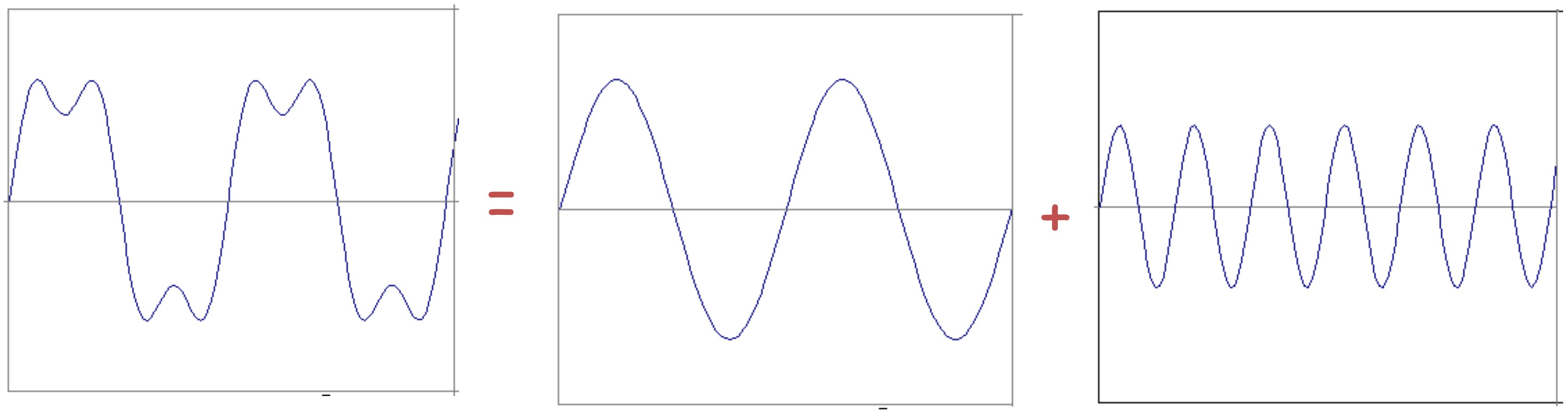
...the manner in which the author arrives at these equations is not exempt of difficulties and...his analysis to integrate them still leaves something to be desired on the score of generality and even rigour.

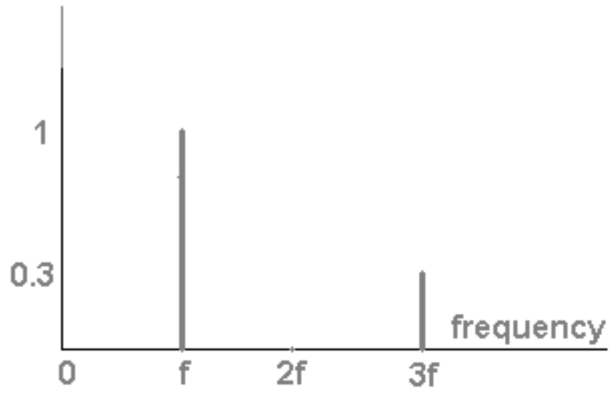
- Don't believe it?
  - Neither did Lagrange, Laplace, Poisson and other big wigs
  - Not translated into English until 1878!
- But it's (mostly) true!
  - called Fourier Series
  - there are some subtle restrictions



Frequency Spectra

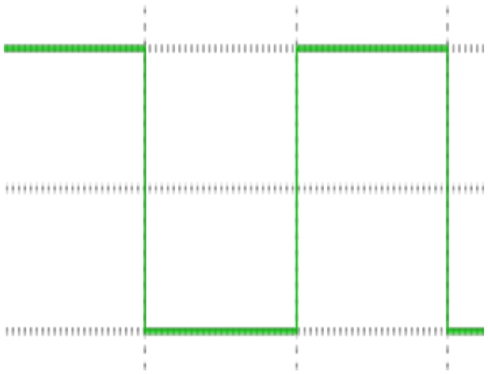
- example : $g(t) = \sin(2\pi f\, t) + (1/3)\sin(2\pi(3f)\, t)$



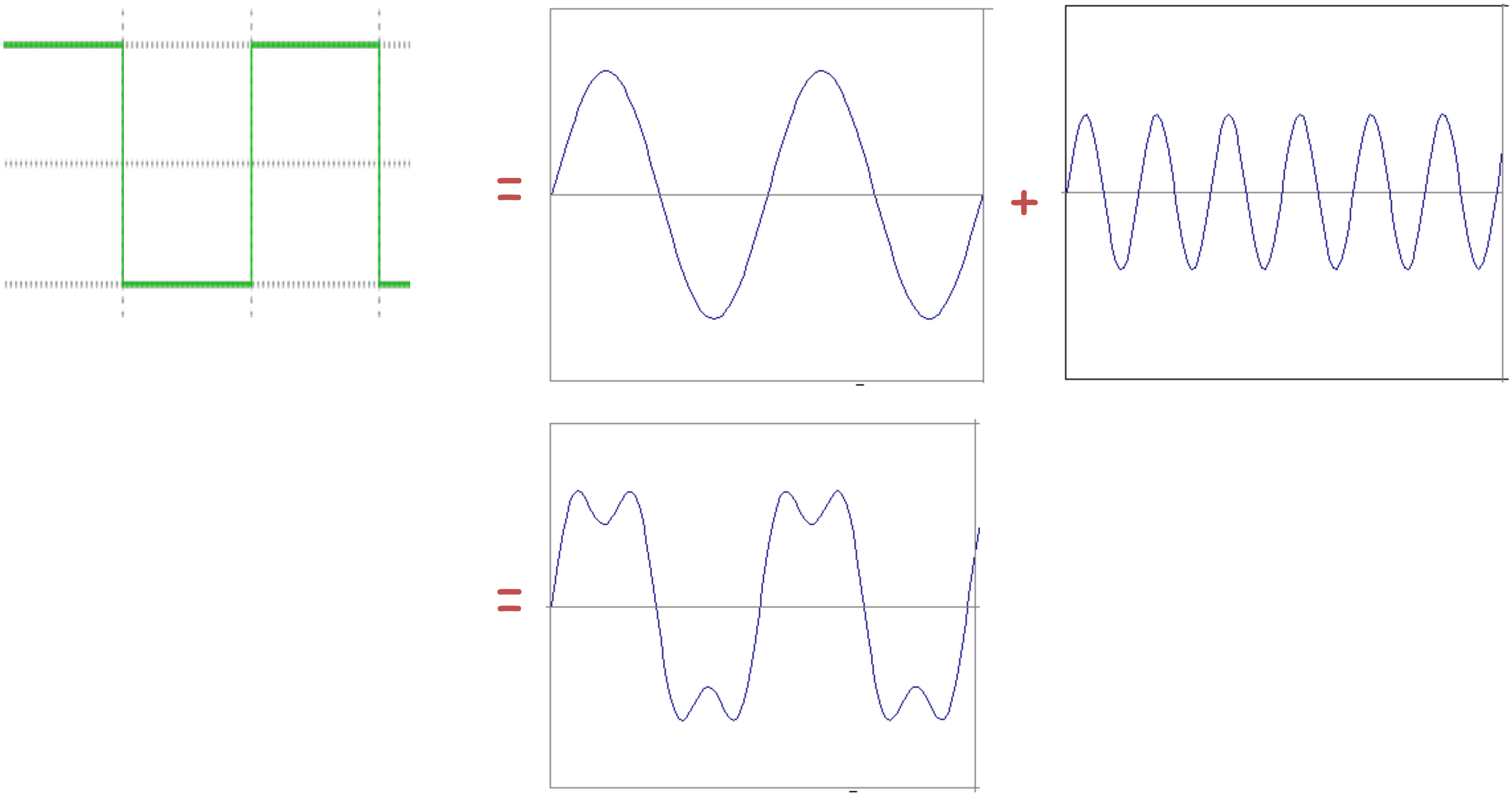


Slides: Efros

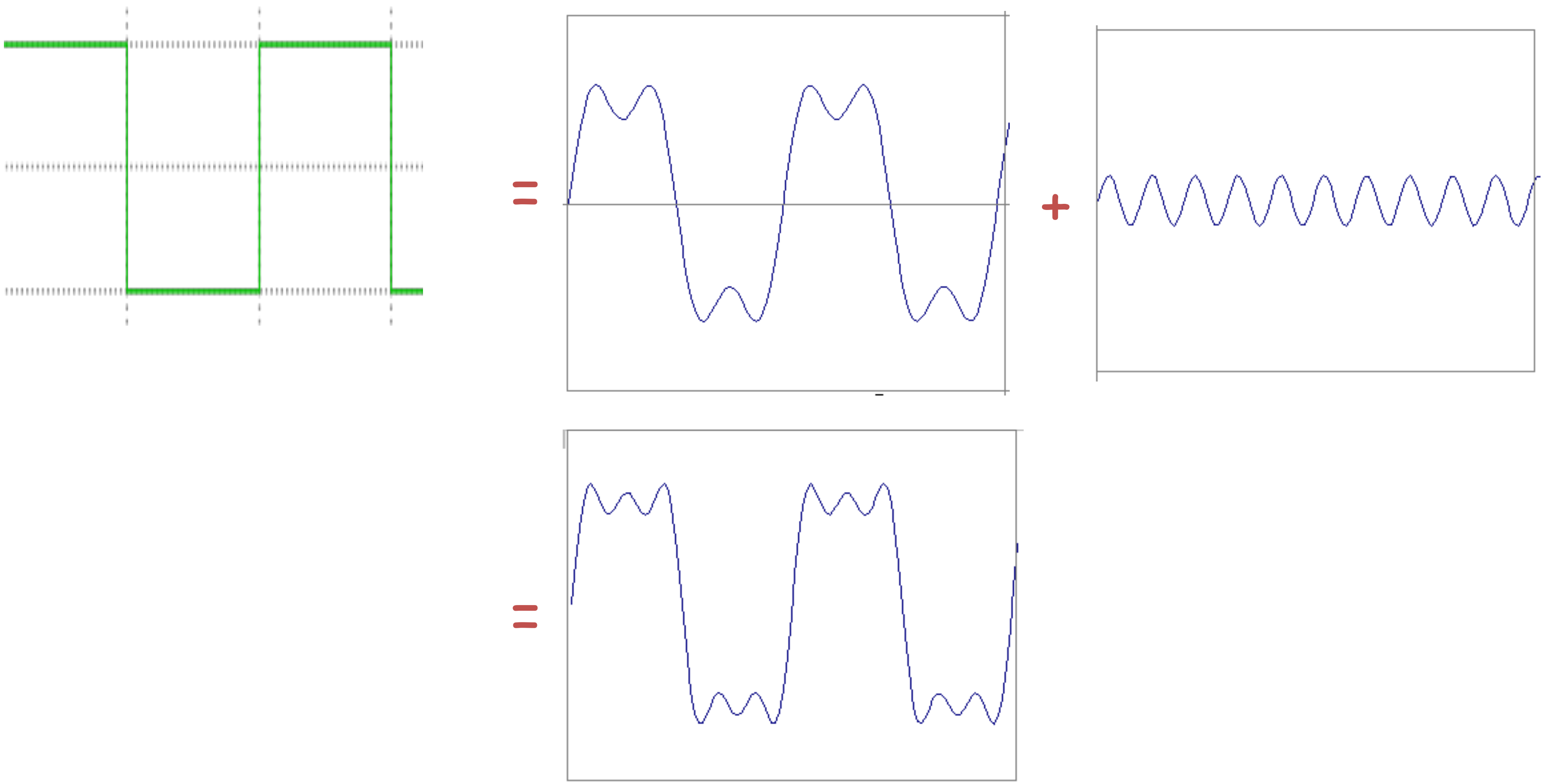
Frequency Spectra



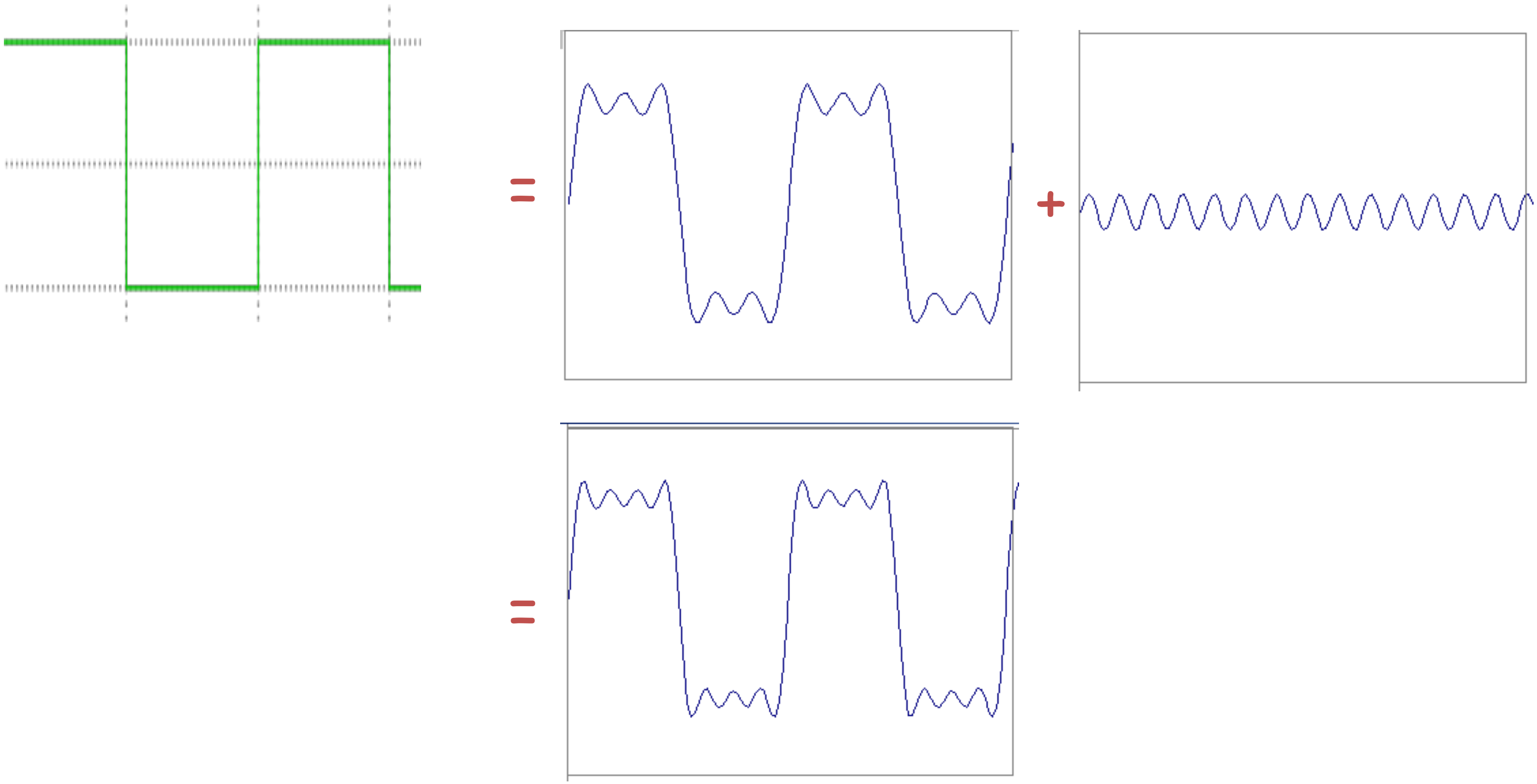
Frequency Spectra



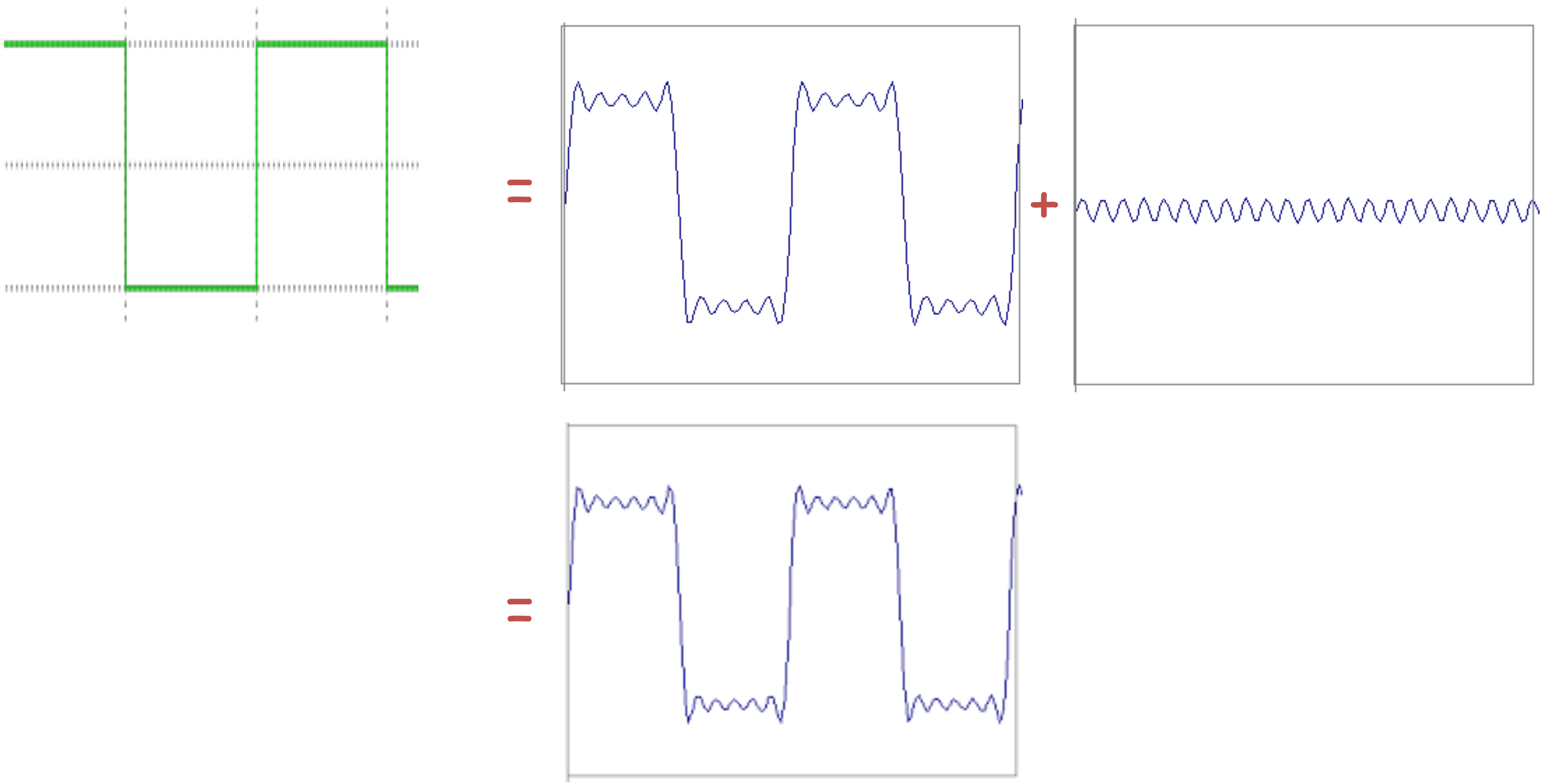
Frequency Spectra



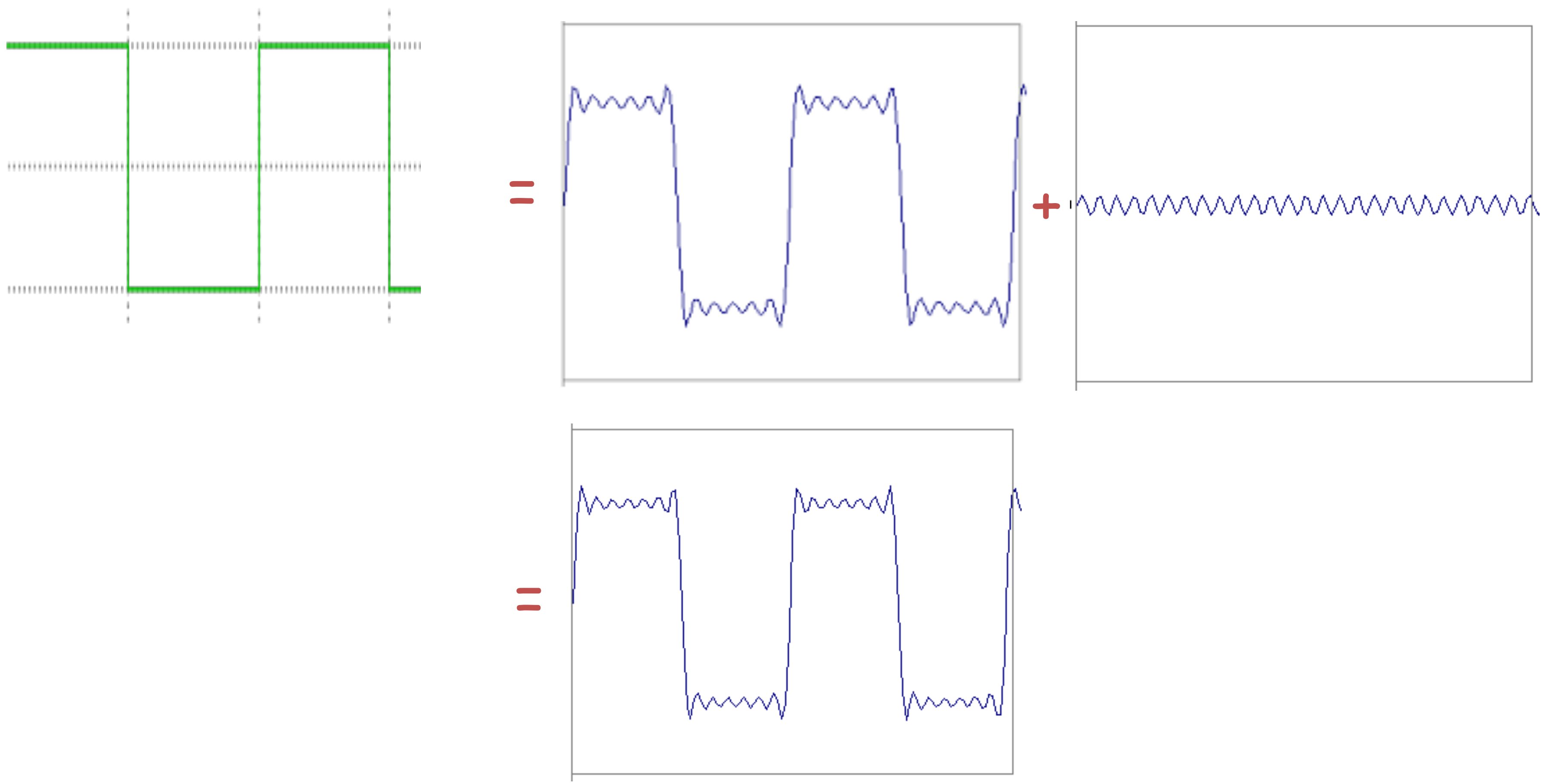
Frequency Spectra



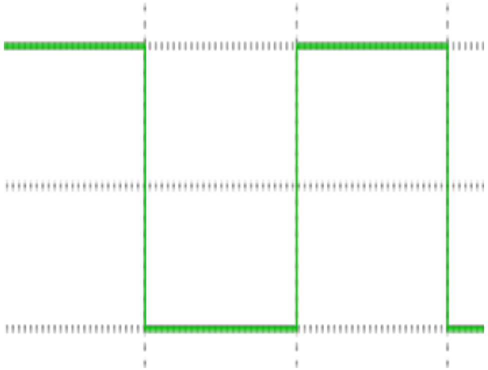
Frequency Spectra



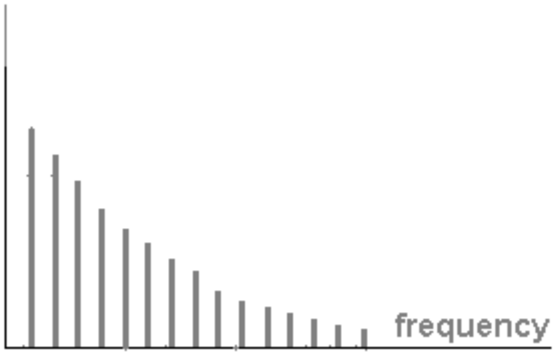
Frequency Spectra



Frequency Spectra

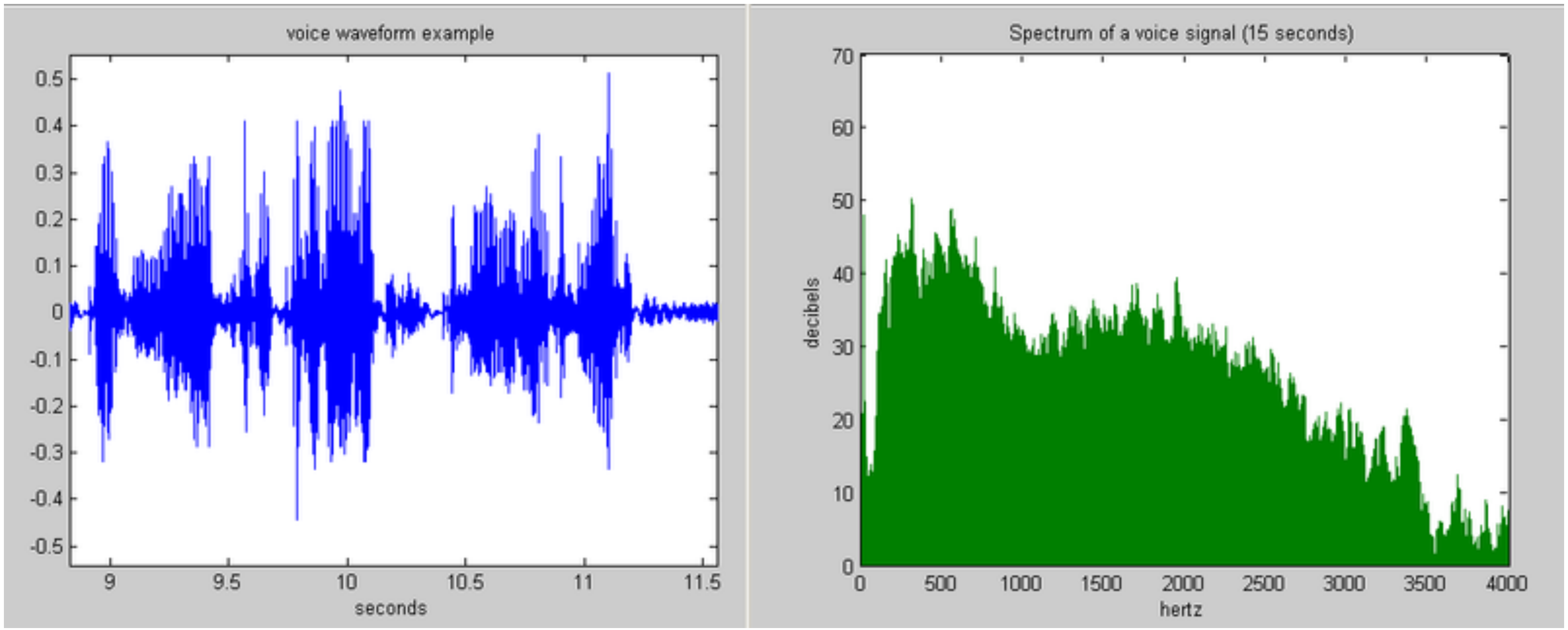


$$= \boxed{A\sum_{k=1}^{\infty}\frac{1}{k}\sin(2\pi kt)}$$



Example: Music

- We think of music in terms of frequencies at different magnitudes

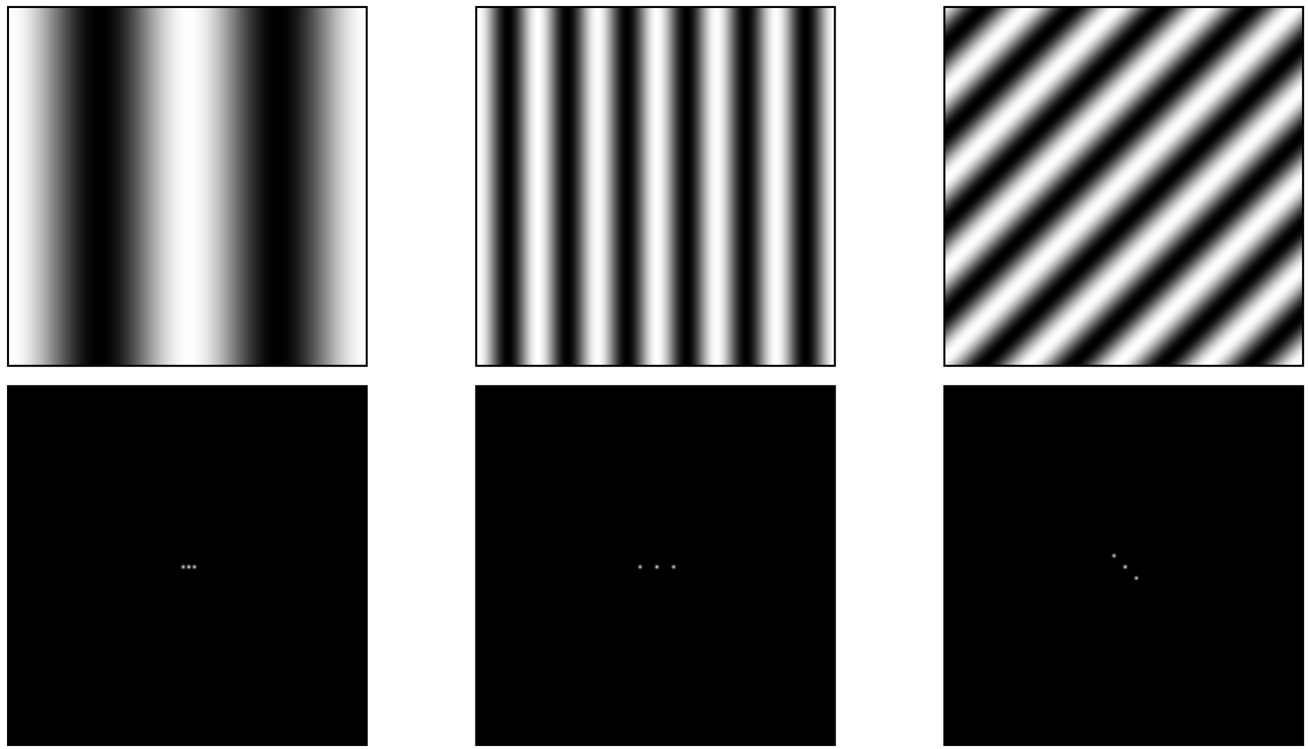


Slide: Hoiem

Fourier analysis in images

Intensity Image

Fourier Image



http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering

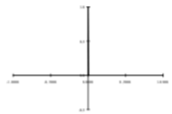
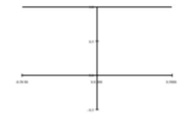
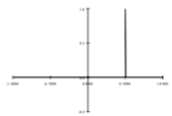
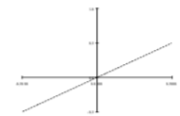
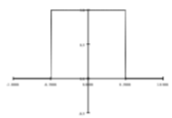
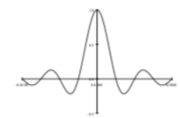
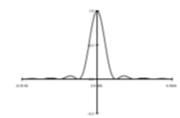
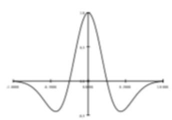
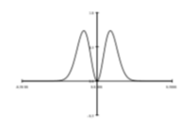
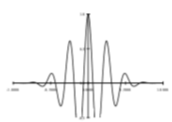
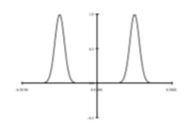
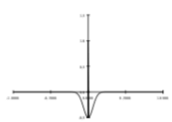
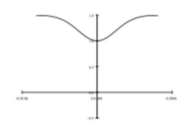
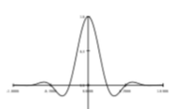
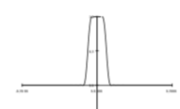
Fourier Transform

- Fourier transform stores the **magnitude** and **phase** at each frequency
  - **Magnitude** encodes how much signal there is at a particular frequency
  - **Phase** encodes spatial information (indirectly)
  - For mathematical convenience, this is often notated in terms of real and complex numbers

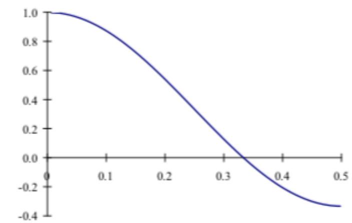
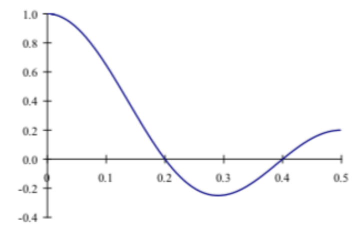
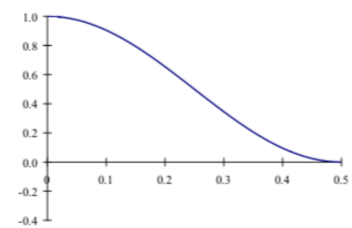
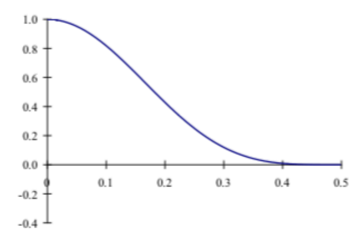
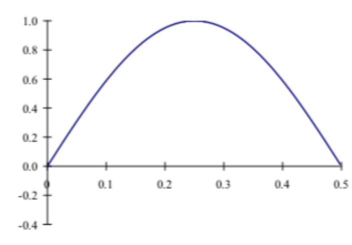
Amplitude: $A = \pm\sqrt{R(\omega)^2 + I(\omega)^2}$

Phase: $\phi = \tan^{-1}\frac{I(\omega)}{R(\omega)}$

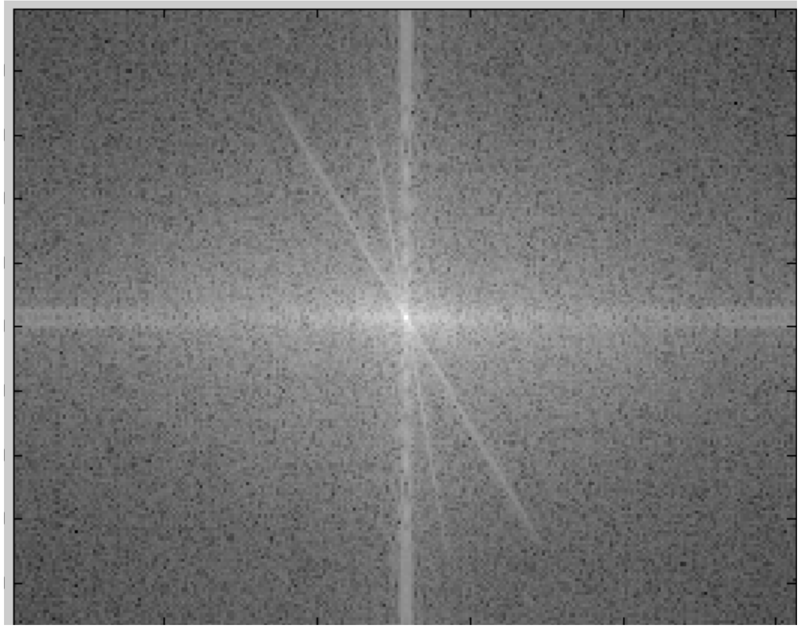
Fourier Transform Pairs

Name	Signal			Transform	
impulse		$\delta(x)$	$\Leftrightarrow$	1	
shifted impulse		$\delta(x-u)$	$\Leftrightarrow$	$e^{-j\omega u}$	
box filter		$\mathrm{box}(x/a)$	$\Leftrightarrow$	$a\mathrm{sinc}(a\omega)$	
tent		$\mathrm{tent}(x/a)$	$\Leftrightarrow$	$a\mathrm{sinc}^2(a\omega)$	
Gaussian		$G(x;\sigma)$	$\Leftrightarrow$	$\frac{\sqrt{2\pi}}{\sigma}G(\omega;\sigma^{-1})$	
Laplacian of Gaussian		$(\frac{x^2}{\sigma^4}-\frac{1}{\sigma^2})G(x;\sigma)$	$\Leftrightarrow$	$-\frac{\sqrt{2\pi}}{\sigma}\omega^2 G(\omega;\sigma^{-1})$	
Gabor		$\cos(\omega_0 x)G(x;\sigma)$	$\Leftrightarrow$	$\frac{\sqrt{2\pi}}{\sigma}G(\omega \pm \omega_0;\sigma^{-1})$	
unsharp mask		$(1+\gamma)\delta(x)$ $-\gamma G(x;\sigma)$	$\Leftrightarrow$	$(1+\gamma)-$ $\frac{\sqrt{2\pi}\gamma}{\sigma}G(\omega;\sigma^{-1})$	
windowed sinc		$\mathrm{rcos}(x/(aW))$ $\mathrm{sinc}(x/a)$	$\Leftrightarrow$	(see Figure 3.29)	

Fourier Transforms of Filters

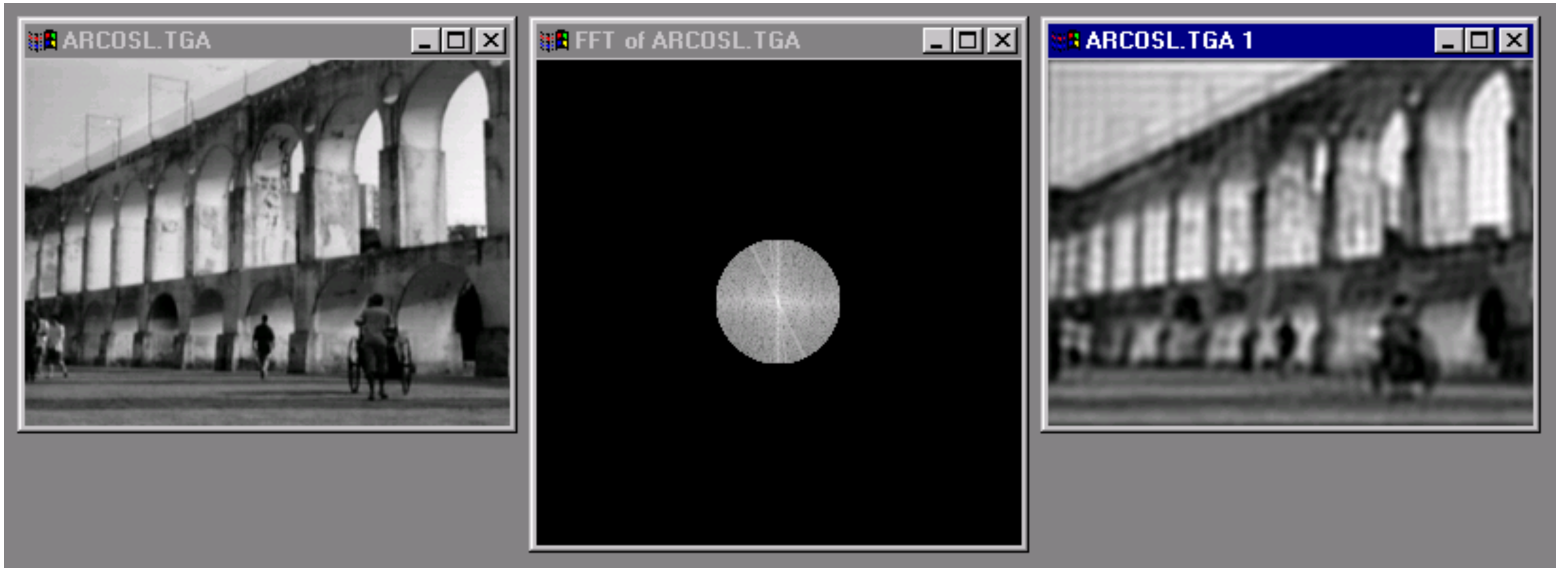
box-3	$\frac{1}{3}\begin{bmatrix}1 & 1 & 1\end{bmatrix}$	$\frac{1}{3}(1 + 2\cos\omega)$	
box-5	$\frac{1}{5}\begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix}$	$\frac{1}{5}(1 + 2\cos\omega + 2\cos 2\omega)$	
linear	$\frac{1}{4}\begin{bmatrix}1 & 2 & 1\end{bmatrix}$	$\frac{1}{2}(1 + \cos\omega)$	
binomial	$\frac{1}{16}\begin{bmatrix}1 & 4 & 6 & 4 & 1\end{bmatrix}$	$\frac{1}{4}(1 + \cos\omega)^2$	
Sobel	$\frac{1}{2}\begin{bmatrix}-1 & 0 & 1\end{bmatrix}$	$\sin\omega$	

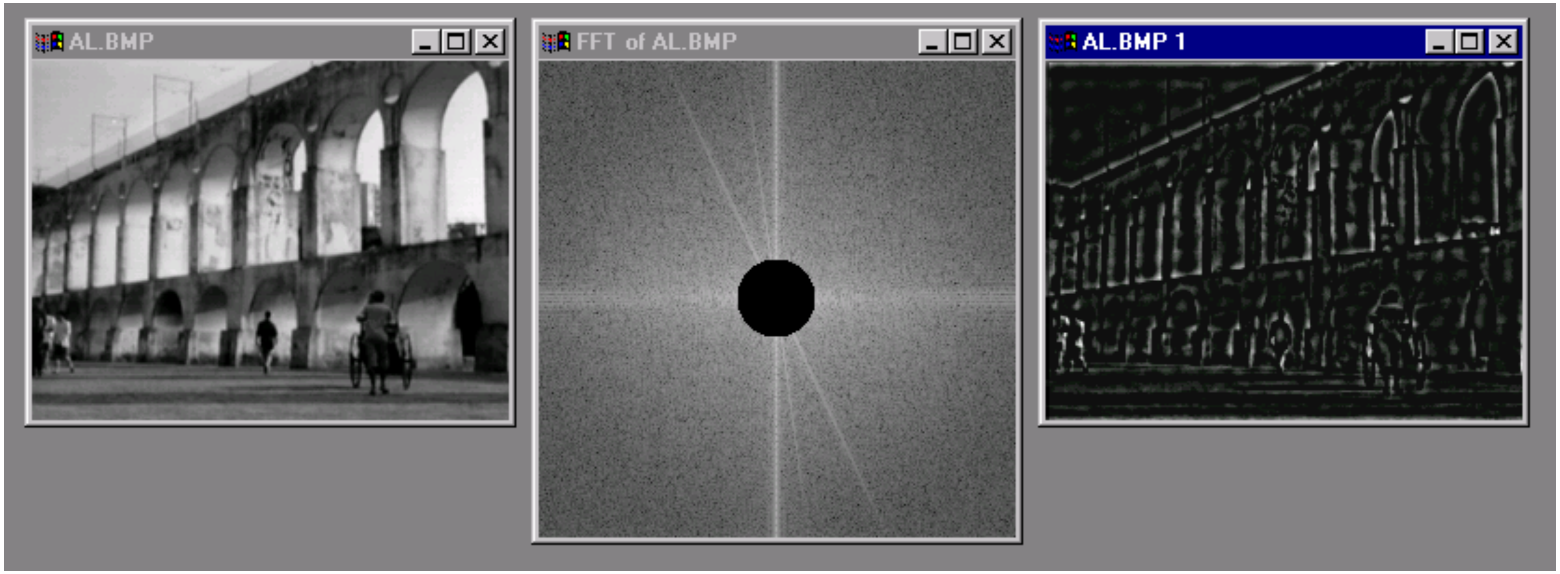
Man-made Scene



Can change spectrum, then reconstruct

Low and High Pass filtering





The Convolution Theorem

- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

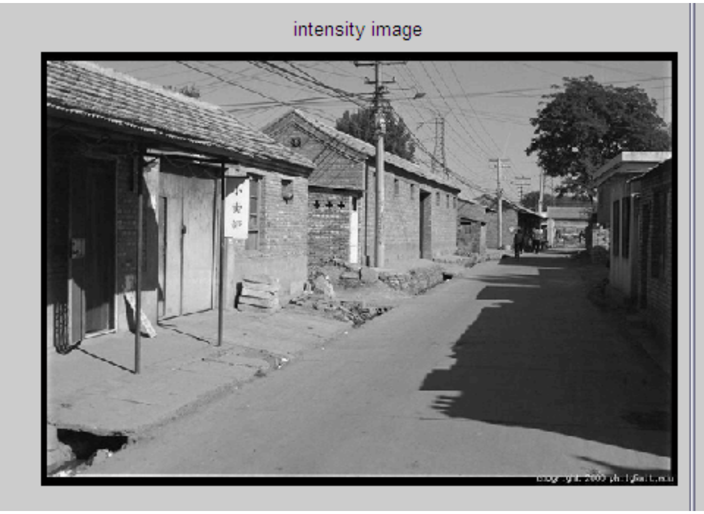
$$\mathrm{F}[g * h] = \mathrm{F}[g]\,\mathrm{F}[h]$$

- **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

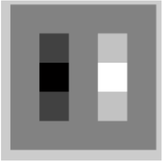
$$g * h = \mathrm{F}^{-1}[\mathrm{F}[g]\,\mathrm{F}[h]]$$

Filtering in spatial domain

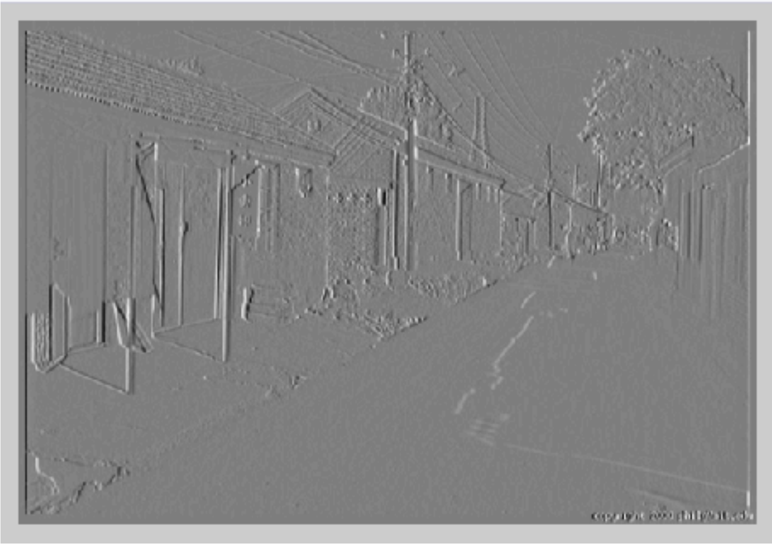
1	0	-1
2	0	-2
1	0	-1



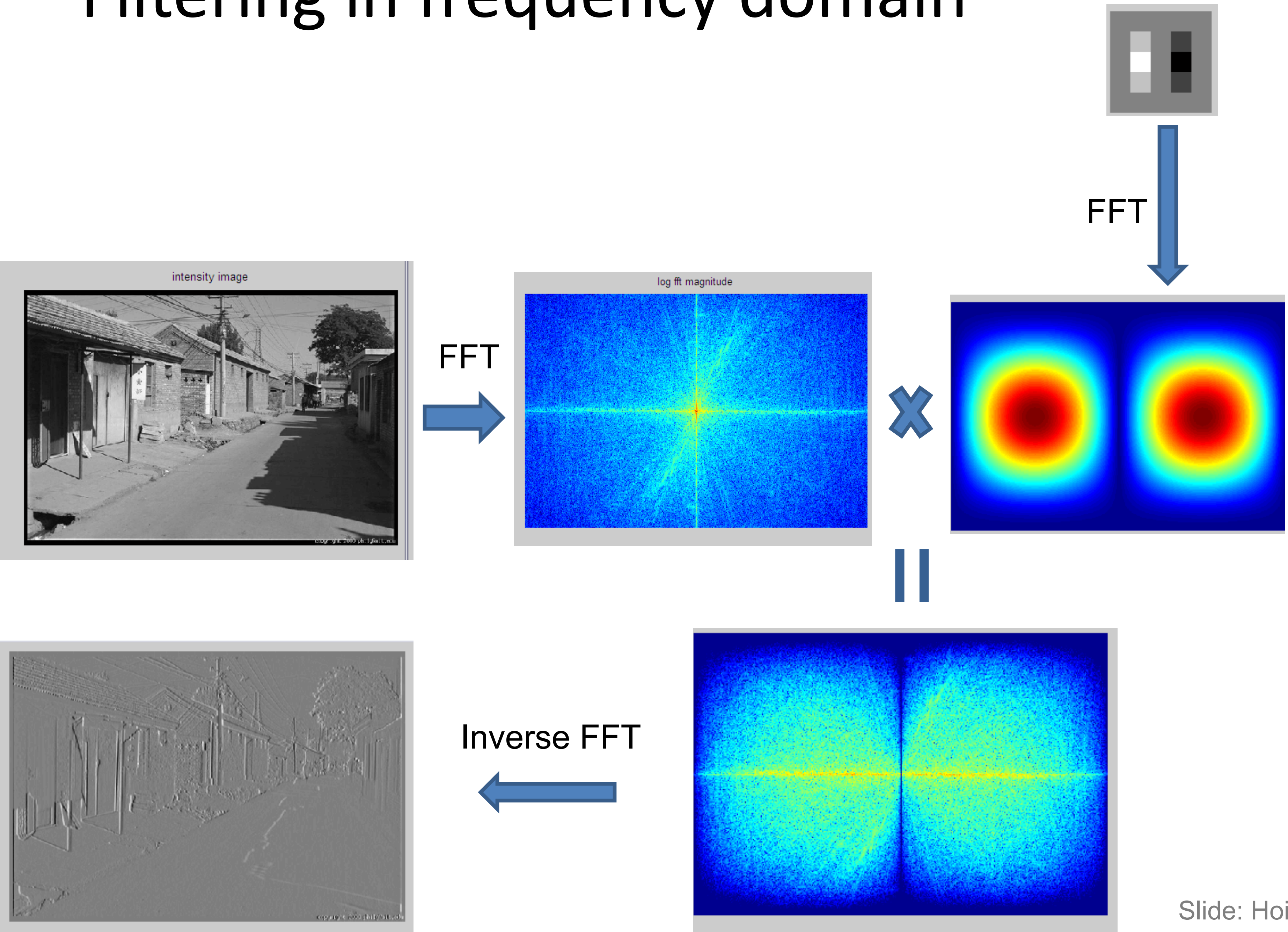
*



=



Filtering in frequency domain



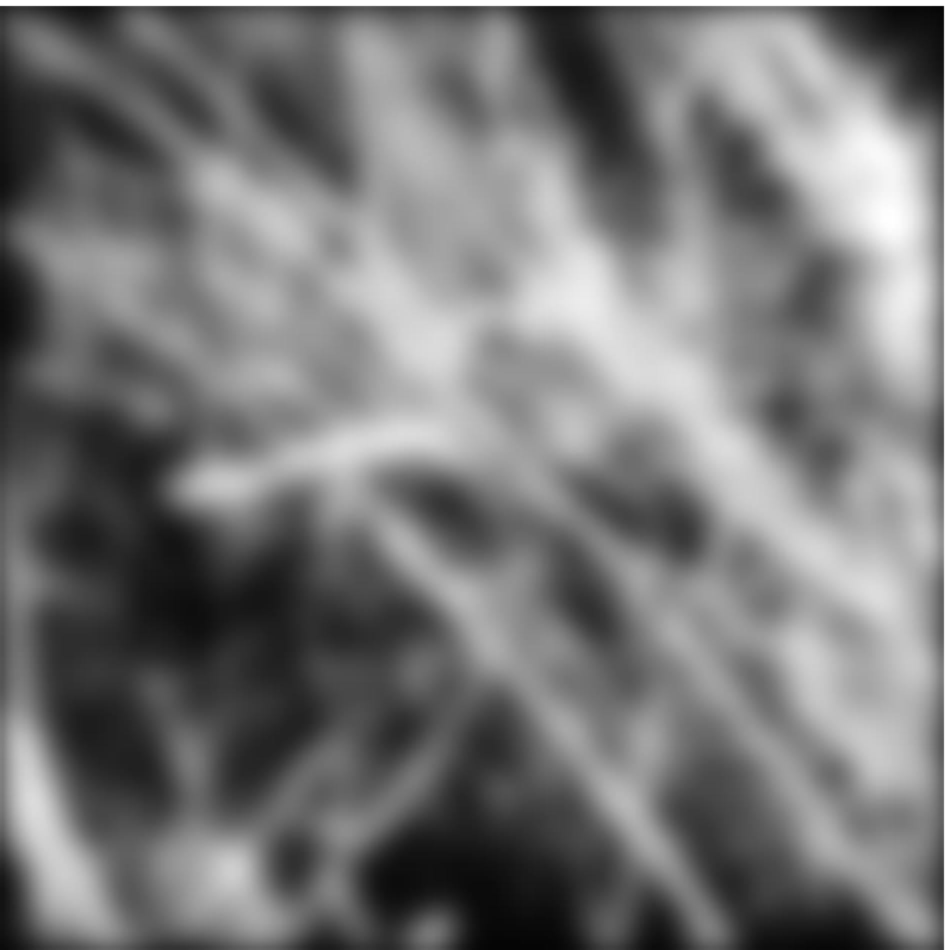
Slide: Hoiem

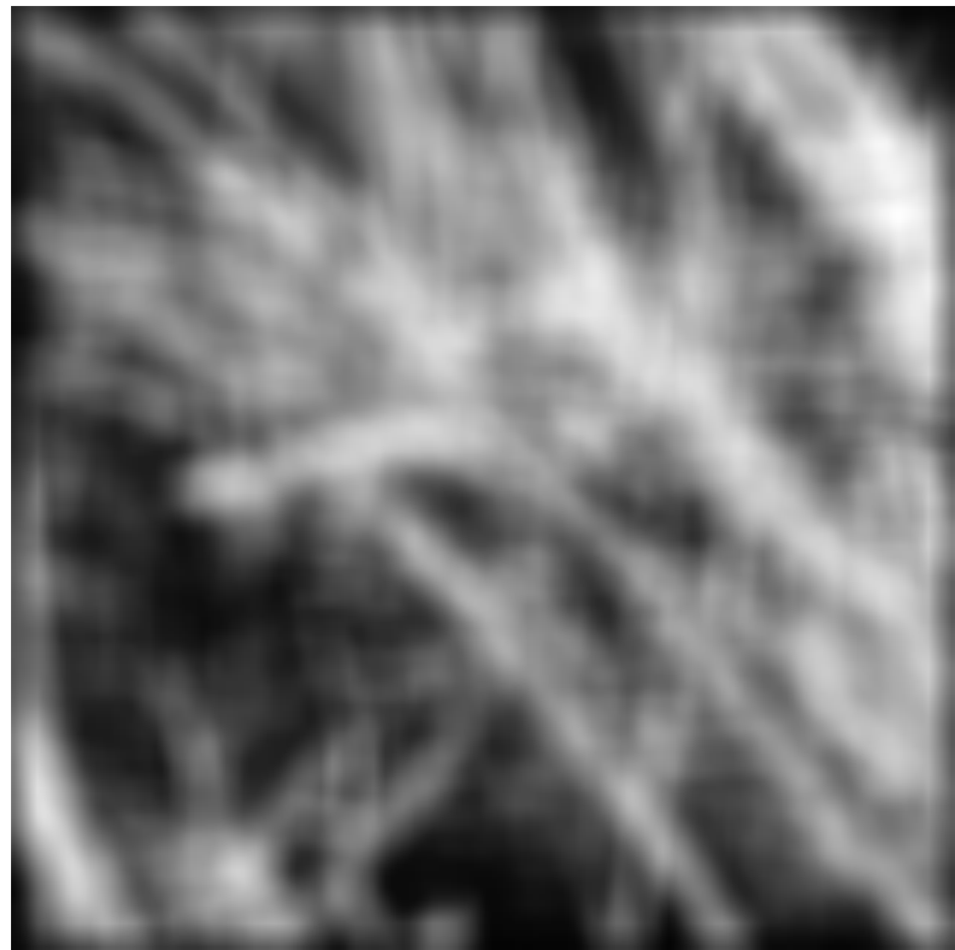
Filtering

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

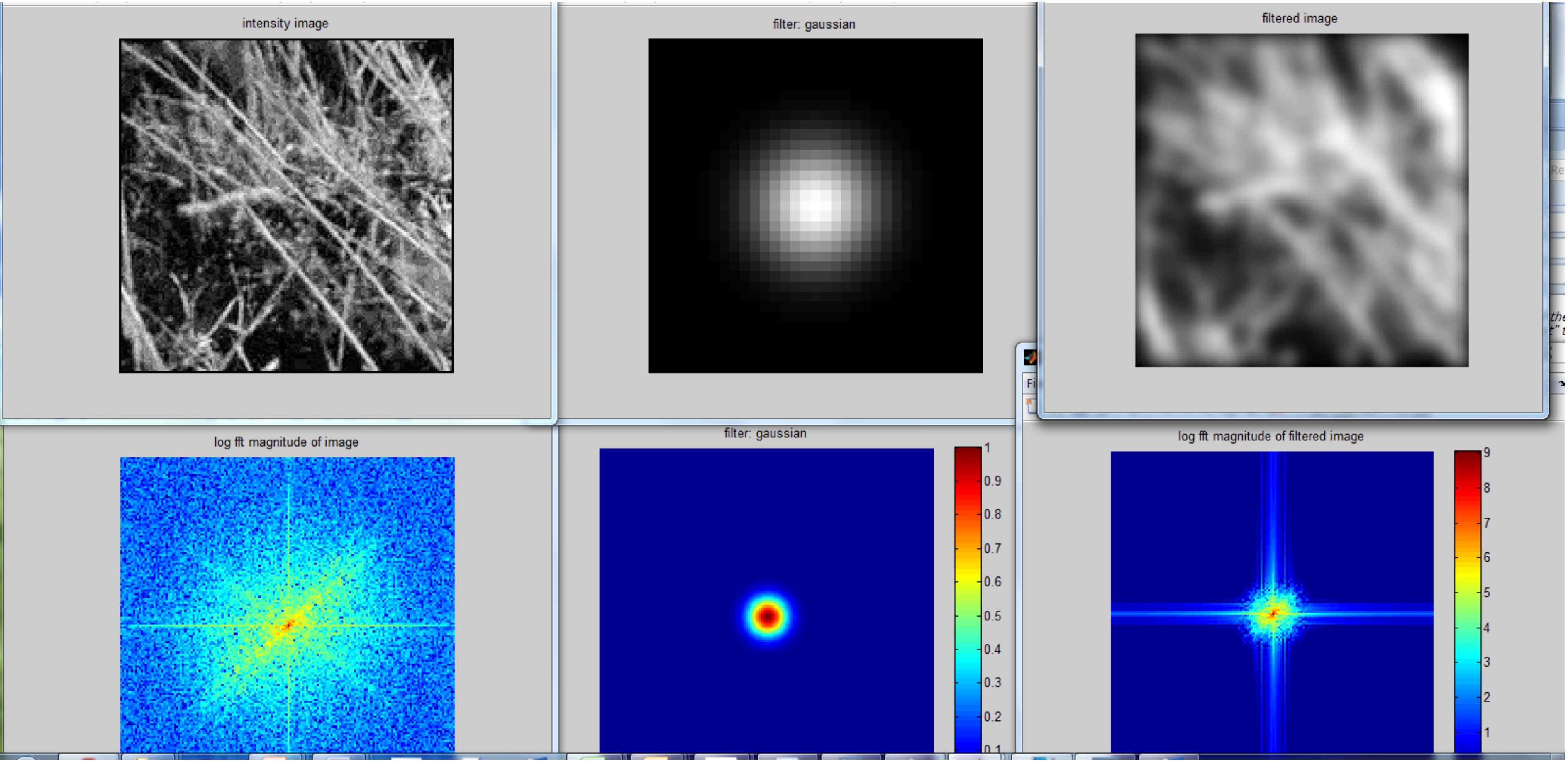
Gaussian 

Box filter 



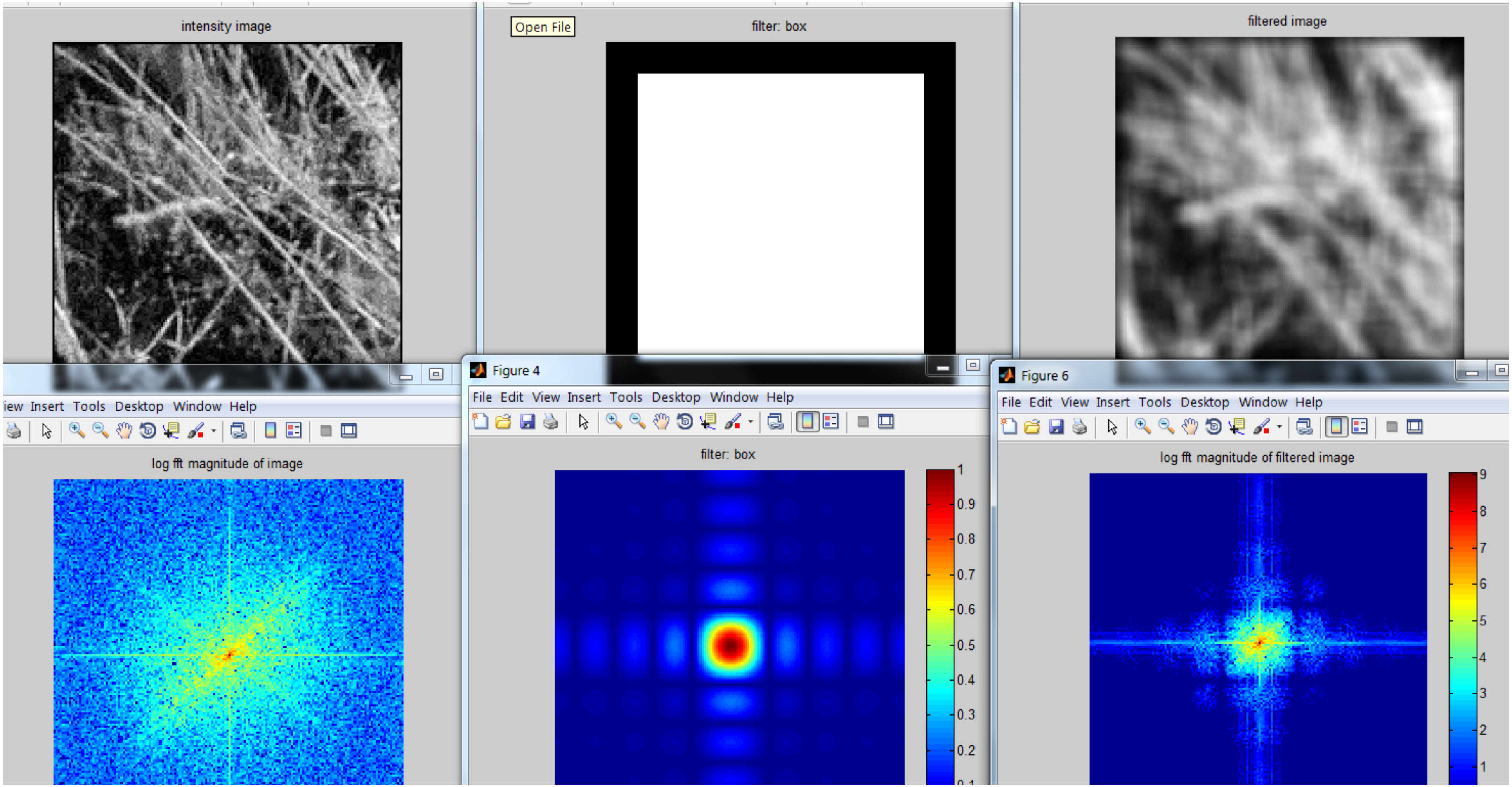


Gaussian



intensity image
filter: gaussian
filtered image
log fft magnitude of image
filter: gaussian
log fft magnitude of filtered image

Box Filter

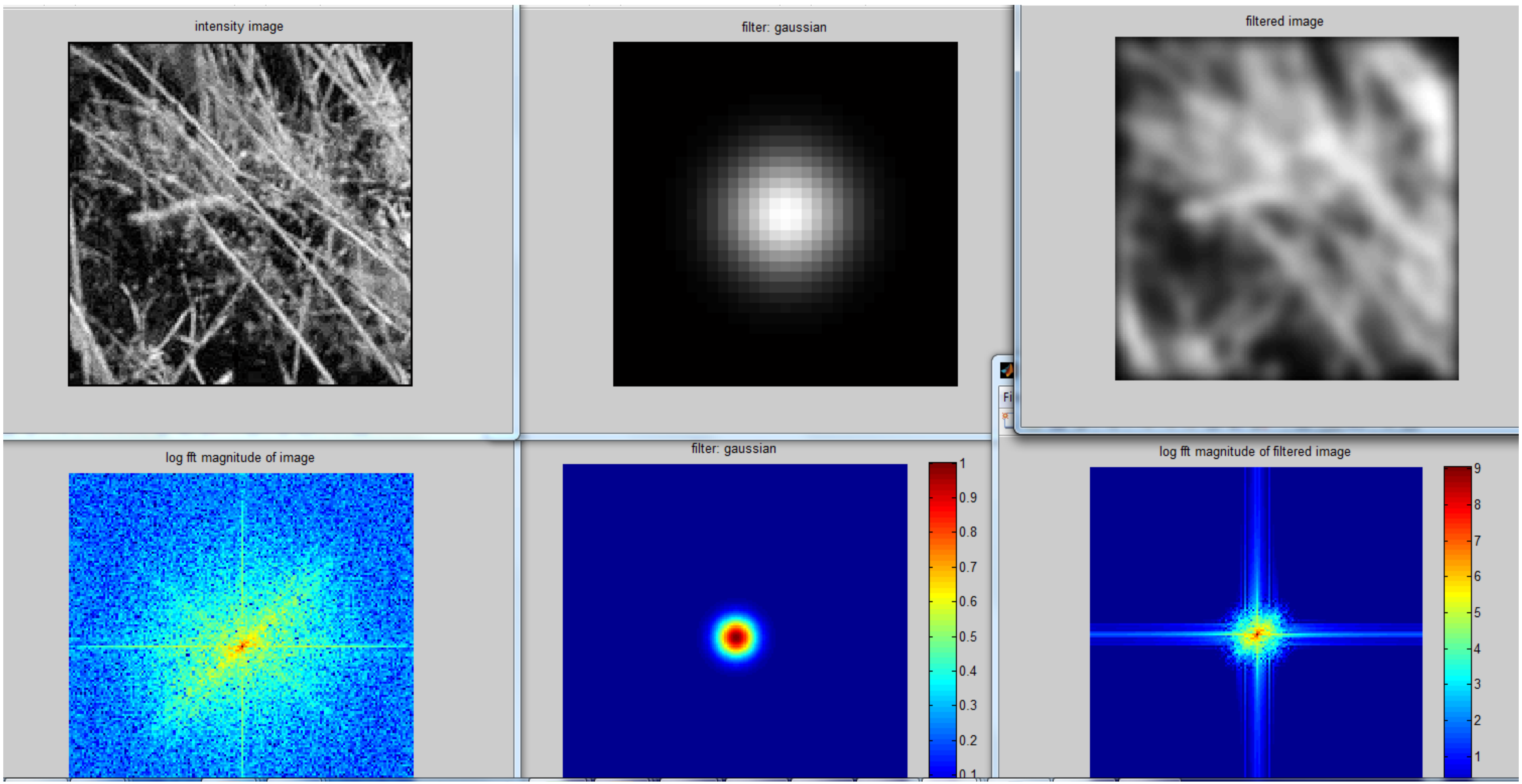

intensity image
Open File
filter: box
filtered image
Figure 4
Figure 6
log fft magnitude of image
filter: box
log fft magnitude of filtered image

Is convolution invertible?

- If convolution is just multiplication in the Fourier domain, isn’t deconvolution just division?
- Sometimes, it clearly is invertible (e.g. a convolution with an identity filter)
- In one case, it clearly isn’t invertible (e.g. convolution with an all zero filter)
- What about for common filters like a Gaussian?

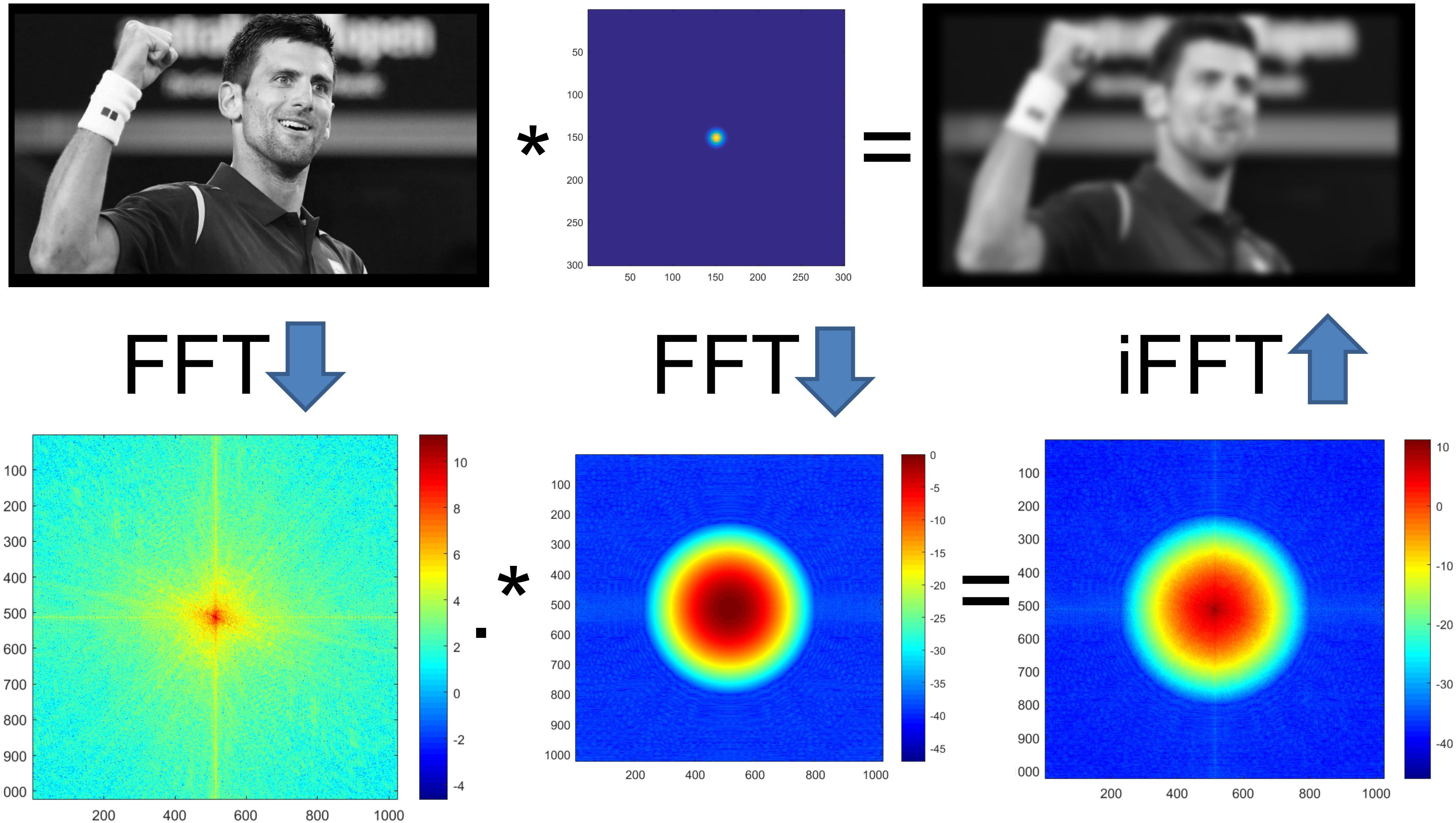
But you can't invert multiplication by 0

- But it's not quite zero, is it...

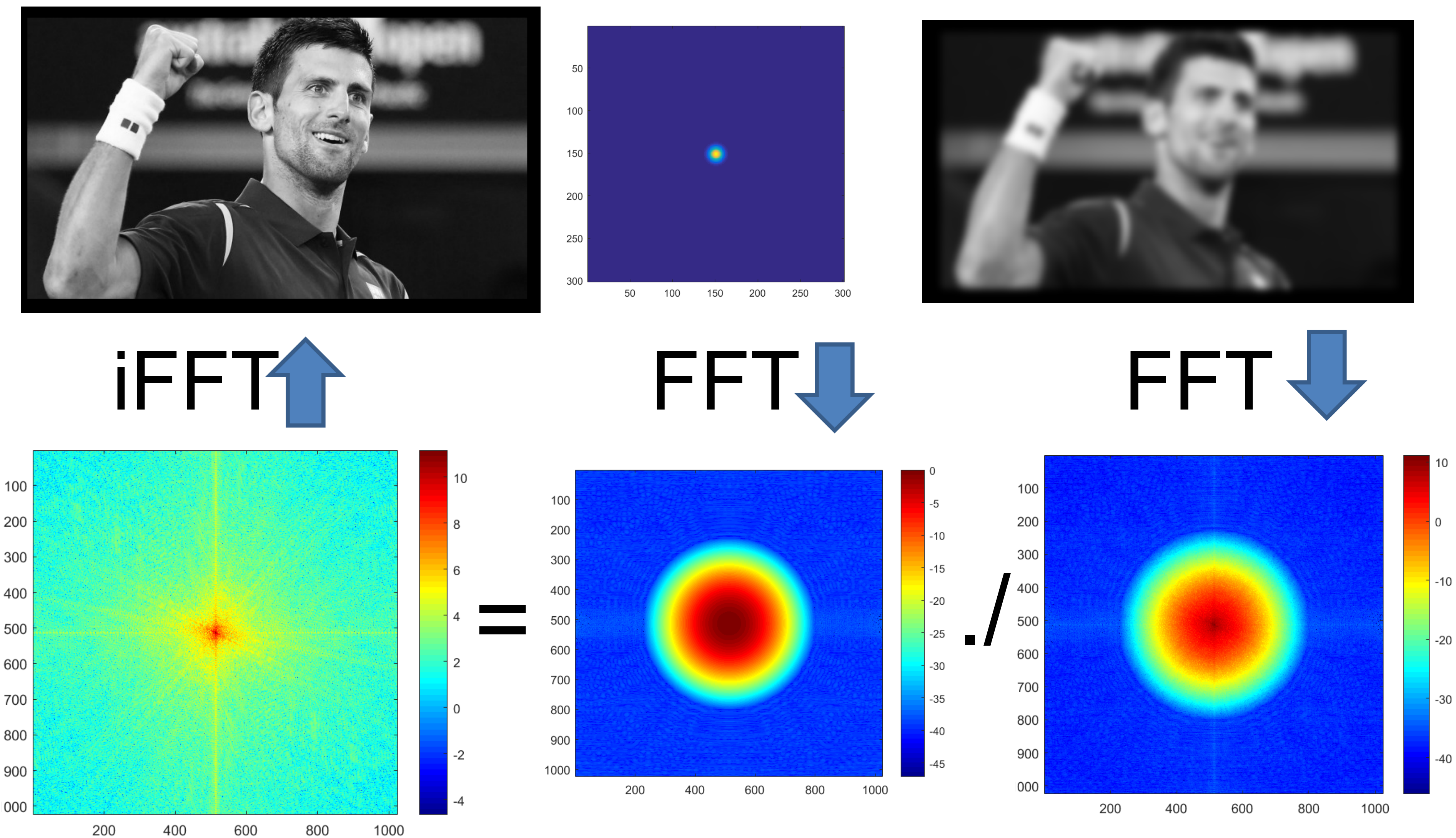


Let’s experiment on Novak

Convolution

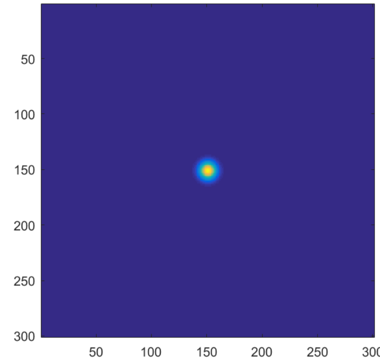


Deconvolution?



But under more realistic conditions

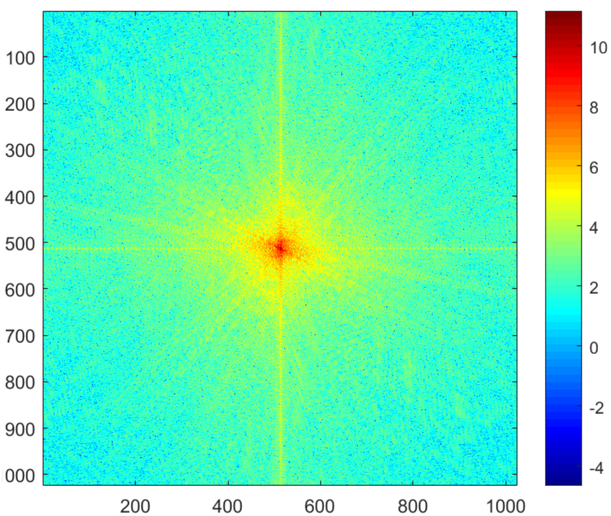
Random noise, .000001 magnitude



iFFT

FFT

FFT



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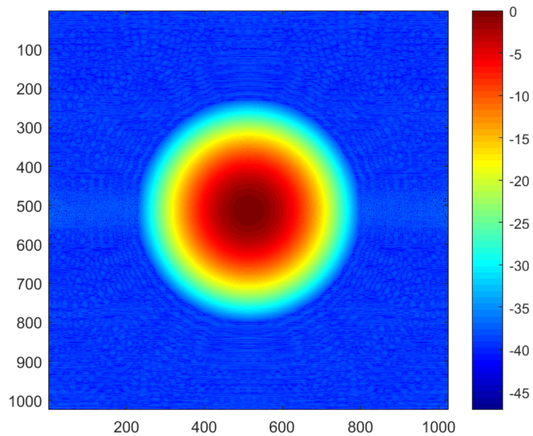

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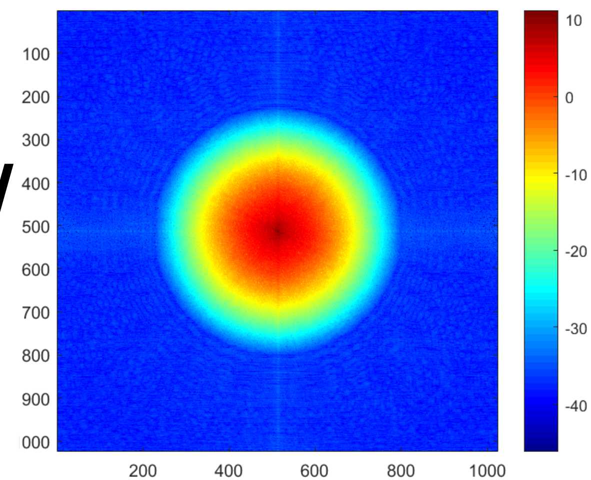
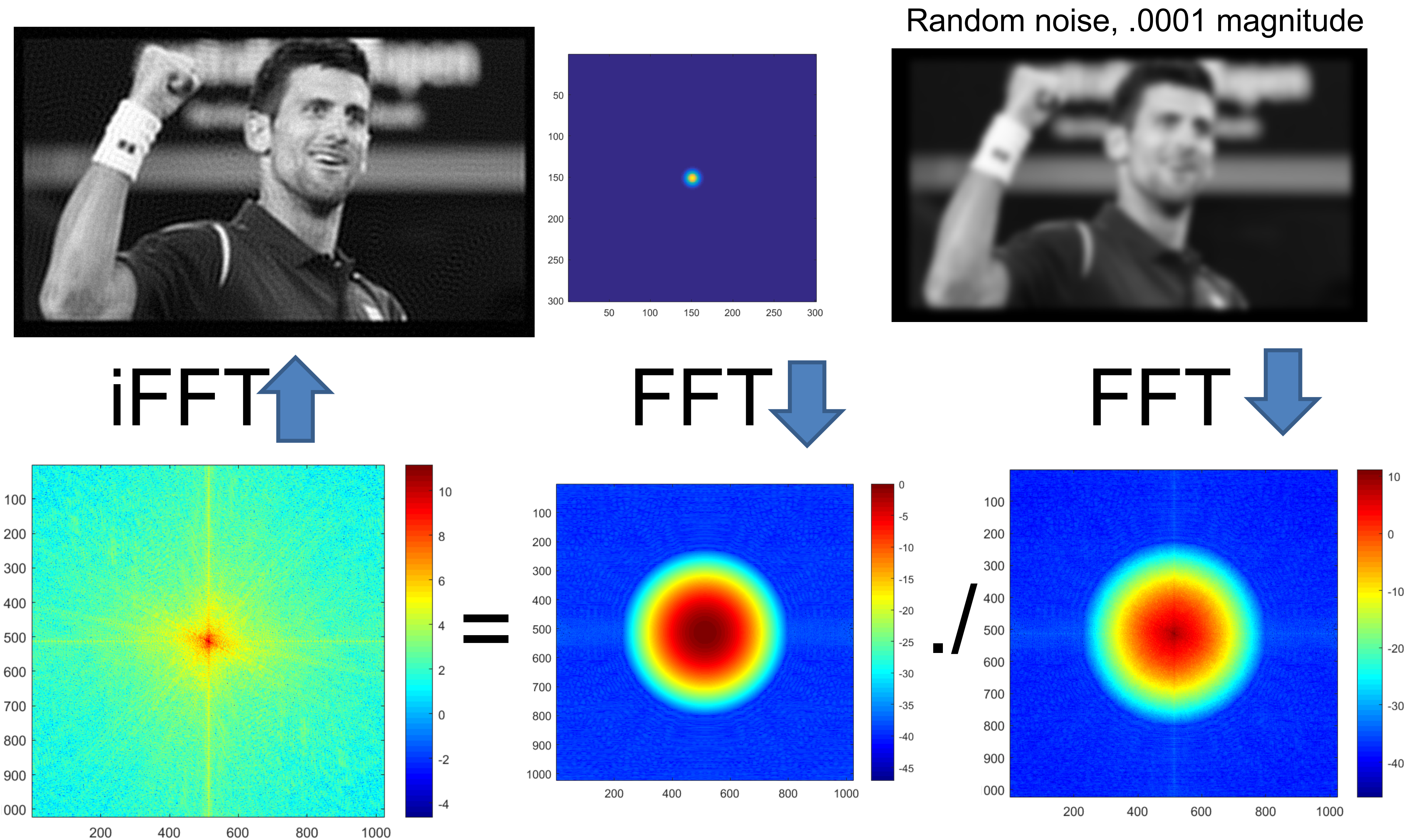
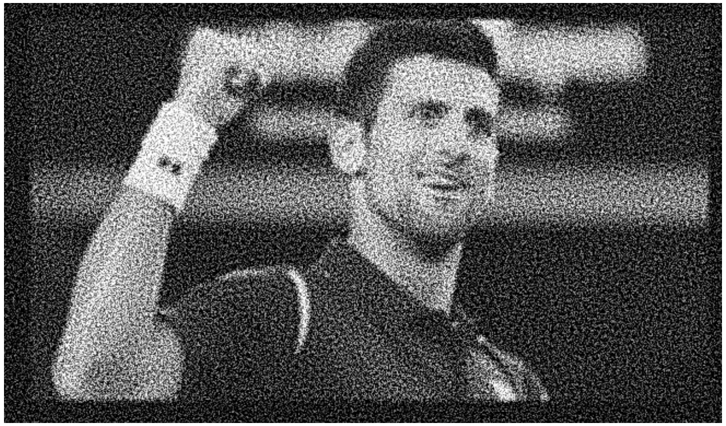
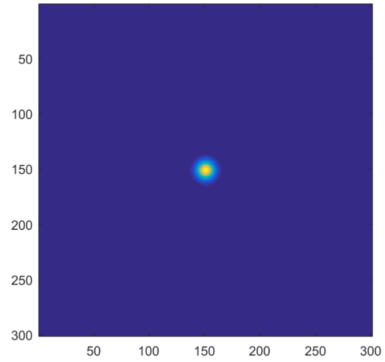


But under more realistic conditions



But under more realistic conditions



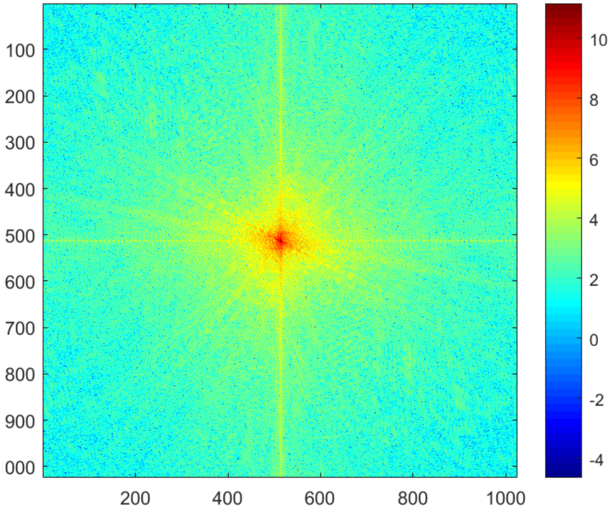


Random noise, .001 magnitude

iFFT

FFT

FFT



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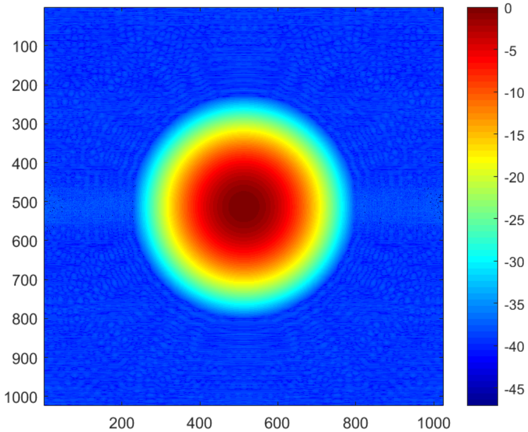

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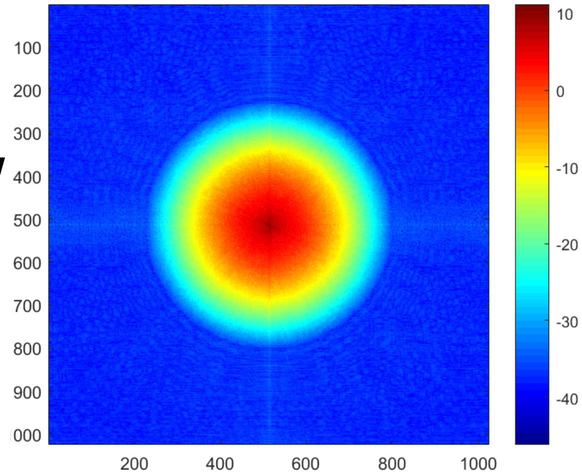
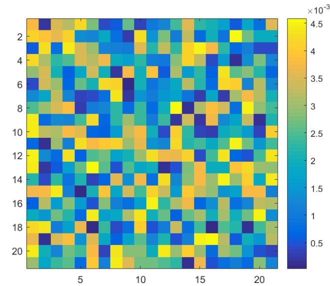


With a random filter...

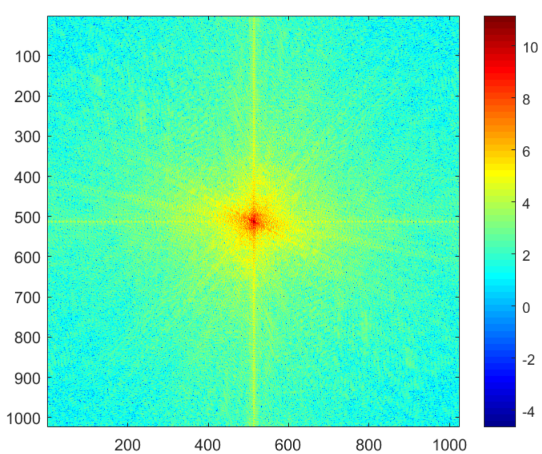


Random noise, .001 magnitude

iFFT

FFT

FFT



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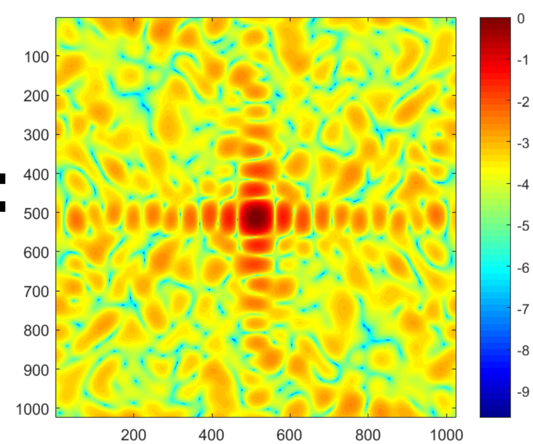

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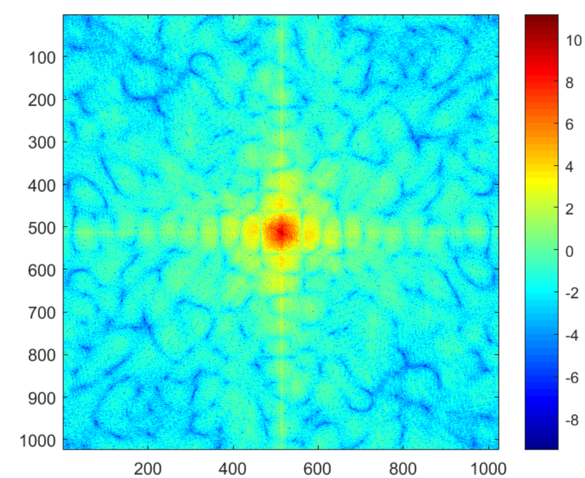

Deconvolution is hard

- Active research area.
- Even if you know the filter (non-blind deconvolution), it is still very hard and requires strong *regularization.*
- If you don’t know the filter (blind deconvolution) it is harder still.